I-74 RELIABILITY ANALYSIS OF A TAINTER GATE

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1. INTRODUCTION A theoretical analysis of reliability of a tainter gate structure is herein presented by applying a Monte Carlo Simulation technique. We assume that the criterion of structural reliability is Euler's coplanar buckling of the structure. The external load on the structure is assumed to be of two parts. These are the deterministic hydrostatic pressure and the random hydrodynamic pressure which follows the surface disturbance of a large storm. Except for the random nature of the hydrodynamic pressure, all the parameters in this analysis are considered as deterministic quantities, since only the distribution with greatest dispersion needs to be considered. The procedure consists of first constructing a probabilistic model of the random hydrodynamic load. Then the reliability of the structure is determined with the criterion of failure as the coplanar instability of at least one of the foot columns of the structure. The reliability obtained serves to logically predict the structural safety, while the concept of the conventional safety factor gives no information of the structural safety in the future. Conversely, given the reliability level associated with the importance of the structure and the other technical and economical reasonings, the size of the structural parameters can be designed. We should note that the method herein presented is applicable to any structural reliability analysis.

2. RELIABILITY ANALYSIS To demonstrate the theoretical method of analyzing the reliability of a tainter gate, we employed a simplified model (Fig. 1). The skin plate is a section of a cylindrical shell with a horizontal width d. Supporting the plate are three columns of length ℓ and are individually attached by pin joints to the plate at one end. The other ends of all three columns are rigidly fastened together and concurrently pin connected to a support shoe. The hydrostatic and hydrodynamic loads on the plate are transferred to the foot columns as axial loads. The external loads to the columns are derived as given in Table 1, where the hydrostatic pressure increases linearly with the depth of the water. The axial load due to the hydrodynamic pressure is assumed to be of the form $X_i^d = \alpha_i^d P$ and α_i^d is a random variable with a joint Gaussian density function. In Table 1, the parameters are given by the following equations.

$$\alpha_1^s = \frac{1}{8}\left(\frac{h_2}{h_1} - 1\right)\left(3 + \frac{h_2}{h_1}\right)$$

$$\alpha_2^s = \frac{1}{8}\left(1 + 2\,\frac{h_2}{h_1} + \frac{h_3}{h_1}\right)\left(\frac{h_3}{h_1} - 1\right)$$

$$\alpha_3^s = \frac{1}{8}\left(3\,\frac{h_3}{h_1} + \frac{h_2}{h_1}\right)\left(\frac{h_3}{h_1} - \frac{h_2}{h_1}\right)$$

and $P = \gamma d h_1^2$ where γ is the density of water. The joint Gaussian density function of α_i^d is

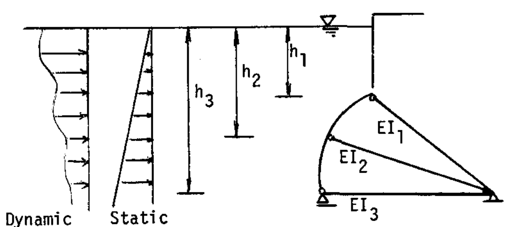

Fig. 1 Model Tainter Gate

Column	Hydrostatic load	Hydrodynamic load	Total load
1	${}^s_1P$	${}^d_1P$	$({}^s_1 + {}^d_1)P$
2	${}^s_2P$	${}^d_2P$	$({}^s_2 + {}^d_2)P$
3	${}^s_3P$	${}^d_3P$	$({}^s_3 + {}^d_3)P$

Table 1

$$f_{\alpha_1^d \alpha_2^d \alpha_3^d}(\alpha_1^d, \alpha_2^d, \alpha_3^d) = \frac{1}{(2\pi)^{3/2}\,|S|^{1/2}} \exp\left[\frac{-1}{2|S|} \sum_{j=1}^{3} \sum_{k=1}^{3} |S|_{jk} (\alpha_j^d - \mu_j)(\alpha_k^d - \mu_k)\right]$$

where S is a covariance matrix given by

$$S = \begin{bmatrix} \sigma_1^2 & K_{12} & K_{13} \\ K_{21} & \sigma_2^2 & K_{23} \\ K_{31} & K_{32} & \sigma_3^2 \end{bmatrix}$$

$K_{ij}(i,j = 1,2,3)$ is a covariance of α_i^d and α_j^d and σ_i^2 is the variance of α_i^d. $|S|_{jk}$ is the cofactor of the element in the j^{th} row and k^{th} column of matrix S. The variance σ_i^2 is assumed to have the following linear variation with depth:

$$\sigma_3^2 = B\sigma_1^2 \; (0 \leq B \leq 1) \qquad \text{and} \qquad \sigma_2^2 = \sigma_1^2 + \frac{h_2 - h_1}{h_3 - h_1}(\sigma_3^2 - \sigma_1^2)$$

The correlation coefficient, ρ_{ij} between α_i^d and α_j^d is assumed to decrease with the distance between level i and j. Thus, we have $\rho_{ij} = 1 - D|h_i - h_j|$ where $0 \leq D \leq \frac{1}{h_3 - h_1}$.

Therefore, the covariance, K_{ij} is given by

$$K_{ij} = \sigma_i \sigma_j \rho_{ij} = \sigma_i \sigma_j [1 - D|h_i - h_j|]$$

In the absence of actual measured data for depth profile of the turbulence force, it is felt that the assumed pattern corresponds to the maximum type of turbulence that follows the surface disturbance of a large storm. The fitness of our model is the load environment should be examined when the data are available. Two basic reasons exist for assuming a Gaussian distribution of the random variables. First, since α_i^d is dependent on several factors, the central limit theorem suggests a Gaussian type. Secondly, we have no data to support any particular *distribution function*.

The stability analysis for elastic buckling of this model tainter gate is given in Reference [1] and the critical value of $P(= \gamma d h_1^2)$ is approximated with high accuracy by

$$P_{cr} = \frac{\pi^2 EI_1}{\ell^2} \left\{ \frac{(\alpha_1^s + \alpha_2^s + \alpha_3^s) + (\alpha_1^d + \alpha_2^d + \alpha_3^d)}{(\alpha_1^s + \alpha_1^d)^{1.5} + \sqrt{\frac{I_1}{I_2}}\,(\alpha_1^s + \alpha_2^d)^{1.5} + \sqrt{\frac{I_1}{I_2}}\,(\alpha_3^s + \alpha_3^d)^{1.5}} \right\}^2$$

We note that this P_{cr} is a random variable as a function of α_i^d (i = 1,2,3). The probability of failure of the tainter gate under a load application is given by the following integrations;

$$P_f = P(P > P_{cr}) = \iiint_D f_{\alpha_1^d \alpha_2^d \alpha_3^d}(\alpha_1^d, \alpha_2^d, \alpha_3^d) d\alpha_1^d \, d\alpha_2^d \, d\alpha_3^d$$

where P( ) reads "probability that" and D is the domain defined by $P > P_{cr}$. The evaluation of this equation is very difficult since the given domain is specified by a three dimensional space $(\alpha_1^d, \alpha_2^d, \alpha_3^d)$. A method of Monte Carlo simulation is therefore developed to solve this equation in the next section. If we are interested in the probability of failure of the tainter gate under several load applications, further calculation and assumptions are required. If the same distribution of the magnitude of surface disturbances repeats N times and if the probability of failure due to a single load, P_f is small, we have

$$P_f(N) \simeq NP_f$$

where $P_f(N)$ is the total probability of failure in N load applications. If the occurance of these disturbances follows the Poisson probability law, then the probability of finding N events in time t with the mean rate of occurence μ is given by [2]

$$P(\tfrac{N}{t}) = \frac{e^{-\mu t}(\mu t)^N}{N!}$$

By applying this Poisson law, we obtain for the mean of the reliability for a given time t as

$$E[P_s(\tfrac{N}{t})] = \sum_{N=0}^{\infty} P_s(N) \, P(\tfrac{N}{t})$$

The variance is

$$Var[P_s(\tfrac{N}{t})] = \sum_{N=0}^{\infty} P_s^2 (N) \, P(\tfrac{N}{t}) - E[P_s(\tfrac{N}{t})]^2$$

3. MONTE CARLO SIMULATION In this section, a Monte Carlo simulation is developed for the evaluation of P_f. Five steps are required as follows.

(Step 1) A set of random variables is *generated from a uniform* distribution functions such that $0 \leq d_i, d_{i+1}, d_{i+2} \leq 1$.

(Step 2) A marginal density function of α_1^d, $f_{\alpha_1^d}(\alpha_1^d)$ is calculated from the joint Gaussian density function. *Then we equate* $d_i = F_{\alpha_1^d}(\alpha_1^d)$ *where* $F_{\alpha_1^d}(\ )$ is the corresponding distribution function. Solving for α_1^d, we have $\alpha_1^d = F^{-1}_{\alpha_1^d}(d_i)$.

(Step 3) A conditional distribution function of α_2^d for the given α_1^d is determined and we equate $d_{i+1} = F_{\alpha_2^d/\alpha_1^d}(d_{i+1})$. Solving for α_2^d, we have $\alpha_2^d = F^{-1}_{\alpha_2^d/\alpha_1^d}(d_{i+1})$.

(Step 4) Given α_1^d and α_2^d, similarly we obtain $\alpha_3^d = F^{-1}_{\alpha_3^d/\alpha_1^d,\alpha_2^d}(d_{i+2})$.

(Step 5) The data set $\{\alpha_1^d, \alpha_2^d, \alpha_3^d\}$ from the previous steps is substituted into the expression for P_f. A sample value of P_{cr} is obtained. By repeating this procedure and keeping a record of the number of trials and the number of times P is greater than P_{cr} we obtained an estimation of the probability of failure, P_f as

$$P_f = P(P > P_{cr}) = \frac{\text{number of data sets where } P > P_{cr}}{\text{total number of data sets}}$$

4. NUMERICAL EXAMPLE In order to demonstrate the Monte Carlo solution, a simulation is performed on a specific model with specific parameters by use of a digital computer. The given values of parameters are: mean of $\alpha_i^d = 0$, standard deviation of $\alpha_i^d = 0.01$(ft.), $I_1 = 6 \times 10^{-2}(\text{ft.})^4$, $I_2 = 13 \times 10^{-2}(\text{ft.})^4$, $I_3 = 7 \times 10^{-2}(\text{ft.})^4$, $h_1 = 20$(ft.), $h_2 = 25$(ft.), $h_3 = 30$(ft.), $E = 2.5 \times 10^6$(psf), $\ell = 20$(ft.), $\gamma = 62.4(\#/\text{ft}^3)$, d = 1(ft.), B = 0.1, 0.5, 0.7 and D = 0.01, 0.05, 0.09. Since we have no accurate values or estimates for B and D, we choose a range of values which allow us to find out how sensitive the overall reliability is to changes in these parameters. The results of this demonstration need not yield reliability levels high enough for an actual tainter gate since this is only a demonstration of a method. The results of the simulation are tabulated in Table 2. As would be expected, an increase in D increases the reliability while an increase in B decreases the reliability. Because of the small numbers of trials (500), no high degree of accuracy is expected in these values.

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D/B	0.1	0.5	0.7
0.01	0.8300	0.7680	0.7580
0.05	0.8520	0.8100	0.8080
0.09	0.8780	0.8320	0.8180

Table 2 Results of Monte Carlo Simulation

REFERENCES

[1] SHIRAISHI, N., "Fundamental Investigations on Structural Instability of Tainter Gate," Proc. of JSCE, Vol. 169, 1969.

[2] PARZEN, E., "Stochastic Processes," pp. 13, Holden-Day.