I－36　格子桁理論による鋼床板の影響面の解析

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断面二次モーメント J で間隔 a なる横リブと，断面二次モーメント J_Q で間隔 b なる縦リブをもつ鋼床板の第Ⅱ系の計算に 格子桁理論を適用するとき，横リブおよび縦リブの捩り剛性を無視すれば，鋼床板の任意点の影響面は次式より計算できる。

横リブの影響面：

$$\left.\begin{aligned}
&k \neq i \quad : \quad \frac{S_{ix,ku}}{a} = \frac{1}{a}\sum_{n=1}^{\infty}\mu_{(n)} S_{ix,i(n)}\gamma_{u(n)} B_{ik(n)} , \\
&k = i \quad : \quad \frac{S_{ix,iu}}{a} = \frac{1}{a}\Big\{ S^{\circ}_{ix,iu} + \sum_{n=1}^{\infty}\mu_{(n)} S_{ix,i(n)}\gamma_{u(n)}(B_{ii(n)}-1)\Big\} , \\
&\quad S = \delta, \varphi, M, Q = \text{撓み，撓角，曲げモーメント，せん断力} , \\
&\quad S^{\circ} = \delta^{\circ}, \varphi^{\circ}, M^{\circ}, Q^{\circ} = \text{基本系の撓み，撓角，曲げモーメント，せん断力} .
\end{aligned}\right\} \cdots (1\ \text{および}\ 2)$$

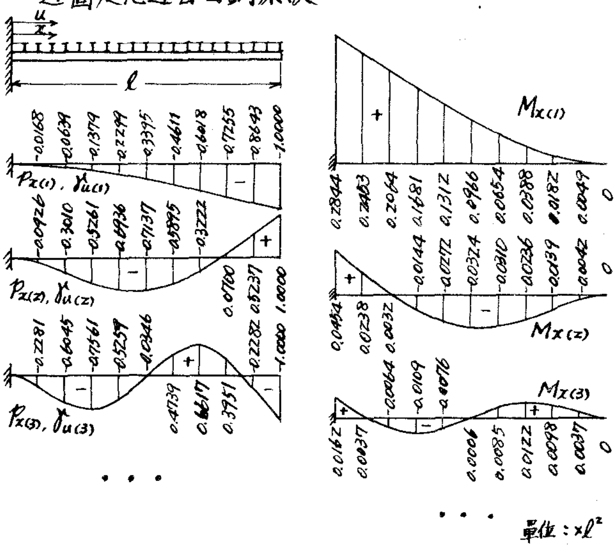
縦リブの影響面：

$$\left.\begin{aligned}
&\frac{S_{xy,ku}}{b} = a\sum_{n=1}^{\infty}\mu_{(n)} p_{x(n)}\gamma_{u(n)}\frac{S_{yk(n)}}{a} , \\
&S = \delta, \varphi, M, Q = \text{撓み，撓角，曲げモーメント，せん断力} .
\end{aligned}\right\} \cdots (3)$$

ここで，$B_{ik(n)}$ は曲げ格子剛度 $z_{(n)}$ に対する弾性沈下可能無限数支点上連続桁の反力，$S_{yk(n)}$ は曲げ格子剛度 $z_{(n)}$ に対する弾性沈下可能無限数支点上連続桁の断面力あるいは撓みである。

$B_{ik(n)}$ および $S_{yk(n)}$ は文献[2]より直接拾うことができるから，式(1),(2),(3) を用いるには，$\mu_{(n)}, S_{x(n)}, \gamma_{u(n)}, p_{x(n)}$ を求めておく必要があるので，筆者は，次のような鋼床板（橋の横断方向に，2径間連続，3径間連続，一辺固定他辺自由）に対してこれらの値を計算した結果を示した。

(1) 一辺固定他辺自由鋼床板



$$p_{x(n)} = \frac{1}{2}\Big[\Big(\cos\frac{\lambda_{(n)}}{\ell}x - \mathrm{Cosh}\frac{\lambda_{(n)}}{\ell}x\Big) - \frac{\cos\lambda_{(n)} + \mathrm{Cosh}\lambda_{(n)}}{\sin\lambda_{(n)} + \mathrm{Sinh}\lambda_{(n)}} \times\Big(\sin\frac{\lambda_{(n)}}{\ell}x - \mathrm{Sinh}\frac{\lambda_{(n)}}{\ell}x\Big)\Big] .$$

$\lambda_{(1)} = 1.875$，$\lambda_{(2)} = 4.694$，$\lambda_{(3)} = 7.855$，

$$\mu_{(n)} = \frac{1}{\int_0^{\ell} p_{x(n)}^2\,dx} = \frac{4}{\ell} . \qquad z_{(n)} = \frac{6\ell^4 J_Q}{a^3 b J}\times\frac{1}{\lambda^4_{(n)}}$$

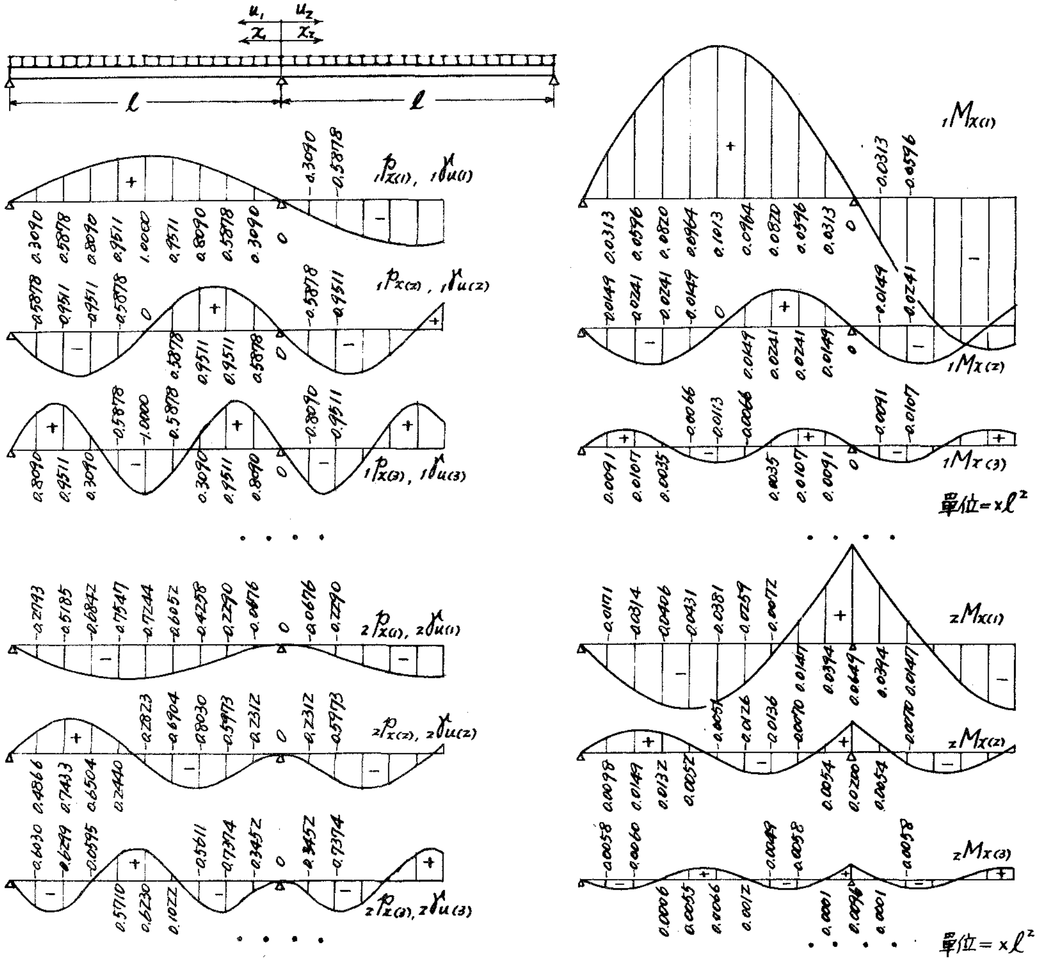
$$\gamma_{u(n)} = \frac{1}{2}\Big[\Big(\cos\frac{\lambda_{(n)}}{\ell}u - \mathrm{Cosh}\frac{\lambda_{(n)}}{\ell}u\Big) - \frac{\cos\lambda_{(n)} + \mathrm{Cosh}\lambda_{(n)}}{\sin\lambda_{(n)} + \mathrm{Sinh}\lambda_{(n)}} \times\Big(\sin\frac{\lambda_{(n)}}{\ell}u - \mathrm{Sinh}\frac{\lambda_{(n)}}{\ell}u\Big)\Big] .$$

$$M_{x(n)} = \frac{\ell^2}{2\lambda^2_{(n)}}\Big[\Big(\cos\frac{\lambda_{(n)}}{\ell}x + \mathrm{Cosh}\frac{\lambda_{(n)}}{\ell}x\Big) - \frac{\cos\lambda_{(n)} + \mathrm{Cosh}\lambda_{(n)}}{\sin\lambda_{(n)} + \mathrm{Sinh}\lambda_{(n)}} \times\Big(\sin\frac{\lambda_{(n)}}{\ell}x + \mathrm{Sinh}\frac{\lambda_{(n)}}{\ell}x\Big)\Big] .$$

$$Q_{x(n)} = \frac{\ell}{2\lambda_{(n)}}\Big[\Big(-\sin\frac{\lambda_{(n)}}{\ell}x + \mathrm{Sinh}\frac{\lambda_{(n)}}{\ell}x\Big) - \frac{\cos\lambda_{(n)} + \mathrm{Cosh}\lambda_{(n)}}{\sin\lambda_{(n)} + \mathrm{Sinh}\lambda_{(n)}} \times\Big(\cos\frac{\lambda_{(n)}}{\ell}x + \mathrm{Cosh}\frac{\lambda_{(n)}}{\ell}x\Big)\Big] .$$

$$\delta_{x(n)} = \frac{\ell^4}{2\lambda^4_{(n)}EJ}\Big[\Big(\cos\frac{\lambda_{(n)}}{\ell}x - \mathrm{Cosh}\frac{\lambda_{(n)}}{\ell}x\Big) - \frac{\cos\lambda_{(n)} + \mathrm{Cosh}\lambda_{(n)}}{\sin\lambda_{(n)} + \mathrm{Sinh}\lambda_{(n)}} \times\Big(\sin\frac{\lambda_{(n)}}{\ell}x - \mathrm{Sinh}\frac{\lambda_{(n)}}{\ell}x\Big)\Big] .$$

(2) 2径間連続鋼床板



(i) 才一の解

$${}_1p_{x_1(n)}=-{}_1p_{x_2(n)}={}_1p_{x(n)}=\sin\frac{\lambda_{(n)}}{\ell}x. \qquad \lambda_{(n)}=n\pi,\ (n=1,2,3,\cdots). \qquad \mu_{(n)}=\frac{1}{2\int_0^\ell {}_1p_{x(n)}^2\,dx}=\frac{1}{\ell}. \qquad Z_{(n)}=\frac{6\ell^4 J_Q}{a^3 b\, J}\times\frac{1}{\lambda_{(n)}^4}$$

$${}_1\gamma_{u_1(n)}=-{}_1\gamma_{u_2(n)}={}_1\gamma_{u(n)}=\sin\frac{\lambda_{(n)}}{\ell}u. \qquad {}_1M_{x_1(n)}=-{}_1M_{x_2(n)}={}_1M_{x(n)}=\frac{\ell^2}{\lambda_{(n)}^2}\sin\frac{\lambda_{(n)}}{\ell}x. \qquad {}_1Q_{x_1(n)}=-{}_1Q_{x_2(n)}={}_1Q_{x(n)}=\frac{\ell}{\lambda_{(n)}}\cos\frac{\lambda_{(n)}}{\ell}x.$$

$${}_1\delta_{x_1(n)}=-{}_1\delta_{x_2(n)}={}_1\delta_{x(n)}=\frac{\ell^4}{\lambda_{(n)}^4 EJ}\sin\frac{\lambda_{(n)}}{\ell}x.$$

(ii) 才二の解

$${}_2p_{x_1(n)}={}_2p_{x_2(n)}={}_2p_{x(n)}=-\frac{\sinh\lambda_{(n)}(1-\frac{x}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{x}{\ell})}{2\sin\lambda_{(n)}}. \qquad \lambda_{(1)}=3.9265\doteqdot(1+\tfrac{1}{4})\pi,\ \ \lambda_{(n)}=(n+\tfrac{1}{4})\pi,\ (n=2,3,4,\cdots).$$

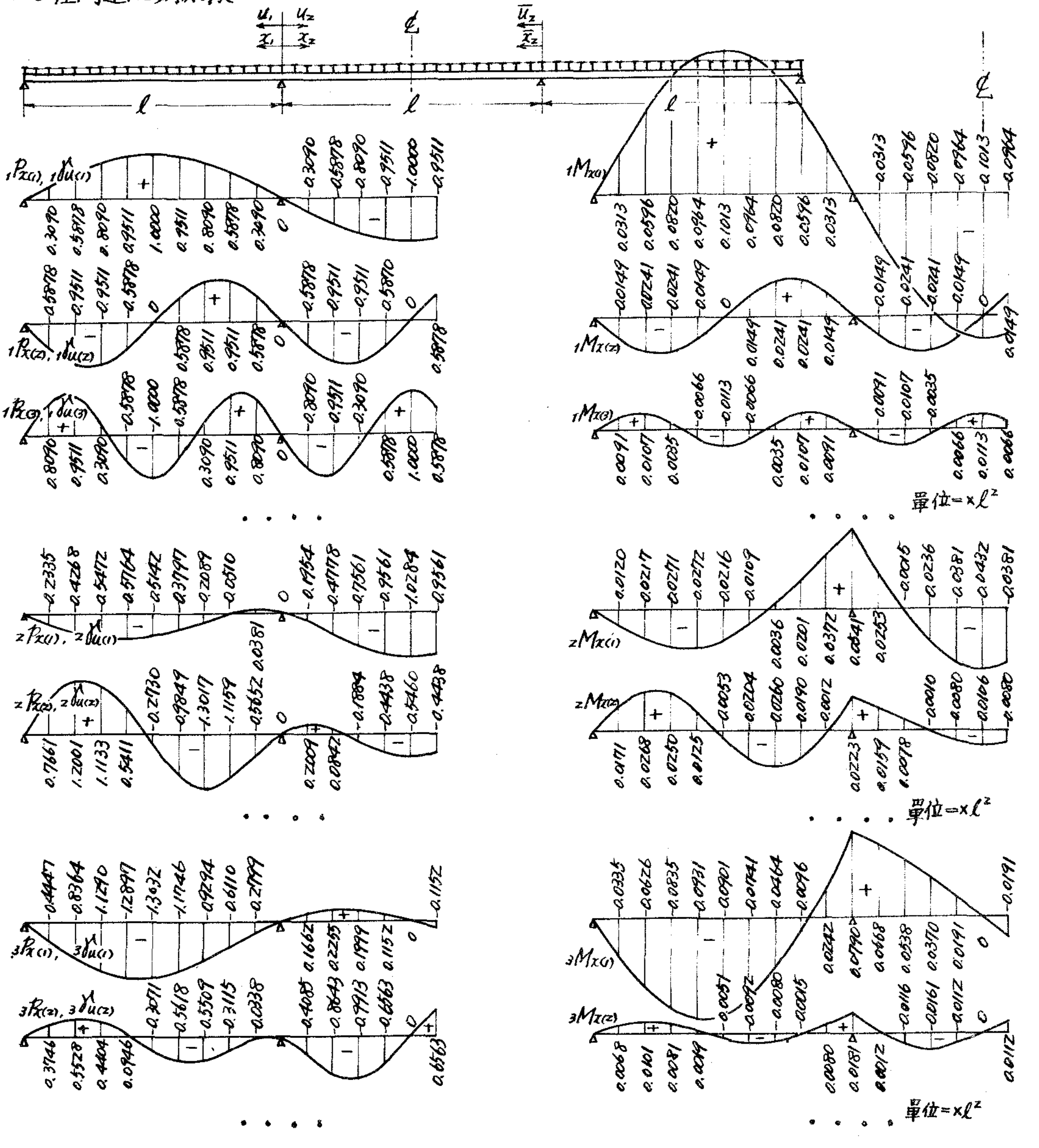
$$\mu_{(n)}=\frac{1}{2\int_0^\ell {}_2p_{x(n)}^2\,dx}=\left[\frac{\ell}{4}\left(\frac{1}{\sin^2\lambda_{(n)}}-\frac{1}{\sinh^2\lambda_{(n)}}\right)\right]^{-1}\doteqdot\frac{2}{\ell}. \qquad Z_{(n)}=\frac{6\ell^4 J_Q}{a^3 b\, J}\times\frac{1}{\lambda_{(n)}^4}.$$

$${}_2\gamma_{u_1(n)}={}_2\gamma_{u_2(n)}={}_2\gamma_{u(n)}=-\frac{\sinh\lambda_{(n)}(1-\frac{u}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{u}{\ell})}{2\sin\lambda_{(n)}}. \qquad {}_2M_{x_1(n)}={}_2M_{x_2(n)}={}_2M_{x(n)}=\frac{\ell^2}{\lambda_{(n)}^2}\left\{\frac{\sinh\lambda_{(n)}(1-\frac{x}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{x}{\ell})}{2\sin\lambda_{(n)}}\right\}.$$

$${}_2Q_{x_1(n)}={}_2Q_{x_2(n)}={}_2Q_{x(n)}=-\frac{\ell}{\lambda_{(n)}}\left\{\frac{\cosh\lambda_{(n)}(1-\frac{x}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\cos\lambda_{(n)}(1-\frac{x}{\ell})}{2\sin\lambda_{(n)}}\right\}.$$

$${}_2\delta_{x_1(n)}={}_2\delta_{x_2(n)}={}_2\delta_{x(n)}=\frac{\ell^4}{\lambda_{(n)}^4 EJ}\left\{-\frac{\sinh\lambda_{(n)}(1-\frac{x}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{x}{\ell})}{2\sin\lambda_{(n)}}\right\}.$$

(3) 3径間連続鋼床板



(i) 第一の解

$$_1P_{x_1(n)} = -{}_1P_{x_2(n)} = {}_1P_{x(n)} = \sin\frac{\lambda_{(n)}}{\ell}x \qquad \lambda_{(n)} = n\pi,\ (n=1,2,3,\cdots).\qquad \mu_{(n)} = \frac{1}{3\int_0^{\ell} {}_1P_{x(n)}^2\,dx} = \frac{2}{3\ell}.\qquad Z_{(n)} = \frac{6\ell^4 J_Q}{a^3 b J}\times\frac{1}{\lambda_{(n)}^4}$$

$$_1\gamma_{u_1(n)} = -{}_1\gamma_{u_2(n)} = {}_1\gamma_{u(n)} = \sin\frac{\lambda_{(n)}}{\ell}u.\qquad {}_1M_{x_1(n)} = -{}_1M_{x_2(n)} = {}_1M_{x(n)} = \frac{\ell^2}{\lambda_{(n)}^2}\sin\frac{\lambda_{(n)}}{\ell}x.\qquad {}_1Q_{x_1(n)} = -{}_1Q_{x_2(n)} = {}_1Q_{x(n)} = \frac{\ell}{\lambda_{(n)}}\cos\frac{\lambda_{(n)}}{\ell}x.$$

$$_1\delta_{x_1(n)} = -{}_1\delta_{x_2(n)} = {}_1\delta_{x(n)} = \frac{\ell^4}{\lambda_{(n)}^4 EJ}\sin\frac{\lambda_{(n)}}{\ell}x.$$

(ii) 第二の解 (対称形)

$$_2P_{x_1(n)}=-\frac{\sinh\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sin\lambda_{(n)}}\ ,\quad {}_2P_{x_2(n)}=\left\{-\frac{\sinh\lambda_{(n)}(1-\frac{x_2}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{x_2}{\ell})}{2\sin\lambda_{(n)}}\right\}+\left\{-\frac{\sinh\lambda_{(n)}(1-\frac{\bar{x}_2}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{\bar{x}_2}{\ell})}{2\sin\lambda_{(n)}}\right\}.$$

$$\lambda_{(1)}=4.3\ ,\quad \lambda_{(2)}=6.7\ ,\cdots\quad \mu_{(n)}=\frac{1}{2\int_0^\ell {}_2P_{x_1(n)}^2dx+\int_0^\ell {}_2P_{x_2(n)}^2dx}=\left[\frac{\ell}{4}\left(\frac{2\cos\lambda_{(n)}-1}{\lambda_{(n)}\sin\lambda_{(n)}}-\frac{2\cosh\lambda_{(n)}-1}{\lambda_{(n)}\sinh\lambda_{(n)}}\right.\right.$$

$$\left.\left.+\frac{\cosh\lambda_{(n)}-2}{\sinh^2\lambda_{(n)}}-\frac{\cos\lambda_{(n)}-2}{\sin^2\lambda_{(n)}}\right)\right]^{-1}\ ,\quad \mu_{(1)}=\frac{1.3448}{\ell}\ ,\quad \mu_{(2)}=\frac{0.6035}{\ell}\ .\quad Z_{(n)}=\frac{6\ell^4J_Q}{a^3bJ}\times\frac{1}{\lambda_{(n)}^4}\ .$$

$$_2\gamma_{u_1(n)}=-\frac{\sinh\lambda_{(n)}(1-\frac{u_1}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{u_1}{\ell})}{2\sin\lambda_{(n)}}\ ,\quad {}_2\gamma_{u_2(n)}=\left\{-\frac{\sinh\lambda_{(n)}(1-\frac{u_2}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{u_2}{\ell})}{2\sin\lambda_{(n)}}\right\}+\left\{-\frac{\sinh\lambda_{(n)}(1-\frac{\bar{u}_2}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{\bar{u}_2}{\ell})}{2\sin\lambda_{(n)}}\right\}.$$

$$_2M_{x_1(n)}=\frac{\ell^2}{\lambda_{(n)}^2}\left\{\frac{\sinh\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sin\lambda_{(n)}}\right\},\quad {}_2M_{x_2(n)}=\frac{\ell^2}{\lambda_{(n)}^2}\left[\left\{\frac{\sinh\lambda_{(n)}(1-\frac{x_2}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{x_2}{\ell})}{2\sin\lambda_{(n)}}\right\}+\left\{\frac{\sinh\lambda_{(n)}(1-\frac{\bar{x}_2}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{\bar{x}_2}{\ell})}{2\sin\lambda_{(n)}}\right\}\right].$$

$$_2Q_{x_1(n)}=-\frac{\ell}{\lambda_{(n)}}\left\{\frac{\cosh\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\cos\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sin\lambda_{(n)}}\right\},\quad {}_2Q_{x_2(n)}=-\frac{\ell}{\lambda_{(n)}}\left[\left\{\frac{\cosh\lambda_{(n)}(1-\frac{x_2}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\cos\lambda_{(n)}(1-\frac{x_2}{\ell})}{2\sin\lambda_{(n)}}\right\}-\left\{\frac{\cosh\lambda_{(n)}(1-\frac{\bar{x}_2}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\cos\lambda_{(n)}(1-\frac{\bar{x}_2}{\ell})}{2\sin\lambda_{(n)}}\right\}\right].$$

$$_2\delta_{x_1(n)}=\frac{\ell^4}{\lambda_{(n)}^4EJ}\left\{\frac{\sinh\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sin\lambda_{(n)}}\right\},\quad {}_2\delta_{x_2(n)}=\frac{\ell^4}{\lambda_{(n)}^4EJ}\left[\left\{-\frac{\sinh\lambda_{(n)}(1-\frac{x_2}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{x_2}{\ell})}{2\sin\lambda_{(n)}}\right\}+\left\{-\frac{\sinh\lambda_{(n)}(1-\frac{\bar{x}_2}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{\bar{x}_2}{\ell})}{2\sin\lambda_{(n)}}\right\}\right].$$

(iii) 第三の解 (逆対称形)

$$_3P_{x_1(n)}=-\frac{\sinh\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sin\lambda_{(n)}}\ ,\quad {}_3P_{x_2(n)}=-\frac{\sinh\lambda_{(n)}(\frac{1}{2}-\frac{x_2}{\ell})}{2\sinh\frac{\lambda_{(n)}}{2}}+\frac{\sin\lambda_{(n)}(\frac{1}{2}-\frac{x_2}{\ell})}{2\sin\frac{\lambda_{(n)}}{2}}\ .\quad \lambda_{(1)}=3.55674\ ,\quad \lambda_{(2)}=7.427\cdots .$$

$$\mu_{(n)}=\left[\frac{\ell}{8}\left\{\left(\frac{\cos\lambda_{(n)}}{\lambda_{(n)}\sin\lambda_{(n)}}-\frac{\cosh\lambda_{(n)}}{\lambda_{(n)}\sinh\lambda_{(n)}}-\frac{1}{2\sinh^2\frac{\lambda_{(n)}}{2}}+\frac{1}{2\sin^2\frac{\lambda_{(n)}}{2}}\right)\right\}\right]^{-1},\quad \mu_{(1)}=\frac{0.6154}{\ell}\ ,\quad \mu_{(2)}=\frac{1.3205}{\ell}\ .\quad Z_{(n)}=\frac{6\ell^4J_Q}{a^3bJ}\times\frac{1}{\lambda_{(n)}^4}\ .$$

$$_3\gamma_{u_1(n)}=-\frac{\sinh\lambda_{(n)}(1-\frac{u_1}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{u_1}{\ell})}{2\sin\lambda_{(n)}}\ ,\quad {}_3\gamma_{u_2(n)}=-\frac{\sinh\lambda_{(n)}(\frac{1}{2}-\frac{u_2}{\ell})}{2\sinh\frac{\lambda_{(n)}}{2}}+\frac{\sin\lambda_{(n)}(\frac{1}{2}-\frac{u_2}{\ell})}{2\sin\frac{\lambda_{(n)}}{2}}\ .$$

$$_3M_{x_1(n)}=\frac{\ell^2}{\lambda_{(n)}^2}\left\{\frac{\sinh\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\sin\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sin\lambda_{(n)}}\right\},\quad {}_3M_{x_2(n)}=\frac{\ell^2}{\lambda_{(n)}^2}\left\{\frac{\sinh\lambda_{(n)}(\frac{1}{2}-\frac{x_2}{\ell})}{2\sinh\frac{\lambda_{(n)}}{2}}+\frac{\sin\lambda_{(n)}(\frac{1}{2}-\frac{x_2}{\ell})}{2\sin\frac{\lambda_{(n)}}{2}}\right\}.$$

$$_3Q_{x_1(n)}=-\frac{\ell}{\lambda_{(n)}}\left\{\frac{\cosh\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sinh\lambda_{(n)}}+\frac{\cos\lambda_{(n)}(1-\frac{x_1}{\ell})}{2\sin\lambda_{(n)}}\right\},\quad {}_3Q_{x_2(n)}=-\frac{\ell}{\lambda_{(n)}}\left\{\frac{\cosh\lambda_{(n)}(\frac{1}{2}-\frac{x_2}{\ell})}{2\sinh\frac{\lambda_{(n)}}{2}}+\frac{\cos\lambda_{(n)}(\frac{1}{2}-\frac{x_2}{\ell})}{2\sin\frac{\lambda_{(n)}}{2}}\right\}.$$

[例] 図 1 c のような断面をもつ図 1 a, b のような一辺固定他辺自由(橋長方向∞長)の鋼床板について、実際に計算した横リブおよび縦リブの曲げモーメント影響面は 図 2 a, b のとおりである。

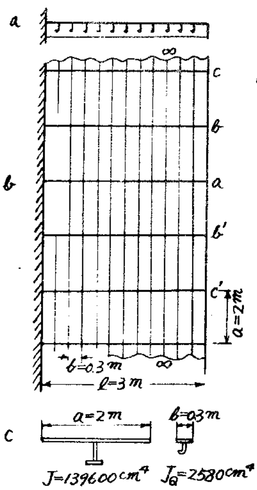


図 1 a-c

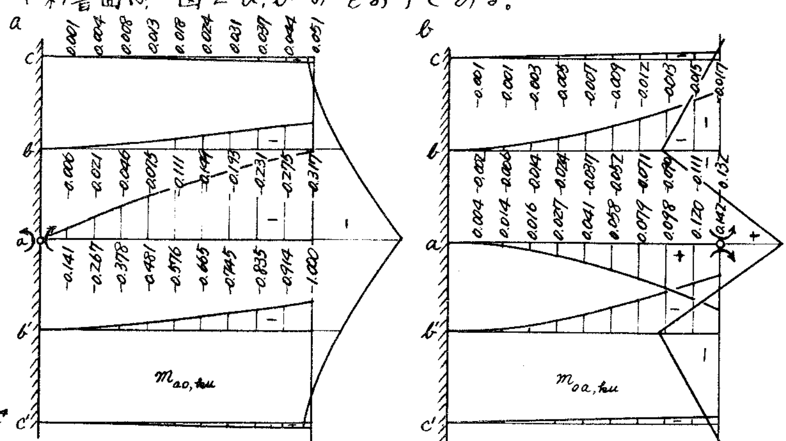


図 2 a, b

なお、2径間, 3径間連続鋼床板の影響面については、文献[3]を参照のこと。

参考文献 : [1] Homberg : Kreuzwerke . [2] Homberg u. Weinmeister : Einflußflächen für Kreuzwerke.
[3] 渡辺昇 : 格子桁理論による連続鋼床板の影響面に関する解析と計算. 技術資料第19号.