論 說 報 告

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THE APPLICATION OF THEORY OF INFLUENCE EQUATIONS FOR THE ANALYSIS OF TALL BUILDING FRAMES.

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Synopsis.

This paper presents the analysis of tall building frames taking thorough consideration of practical methods of construction and theoretical investigations.

INTRODUCTION.

The utilization of vertical space in building is gaining momentum in this twentieth century. This fact leads us to consider a more rational solution of building frame, a subject which is being recognized more and more because of its importance in tall buildings.

Here the writer presents his method which is based on his Influence Equation Theory, as fully explained in previous papers, and is simple and accurate enough for practical use.

The solution of the building frame will be divided into two groups according to the condition of loadings;

(I) Vertical loads (Floor loads)

(II) Horizontal loads (Wind or Earthquake loads)

Before going into a discussion, the following assumptions will be made:

A. The connections between the columns and beams are perfectly rigid.

B. The change in the length of a member due to direct stress will be negligible.

C. The length of a beam is the distance between the neutral axes of the columns which it connects, and the length of a columns is the distance between the neutral axes of the beam which it connects.

D. The deflection of a member due to the internal shearing stresses is equal to zero.

E. The loads, vertical or horizontal, are resisted entirely by the frame only.

THE FRAME UNDER VERTICAL LOADS.

The solution of a building frame with vertical loads has been published by the

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author in 1927 and others.*

The analysis of this structure can be carried out by taking the entire structure as a whole and establishing the numerous simultaneous equations but it takes too much labor and is far from practical use.

The method here presented is simpler and most practical.

In the analysis of a vertical loading we have two loadings, a dead and a live load. Some methods simply assume that the loading on floor beams is the sum of dead and live loads distributed over the entire floor as though all floor beams carried dead load with increased intensity of loading having the sum of dead and live loads. Actually, however, a live load does not always meet these assumed conditions. Sometime we may have a live load on the exterior bay and no live load on interior bay or vice versa. Therefore for a live load it is necessary to find the possible critical loading for a beam or column under consideration because the ratio of dead and live load intensity varies approximately from 1 : 1 to 1 : 4.

For finding the possible critical loading, moment and shear it is necessary to have the Influence values of these. For this rational consideration the following method will be employed for the analysis.

Take a building frame and apply any load as shown in **Fig. 1**

We have bending moments at various points in the frame. The moment in the member is largest in the beam on which the load is applied and diminishes gradually as the distance increases from the beam. Therefore the problem can be solved if it is possible to establish some method of finding the distribution and character of moment at various points.

The distribution and the character of the moment of members will be found by considering a portion of the building. This method may not be exact but practically satisfactory with good accuracy.

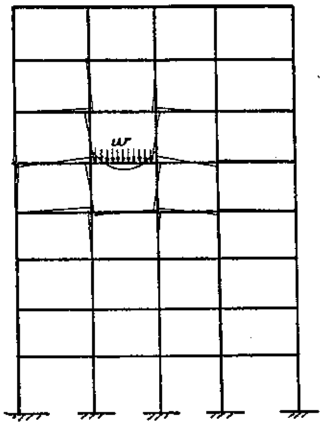


Fig. 1

The distribution of moment among members will be found in three steps, (a) End moment of a beam with a load, (b) Distribution of the end moment among adjacent members, (c) Ratio of moments at both ends of an adjacent member.

(a) End Moment of a beam with load.

Take a portion of the building as shown in **Fig. 2**, and all the supports assumed

* Messrs, W. M. Wilson, F. E. Richart and Camillo Weiss in bulletin No. 108 of University of Illinois 1918.
Dr. Fukuhei Takabeya, 1927.
Prof. Cross, 1929.

as fixed. Practically this assumption is satisfactory because under the vertical loading the final axis of entire building remains relatively the same as the original axis, in other words, the values of "φ" under vertical loading are zero although there does not exists a slight horizontal deflection which reduces the moments somewhat but which for practical purposes is negligible.

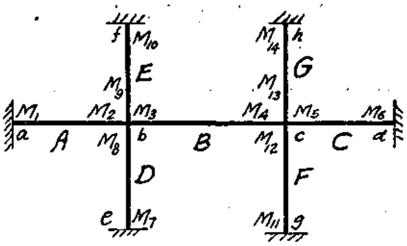


Fig. 2

However, this assumption is not applicable in the case of a horizontal loading because the entire building sways horizontally with certain values in "φ" in vertical direction.

By the usual procedure, establish moment equations.

$$AM_1 = -(2m_1+m_2) - A\alpha_{ab} \qquad DM_8 = -(2m_8+m_7)$$
$$AM_2 = -(2m_2+m_1) + A\beta_{ab} \qquad EM_9 = -(2m_9+m_{10})$$
$$BM_3 = -(2m_3+m_4) - B\alpha_{bc} \qquad EM_{10} = -(2m_{10}+m_9)$$
$$BM_4 = -(2m_4+m_3) + B\beta_{bc} \qquad FM_{11} = -(2m_{11}+m_{12})$$
$$CM_5 = -(2m_5+m_6) - C\alpha_{cd} \qquad FM_{12} = -(2m_{12}+m_{11})$$
$$CM_6 = -(2m_6+m_5) + C\beta_{cd} \qquad GM_{13} = -(2m_{13}+m_{14})$$
$$DM_7 = -(2m_7+m_8) \qquad GM_{14} = -(2m_{14}+m_{13})$$

Angular Relations:

From the assumption that all supports are fixed the value of all "m" at supports are zero, and all "φ" are zero.

$$m_1 = m_7 = m_{10} = m_{11} = m_{14} = m_6 = 0$$
$$m_2 - \varphi_{ab} = m_8 - \varphi_{be} = m_9 - \varphi_{bf} = m_3 - \varphi_{bc} \qquad \varphi_{ab} = \varphi_{bc} = \varphi_{bf} = \varphi_{bc} = 0$$
$$\therefore \quad m_2 = m_3 = m_8 = m_9, \qquad m_4 = m_5 = m_{12} = m_{13}$$

Substituting these relations into above moment equations we have,

$$AM_1 = -m_2 - A\alpha_{ab} \qquad CM_5 = -2m_4 - C\alpha_{cd} \qquad EM_{10} = -m_2$$
$$AM_2 = -2m_2 + A\beta_{ab} \qquad CM_6 = -m_4 + C\beta_{cd} \qquad FM_{11} = -m_4$$
$$BM_3 = -(2m_2+m_4) - B\alpha_{bc} \qquad DM_7 = -m_2 \qquad FM_{12} = -2m_4$$
$$BM_4 = -(2m_4+m_2) + B\beta_{bc} \qquad DM_8 = -2m_2 \qquad GM_{13} = -2m_4$$
$$EM_9 = -2m_2 \qquad GM_{14} = -m_4$$

To solve the moments at joints b and c it requires 10 equations, 8 for the unknown moments and 2 for unknown "m".

Arrange the moment equations in tabulated form by taking the equations which include only unknowns at joints b and c and eliminate "m". After eliminating "m" eliminate all moments except the moments of the center beam M_3 and M_4 by combining with the conditions

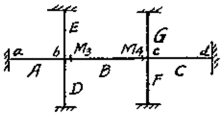
$$M_2 + M_3 + M_8 + M_9 = 0, \qquad M_4 + M_5 + M_{12} + M_{13} = 0$$

at joints b and c then we have the Influence Equations for the moments of beam

bc as shown in **Table 1**.

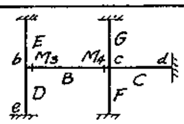
In the same manner the Influnence Equations for other portions of the building will be obtained as shown in the same table.

Table 1



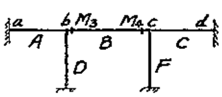
M_3	M_4	β_{ab}	α_{bc}	β_{bc}	α_{cd}
$QS-TR$		US	$R(2S-T)$	$R(S-2T)$	$-UR$
	$QS-TR$	UT	$T(2R-Q)$	$T(R-2Q)$	$-UQ$

$Q=2(\frac{1}{A}+\frac{1}{D}+\frac{1}{E})+1.5\frac{1}{B}$ $R=(\frac{1}{A}+\frac{1}{D}+\frac{1}{E})$
$S=2(\frac{1}{C}+\frac{1}{F}+\frac{1}{G})+1.5\frac{1}{B}$ $T=(\frac{1}{C}+\frac{1}{F}+\frac{1}{G})$
$U=1.5\frac{1}{B}$



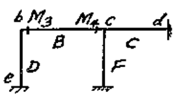
M_3	M_4	β_{eb}	α_{bc}	β_{bc}	α_{cd}
$QS-TR$		US	$R(2S-T)$	$R(S-2T)$	$-UR$
	$QS-TR$	UT	$T(2R-Q)$	$T(R-2Q)$	$-UQ$

$Q=2(\frac{1}{D}+\frac{1}{E})+1.5\frac{1}{B}$ $R=(\frac{1}{D}+\frac{1}{E})$
$S=2(\frac{1}{C}+\frac{1}{F}+\frac{1}{G})+1.5\frac{1}{B}$ $T=(\frac{1}{C}+\frac{1}{F}+\frac{1}{G})$
$U=1.5\frac{1}{B}$



M_3	M_4	β_{ab}	α_{bc}	β_{bc}	α_{cd}
$QS-TR$		US	$R(2S-T)$	$R(S-2T)$	$-UR$
	$QS-TR$	UT	$T(2R-Q)$	$T(R-2Q)$	$-UQ$

$Q=2(\frac{1}{A}+\frac{1}{D})+1.5\frac{1}{B}$ $R=(\frac{1}{A}+\frac{1}{D})$
$S=2(\frac{1}{C}+\frac{1}{F})+1.5\frac{1}{B}$ $T=(\frac{1}{C}+\frac{1}{F})$
$U=1.5\frac{1}{B}$



M_3	M_4	β_{eb}	α_{bc}	β_{bc}	α_{cd}
$QS-TR$		US	$R(2S-T)$	$R(S-2T)$	$-UR$
	$QS-TR$	UT	$T(2R-Q)$	$T(R-2Q)$	$-UQ$

$Q=2(\frac{1}{D})+1.5\frac{1}{B}$ $R=\frac{1}{D}$
$S=2(\frac{1}{C}+\frac{1}{F})+1.5\frac{1}{B}$ $T=(\frac{1}{C}+\frac{1}{F})$
$U=1.5\frac{1}{B}$

(b) Distribution of the End Moments among adjacent members.

From the equation in **Table 1** the moments at both ends of a beam are found. If a load is applied on beam bc as shown in **Fig. 3** and considering the left hand side of the center line only the end moment of the beam bc must be distributed among adjacent members to keep the equilibrium at the joint.

The ratio of distribution and the nature of moments will be determined by considering the equilibrium of a joint.

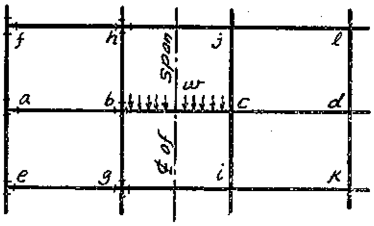


Fig. 3

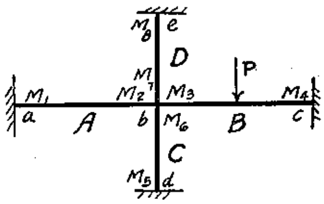
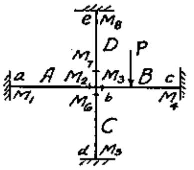


Fig. 4

At joint b we assume four members joined together having other ends of members fixed as shown in **Fig. 4**. Apply a load on beam bc and by the usual procedure establish Influence Equations for this structure and find the ratio of the distribution of the moment among members which are given in **Table 2** together with the case of three members at a joint. Examining the equations in **Table 2** we find that the moment at the left end of beam bc will be distributed among other three members in a reciprocal ratio of member ratio.

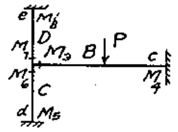
Table 2



M_3	M_2	M_6	M_7	α_{bc}
$\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+\frac{1}{D}$				$\frac{1}{A}+\frac{1}{C}+\frac{1}{D}$
	$\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+\frac{1}{D}$			$-\frac{1}{A}$
		$\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+\frac{1}{D}$		$-\frac{1}{C}$
			$\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+\frac{1}{D}$	$-\frac{1}{D}$

$M_3 : M_2 : M_6 : M_7 = \left(\frac{1}{A}+\frac{1}{C}+\frac{1}{D}\right) : -\frac{1}{A} : -\frac{1}{C} : -\frac{1}{D}$

$M_2 : M_6 : M_7 = \frac{1}{A} : \frac{1}{C} : \frac{1}{D}$



M_3	M_6	M_7	α_{bc}
$\frac{1}{B}+\frac{1}{C}+\frac{1}{D}$			$\frac{1}{C}+\frac{1}{D}$
	$\frac{1}{B}+\frac{1}{C}+\frac{1}{D}$		$-\frac{1}{C}$
		$\frac{1}{B}+\frac{1}{C}+\frac{1}{D}$	$-\frac{1}{D}$

$M_3 : M_6 : M_7 = \left(\frac{1}{C}+\frac{1}{D}\right) : -\frac{1}{C} : -\frac{1}{D}$

$M_6 : M_7 = \frac{1}{C} : \frac{1}{D}$

$$M_2 : M_6 : M_7 = \frac{1}{A} : \frac{1}{C} : \frac{1}{D}$$

The nature of these moments are opposite to that moment at the left end of beam *bc*. Because from the equilibrium of the joint *b* we have

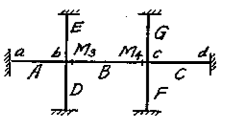
$$M_2+M_6+M_7+M_3=0 \quad \text{or} \quad M_2+M_6+M_7=-M_3$$

(c) Ratio of moments at both ends of an adjacent member.

Refering to **Fig. 3** as soon as a load is applied on beam *bc* certain moments occur at the right and the left end of the adjacent members *ba*, *bg* and *bh* having a certain ratio. The ratio of moments at both ends of a member will be found by the following consideration;

Take a portion of the building as shown in **Fig. 2** and apply a load on beam *cd*. From the Influence Equations in **Table 1** we have the values of the moments of beam *bc* at both ends. We can find the ratio of distribution of moments at both ends of the beam by comparing their values. The ratio of these moments are given on **Table 3** together with other cases.

Table 3

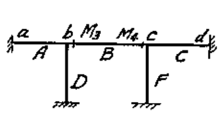


Load on ab

$M_3 : M_4 = \left\{2+\frac{3\frac{1}{B}}{2\left(\frac{1}{C}+\frac{1}{F}+\frac{1}{G}\right)}\right\} : 1$

Load on cd

$M_3 : M_4 = 1 : \left\{2+\frac{3\frac{1}{B}}{2\left(\frac{1}{A}+\frac{1}{D}+\frac{1}{E}\right)}\right\}$

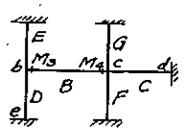


Load on ab

$M_3 : M_4 = \left\{2+\frac{3\frac{1}{B}}{2\left(\frac{1}{C}+\frac{1}{F}\right)}\right\} : 1$

Load on cd

$M_3 : M_4 = 1 : \left\{2+\frac{3\frac{1}{B}}{2\left(\frac{1}{A}+\frac{1}{D}\right)}\right\}$

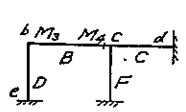


Load on eb

$M_3 : M_4 = \left\{2+\frac{3\frac{1}{B}}{2\left(\frac{1}{C}+\frac{1}{F}+\frac{1}{G}\right)}\right\} : 1$

Load on cd

$M_3 : M_4 = 1 : \left\{2+\frac{3\frac{1}{B}}{2\left(\frac{1}{D}+\frac{1}{E}\right)}\right\}$



Load on eb

$M_3 : M_4 = \left\{2+\frac{3\frac{1}{B}}{2\left(\frac{1}{C}+\frac{1}{F}\right)}\right\} : 1$

Load on cd

$M_3 : M_4 = 1 : \left\{2+\frac{3\frac{1}{B}}{2\frac{1}{D}}\right\}$

After determining the distribution of the moments at each point the nature of the moment will be decided according to the rules of the convention.

Illustrative Problem—(1) Find moments at each points in a building frame with constant member ratios when a uniform load of 120-lbs. per linear unit is applied on an beam ab in **Fig. 5**.

(2) Find the maximum moments at points S and T under uniform live loads.

(1) $A=B=C=D=E=F=G=1,\quad \alpha=\beta=\frac{1}{12}wl_1^2=\frac{1}{12}\times 120\times 20^2=4\,000'\#$

$Q=5.5,\quad R=2,\quad S=7.5,\quad T=3,\quad U=1.5$ **(Table 1)**

Step (a)

$$\begin{cases} M_3=-\dfrac{R(2S-T)\alpha_{bc}+R(S-2T)\beta_{bc}}{QS-TR}=-\dfrac{3\times 2(7.5-3)}{5.5\times 7.5-3\times 2}\times 4\,000=-3\,065'\# \\ M_4=-\dfrac{T(2R-Q)\alpha_{bc}+T(R-2Q)\beta_{bc}}{QS-TR}=\dfrac{-3\times 3(2-5.5)}{5.5\times 7.5-3\times 2}\times 4\,000=+3\,575'\# \end{cases}$$

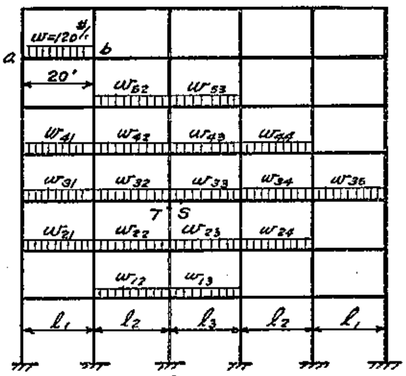


Fig. 5

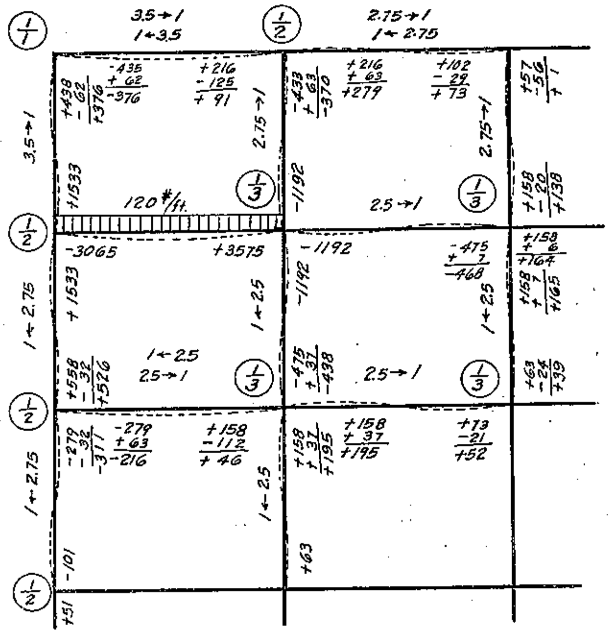


Fig. 6

After step (a) the computation of moments of members will be done by drawing a sketch with the distribution ratio of moments by means of a circle, 1/3 for step (b), and arrow, (2.5→1) for step (c), as shown in **Fig. 6**. After the computation of moments the elastic curve of members will be drawn according to convention which will clearly indicate the nature of moments in members if desired.

(2) In a similar manner the moments at S and T are determined and the results are given in **Table 4**.

Maximum Moment at S

$$M_S=-1.261\,\alpha=-1.261\times\frac{wl^2}{12}\fallingdotseq-\frac{w}{10}l^2$$

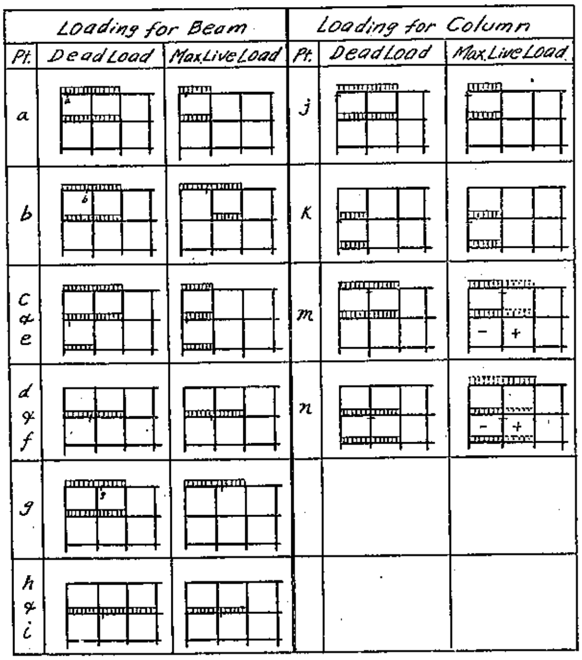
Maximum Moment at T

$$M_T=\pm\ 0.52\alpha=\pm\ 0.52\frac{wl^2}{12}\fallingdotseq\pm\frac{wl^2}{24}$$

From **Table 4** if a building is uniformly loaded, such as dead, the moment at the end of the interior beams is appoximately $-\dfrac{wl^2}{12}$ provided the member ratio of all members is constant, but for a uniform live load the moment at the end of the beam is approximately $-\dfrac{wl^2}{10}$ and column moment is about $\pm\dfrac{wl^2}{24}$.

All loads contribute towards an increase or decrease of the moments at S and T but the greatest effect comes from the neighbouring loads, therefore the following loading is good enough for practical purposes.

Table 5



SUGGESTION FOR DESIGN

When actually making a preliminary design it is impossible to go through this method before determining the member ratio therefore some approximate values must be used. The writer suggests the values as shown in **Table 6** for moments at various points in a building (**Fig. 7**) assuming the uniform loads intensities are w_D and w_L for dead and live load respectively.

For the solution of the floor beam some

Table 4

Coeff. of $\alpha = \frac{1}{12}w l^2$		
Load	At S	At T
w_{21}	-.005	+.015
w_{31}	+.038	+.038
w_{41}	-.005	-.005
w_{12}	-.006	+.017
w_{22}	+.043	-.109
w_{32}	-.286	-.285
w_{42}	+.043	+.043
w_{52}	-.002	-.005
w_{13}	+.003	-.017
w_{23}	-.023	+.109
w_{33}	-.858	+.285
w_{43}	-.023	-.043
w_{53}	-.005	+.005
w_{24}	-.017	-.015
w_{34}	+.114	-.038
w_{44}	-.017	+.005
w_{35}	-.015	+.005
Total	-1.021	0
Maximum	-1.261	±.517

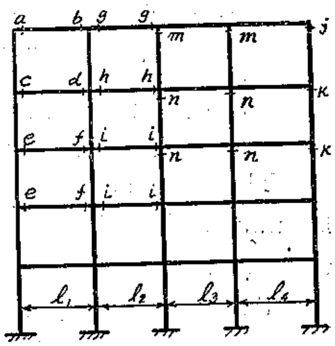


Fig. 7

Table 6

Approximate Moments in Building		
Pt.	D.L. Mom.	Max. L. L. Mom.
a	$-\frac{1}{19}w_D l_1^2$	$-\frac{1}{19}w_L l_1^2$
b	$-\frac{1}{10}w_D l_1^2$	$-(\frac{1}{15}w_L l_1^2+\frac{1}{30}w_L l_2^2)$
c	$-\frac{1}{15}w_D l_1^2$	$-\frac{1}{13}w_L l_1^2$
d	$-\frac{1}{11}w_D l_1^2$	$-(\frac{1}{12}w_L l_1^2+\frac{1}{40}w_L l_2^2)$
e	$-\frac{1}{15}w_D l_1^2$	$-\frac{1}{13}w_L l_1^2$
f	$-\frac{1}{11}w_D l_1^2$	$-(\frac{1}{12}w_L l_1^2+\frac{1}{40}w_L l_2^2)$
g	$-\frac{1}{10}w_D l_2^2$	$-(\frac{w_L l_1^2}{30}+\frac{w_L l_2^2}{15})$ or $-(\frac{w_L l_2^2}{15}+\frac{w_L l_3^2}{30})$
h	$-\frac{1}{12}w_D l_2^2$	$-(\frac{w_L l_1^2}{40}+\frac{w_L l_2^2}{11})$ or $-(\frac{w_L l_2^2}{11}+\frac{w_L l_3^2}{40})$
i	$-\frac{1}{12}w_D l_2^2$	$-(\frac{w_L l_1^2}{40}+\frac{w_L l_2^2}{11})$ or $-(\frac{w_L l_2^2}{11}+\frac{w_L l_3^2}{40})$
j	$+\frac{1}{19}w_D l_1^2$	$+\frac{w_L}{19}l_1^2$
K	$+\frac{1}{30}w_D l_1^2$	$+\frac{1}{26}w_L l_1^2$
m	$\pm\frac{1}{10}w_D(l_1^2-l_2^2)$	$-\frac{1}{23}w_L l_1^2$ or $+\frac{1}{23}w_L l_2^2$
n	$\pm\frac{1}{24}w_D(l_1^2-l_2^2)$	$-\frac{1}{20}w_L l_1^2$ or $+\frac{1}{20}w_L l_2^2$

attempt to solve the problem assuming that the floor beam is a mere continuous beam, however, this is quite contrary to the actual condition because the distribution of the moment is different due to the the existence of columns at each joint.

SHEARS IN BEAMS AND COLUMNS.

The shears at the ends of beams will be transmitted as the vertical load on columns. The shears of beams will be found by taking the moment about the each end of the beam passing through the section.

For instance, in **Fig. 8**, the shear of beam ef at the left end e will be

$$M_{ef}+M_{fe}+V_e l_1+M_0=0$$

$$V_e=\frac{-1}{l_1}(M_{ef}+M_{fe}+M_0)$$

Where V_e=shear in the beam at the left end e.

M_{ef}, M_{fe} are end moment of the beam at e and f respectively.

$M_0=-\frac{wl_1^2}{12}$ is the moment due to the external uniform load on the beam by taking the right end f of the beam as the origin of the moment.

l_1=the span length between e and f.

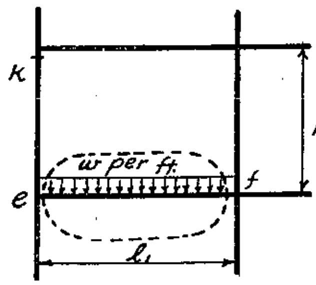


Fig. 8

The shear at the right end will be found in a similar manner by taking the moment about the left end of the beam.

The shear in a column will be found in the same way and will be

$$H_k=-\frac{M_{ke}+M_{ek}}{h_1}$$

Where H_k shear in the column, at top end k as well as the shear at any point on the column between e and k.

M_{ke} and M_{ek} are moments of the column at top k and bottom e respectively.

h_1=Column height.

THE FRAME UNDER HORIZONTAL LOADS.

In conjunction with this subject there are several factors which must be taken into consideration in designing tall structures. They are:

(1) Intensity of wind pressure, or rate of accelaration in case of earthquake vibration.

(2) Distribution of wind pressure.

(3) Location of building.

(4) Stiffening effect of walls and floors.

(5) Method of calculation.

The first four items depend upon the judgement of engineers and the building code, therefore they will not be considered in this paper. The following pages will endeavor to illustrate the fifth factor, i. e, the method of calculation.

The method of calculating wind stresses have been published by several authors. (Mr. Ernest F. Johnson, 1905; Dr. C. A. Melick, 1908; Professor Albert Smith; Mr. R. Fleming 1913; Professors W. M. Wilson and G. A. Maney, 1915; Mr. A. W. Ross; Mr. Spar; Dr. F. Takabeya, 1928 and myself, 1927.)

All these method may be divided into two distinct types according whether the "Member Ratio" or Reciprocal of member Ratio is taken into consideration or not. The methods which do consider the member ratio or reciprocal of member ratio are more accurate but some are too long in analysis and impractical; the ones that do not are a little more convenient for practical design, but unfortunately the errors are too great, sometime as high as 100 %.

I am not in favor of the latter group because it assume moment of inertia of members as being alike throughout a building when in actual practice they vary. It is dangerous to use this method especially where beams and girders with different member ratios join with columns from both sides, or where member ratios are so different between two floors.

The truth of this statement can be readly seen from the illustrative numerical example and the table which compares the results of various methods.

Before going into a discussion, the same five assumptions as in the previous subject will be made.

A. The connections between columns and beams are rigid.

And so on.

Should the wind exert upon a building as shown then the building will take such a tendency as indicated in **Fig. 9**.

Now if we consider a portion of the building between two contraflexure lines as shown in **Fig. 9**. Then that will be considered as simple frame structure with hinged suppports as shown in **Fig. 10**.

From the theory the deflection moment equations and angular relations are written as other problems, the angular relations of this simple structure is the same as the whole building being considered as a structure because this simple structure is the partial structure of the building after wind effect. Suppose all correct contra-flexure points in the columns were

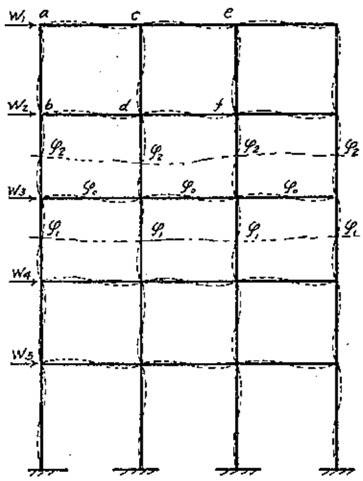


Fig. 9

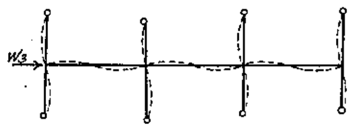


Fig. 10

located then the problem will be solved by following the steps refered to **Fig. 11.**

In **Fig. 11.**

I_1, I_2, I_3 etc., are moment of inertia of the members.

H_1, H_2, H_3 etc., are the members in the columns.

$$A=\frac{a}{I_1},\quad B=\frac{b}{I_2},\quad C=\frac{c}{I_3},\quad D=\frac{d}{I_4},\quad E=\frac{e}{I_5}\quad F=\frac{f}{I_6},\quad G=\frac{g}{I_7},$$

$$H=\frac{h}{I_8},\quad I=\frac{l_1}{I_9},\quad J=\frac{l_2}{I_{10}},\quad K=\frac{l_3}{I_{11}}$$

From the deflection moment equations without intervening loads,

$$AM_1=-\frac{3}{2}m_1 \qquad HM_8=-\frac{3}{2}m_8$$

$$BM_2=-\frac{3}{2}m_2 \qquad IM_9=(2m_9+m_{10})$$

$$CM_3=-\frac{3}{2}m_3 \qquad IM_{10}=(2m_{10}+m_9)$$

$$DM_4=-\frac{3}{2}m_4 \qquad JM_{11}=(2m_{11}+m_{12})$$

$$EM_5=-\frac{3}{2}m_5 \qquad JM_{12}=(2m_{12}+m_{11})$$

$$FM_6=-\frac{3}{2}m_6 \qquad KM_{13}=(2m_{13}+m_{14})$$

$$GM_7=-\frac{3}{2}m_7 \qquad KM_{14}=(2m_{14}+m_{13})$$

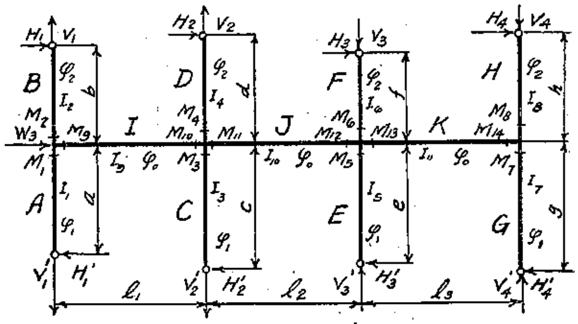


Fig. 11

From the Angular Relations,

$$m_2=m_1-\varphi_1+\varphi_2 \qquad \varphi_0=0$$

$$m_4=m_3-\varphi_1+\varphi_2 \qquad m_9=m_1-\varphi_1$$

$$m_6=m_5-\varphi_1+\varphi_2 \qquad m_{10}=m_{11}=m_3-\varphi_1$$

$$m_8=m_7-\varphi_1+\varphi_2 \qquad m_{12}=m_{13}=m_5-\varphi_1$$

$$m_{14}=m_7-\varphi_1$$

Substituting the angular relations into moment equations,

$$AM_1=-\frac{3}{2}m_1, \qquad HM_8=-\frac{3}{2}(m_7-\varphi_1+\varphi_2)$$

$$BM_2=-\frac{3}{2}(m_1-\varphi_1+\varphi_2) \qquad IM_9=-(2m_1+m_3-3\varphi_1)$$

$$CM_3=-\frac{3}{2}m_3 \qquad IM_{10}=-(2m_3+m_1-3\varphi_1)$$

$$DM_4=-\frac{3}{2}(m_3-\varphi_1+\varphi_2) \qquad JM_{11}=-(2m_3+m_5-3\varphi_1)$$

$$EM_5=-\frac{3}{2}m_5 \qquad JM_{12}=-(2m_5+m_3-3\varphi_1)$$

$$FM_6=-\frac{3}{2}(m_5-\varphi_1+\varphi_2) \qquad KM_{13}=-(2m_5+m_7-3\varphi_1)$$

$$GM_7=-\frac{3}{2}m_7 \qquad KM_{14}=-(2m_7+m_5-3\varphi_1)$$

Arranging the moment equations into tabulated form for the elimination of angles "m"

Table 7

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_{11}	M_{12}	M_{13}	M_{14}	m_1	m_3	m_5	m_7	φ_1	φ_2
1	A														3/2					
2		B													3/2				-3/2	3/2
3			C													3/2				
4				D												3/2			-3/2	3/2
5					E												3/2			
6						F											3/2		-3/2	3/2
7							G											3/2		
8								H										3/2	-3/2	3/2
9									I						2	1			-3	
10										I					1	2			-3	
11											J					2	1		-3	
12												J				1	2		-3	
13													K				2	1	-3	
14														K			1	2	-3	
15	A	-B	-C	D																
16			C	-D	-E	F														
17					E	-F	-G	H							Eliminating "m" and "φ" from above equations.					
18	2A		-2C						-3I	3I										
19	2A				-2E					-3I	3J									
20			2C		-2E						-3J	3J								
21			2C				-2G					-3J	3K							
22					2E		-2G						-3K	3K						
23	1	1							1						Applying ΣM=0 around each joint.					
24			1	1						1	1									
25					1	1						1	1							
26							1	1						1						
27	A	-B	-C	D					Eliminating M_9, M_{10}, M_{11}, M_{12} M_{13} and M_{14} from equations (15-26)											
28			C	-D	-E	F														
29					E	-F	-G	H												
30	2A(2I+J)+3I(I+J)	3I(I+J)	-2C(I+J) -3IJ	-3IJ	-2EI															
31	2AJ(J+K)		2C(2J+K)×(I+J)+3IJ(J+K)	3IJ(J+K)	-2E(2J+I)×(J+K) -3KJ(I+J)	-3KJ(I+J)	-2GJ(I+J)													
32			2CK		2E(J+K) +3KJ	3KJ	-2G(2K+J) -3K(J+K)	-3K(J+K)												

and "φ", we have.

We have another conditions which are $M_1+H_1'a=0$ etc. as shown on **Table 8** and $\Sigma H=\Sigma W$. $\Sigma H=\Sigma W$ mean that total summation of the shears in columns is equal to the total summation of the wind loads above the section being considered. In the following tables ΣW_U and ΣW_L are the summation of wind loads above the sections which are taken above and below the floor line respectively. Refering to **Fig. 11** we have following equations.

Table 8

	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	H_1	H_2	H_3	H_4	H_1'	H_2'	H_3'	H_4'	ΣW_L	ΣW_U
1	1												a					
2		1							b									
3			1											c				
4				1						d								
5					1										e			
6						1					f							
7							1									g		
8								1				h						
9									1	1	1	1						1
10													1	1	1	1	1	
11	$\frac{1}{a}$		$\frac{1}{c}$		$\frac{1}{e}$		$\frac{1}{g}$		Eliminating H_1, H_2 etc								-1	
12		$\frac{1}{b}$		$\frac{1}{d}$		$\frac{1}{f}$		$\frac{1}{h}$										-1

Table 9

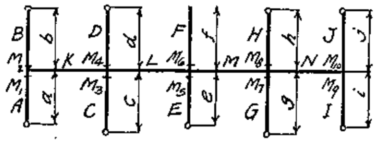
	M_1	M_3	M_5	M_7	M_2	M_4	M_6	M_8	ΣW_L	ΣW_U
1	A	-C			-B	D				
2		C	-E			-D	F			
3			E	-G			-F	H		
4	2A(2I+J) +3I(I+J)	-2C(I+J) -3IJ	-2EI		3I(I+J)	-3IJ				
5	2AJ(J+K)	2C(2J+K) ×(I+J) +3IJ(J+K)	-2E(2J+I) ×(J+K) -3KJ(I+J)	-2GJ(I+J)		3IJ(J+K)	-3KJ(I+J)			
6		2CK	2E(J+K) +3JK	-2G(2K+J) -3K(J+K)			3KJ	-3K(J+K)		
7	$\frac{1}{a}$	$\frac{1}{c}$	$\frac{1}{e}$	$\frac{1}{g}$					-1	
8					$\frac{1}{b}$	$\frac{1}{d}$	$\frac{1}{f}$	$\frac{1}{h}$		-1

Combing equations (27–32) in **Table 7** and (11–12) in **Table 8** we get final equations as shown in **Table 9.**

Equations for a building with 4-bays are obtained in a similar manner and are shown in **Table 10.**

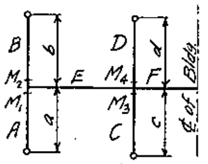
If we compare the equations of these two cases it will be noticed that they are

Table 10



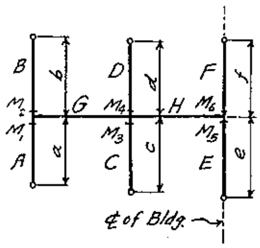
	M_1	M_3	M_5	M_7	M_9	M_2	M_4	M_6	M_8	M_{10}	ΣW_L	ΣW_U
1	A	$-C$				$-B$	D					
2		C	$-E$				$-D$	F				
3			E	$-G$				$-F$	H			
4				G	$-I$				$-H$	J		
5	$2A(2K+L)+3K(K+L)$	$-2C(K+L)-3KL$	$-2EK$			$3K(K+L)$	$-3KL$					
6	$2AL(L+M)$	$2C(2L+M)\times(K+L)+3KL(L+M)$	$-2E(2L+K)\times(L+M)-3LM(K+L)$	$-2GL(K+L)$			$3KL(L+M)$	$-3ML(K+L)$				
7		$2CM(M+N)$	$2E(2M+N)\times(L+M)+3ML(M+N)$	$-2G(2M+L)\times(M+N)-3MN(L+M)$	$-2IM(L+M)$			$3ML(M+N)$	$-3MN(L+M)$			
8			$2EN$	$2G(M+N)+3MN$	$-2I(2N+M)-3N(M+N)$				$3MN$	$-3N(M+N)$		
9	$\frac{1}{a}$	$\frac{1}{c}$	$\frac{1}{e}$	$\frac{1}{g}$	$\frac{1}{i}$						-1	
10						$\frac{1}{b}$	$\frac{1}{d}$	$\frac{1}{f}$	$\frac{1}{h}$	$\frac{1}{j}$		-1

Table 11



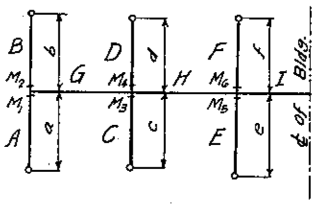
	M_1	M_3	M_2	M_4	ΣW_L	ΣW_U
1	A	$-C$	$-B$	D		
2	$2A(2E+F)+3E(E+F)$	$-2C(2E+F)-3EF$	$3E(E+F)$	$-3EF$		
3	$\frac{1}{a}$	$\frac{1}{c}$			$-\frac{1}{2}$	
4			$\frac{1}{b}$	$\frac{1}{d}$		$-\frac{1}{2}$

Table 12



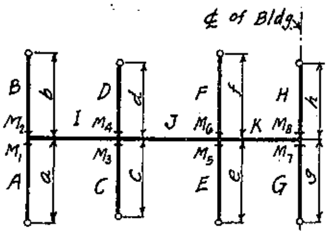
	M_1	M_3	M_5	M_2	M_4	M_6	ΣW_L	ΣW_U
1	A	$-C$		$-B$	D			
2		C	$-E$		$-D$	E		
3	$2A(2G+H)+3G(G+H)$	$-2C(G+H)-3GH$	$-2EG$	$3G(G+H)$	$-3GH$			
4	$4AH$	$4C(G+H)+6GH$	$-4E(2G+H)-3H(G+H)$		$6GH$	$-3H(G+H)$		
5	$\frac{2}{a}$	$\frac{2}{c}$	$\frac{1}{e}$				-1	
6				$\frac{2}{b}$	$\frac{2}{d}$	$\frac{1}{f}$		-1

Table 13



	M_1	M_3	M_5	M_2	M_4	M_6	ΣW_L	ΣW_U
1	A	$-C$		$-B$	D			
2		C	$-E$		$-D$	F		
3	$2A(2G+H)+3G(G+H)$	$-2C(G+H)-3GH$	$-2EG$	$3G(G+H)$	$-3GH$			
4	$2AH(H+I)$	$2C(2H+I)\times(G+H)+3GH(H+I)$	$-2E(2H+G)\times(H+I)-H(2E+3I)(G+H)$		$3GH(H+I)$	$-3IH(G+H)$		
5	$\frac{1}{a}$	$\frac{1}{c}$	$\frac{1}{e}$				$-\frac{1}{2}$	
6				$\frac{1}{b}$	$\frac{1}{d}$	$\frac{1}{f}$		$-\frac{1}{2}$

Table 14



	M_1	M_3	M_5	M_7	M_2	M_4	M_6	M_8	ΣW_L	ΣW_U
1	A	$-C$			$-B$	D				
2		C	$-E$			$-D$	F			
3			E	$-G$			$-F$	H		
4	$2A(2I+J)+3I(I+J)$	$-2C(I+J)-3IJ$	$-2EI$		$3I(I+J)$	$-3IJ$				
5	$2AJ(J+K)$	$2C(2J+K)\times(I+J)+3IJ(J+K)$	$-2E(2J+I)\times(J+K)-3JK(I+J)$	$-2GJ(I+J)$		$3IJ(J+K)$	$-3JK(I+J)$			
6		$4CK$	$6E(J+K)+6JK$	$-4G(2K+J)-(J+K)(3K+2G)$			$6JK$	$-3K(J+K)$		
7	$\frac{2}{a}$	$\frac{2}{c}$	$\frac{2}{e}$	$\frac{1}{g}$					-1	
8					$\frac{2}{b}$	$\frac{2}{d}$	$\frac{2}{f}$	$\frac{1}{h}$		-1

systematically arranged and that the equations for a building with any number of bays may be written by inspection.

If a structure become symmetry about the center line of the building then the equations will be much simplified. The equations for various cases are given in **Tables 11～14.**

MOMENT EQUATIONS FOR THE ROOF.

The moment equations for the roof will be established by the same manner taking the portion of the building.

Take a structure as shown in **Fig. 12.** From the deflection moment eqations we have,

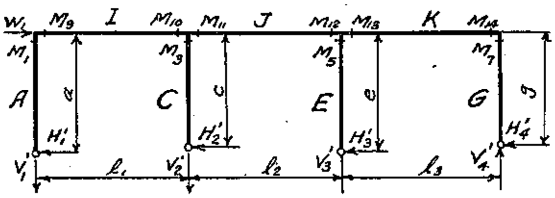


Fig. 12

$$AM_1=-\frac{3}{2}m_1 \qquad IM_9=-(2m_9+m_{10})$$

$$CM_3=-\frac{3}{2}m_3 \qquad IM_{10}=-(2m_{10}+m_9)$$

$$EM_5=-\frac{3}{2}m_5 \qquad JM_{11}=-(2m_{11}+m_{12}) \qquad KM_{13}=-(2m_{13}+m_{14})$$

$$GM_7=-\frac{3}{2}m_7 \qquad JM_{12}=-(2m_{12}+m_{11}) \qquad KM_{14}=-(2m_{14}+m_{13})$$

From the angular relations:

$$m_9=m_1-\varphi_1 \qquad m_{12}=m_{13}=m_5-\varphi_1$$

$$m_{10}=m_{11}=m_3-\varphi_1 \qquad m_{14}=m_7-\varphi_1$$

Substituting these angular relations into moment equations we have,

$$AM_1=-\frac{3}{2}m_1 \qquad IM_9=-(2m_1+m_3-3\varphi_1)$$

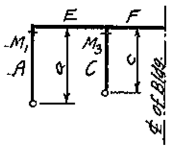
$$CM_3=-\frac{3}{2}m_3 \qquad IM_{10}=-(2m_3+m_1-3\varphi_1)$$

$$EM_5=-\frac{3}{2}m_5 \qquad JM_{11}=-(2m_3+m_5-3\varphi_1)$$

Table 15

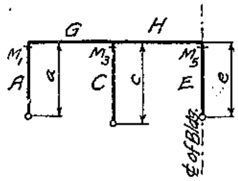
	M_1	M_3	M_5	M_7	M_9	M_{10}	M_{11}	M_{12}	M_{13}	M_{14}	ΣW_L
1	2A	-2C			-3I	3I					
2	2A		-2E			-3I	3J				
3		2C	-2E				-3J	3J			
4		2C		-2G				-3J	3K		
5			2E	-2G					-3K	3K	
6	1				1						
7		1				1	1				
8			1					1	1		
9				1						1	
10	2A(2I+J) +3I(I+J)	-2C(I+J) - 3IJ	-2EI		Eliminating M_9, M_{10}, M_{11}, M_{12}, M_{13} and M_{14}.						
11	2AJ(J+K)	2C(2J+K)(I+J) +3IJ(J+K)	-2E(2J+I)(J+K) -3KJ(I+J)	-2GJ(I+J)							
12		2CK	2E(J+K) +3KJ	-2G(2K+J) -3K(J+K)							
13	$\frac{1}{a}$	$\frac{1}{c}$	$\frac{1}{e}$	$\frac{1}{g}$							-1

Table 16



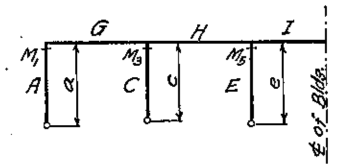
	M_1	M_3	ΣW_L
1	2A(2E+F) +3E(E+F)	-2C(2E+F) -3EF	
2	$\frac{1}{a}$	$\frac{1}{c}$	$-\frac{1}{2}$

Table 17



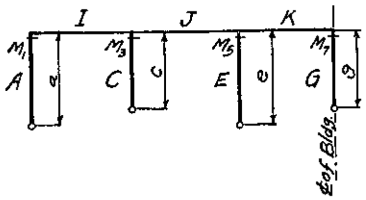
	M_1	M_3	M_5	ΣW_L
1	2A(2G+H) +3G(G+H)	-2C(G+H) -3GH	-2EG	
2	4AH	4C(G+H) +6GH	-4E(2G+H) -3H(G+H)	
3	$\frac{2}{a}$	$\frac{2}{c}$	$\frac{1}{e}$	-1

Table 18



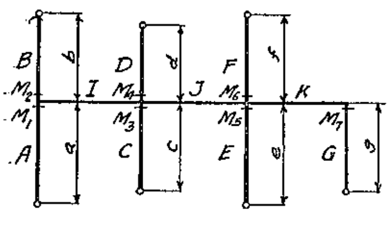
	M_1	M_3	M_5	ΣW_L
1	2A(2G+H) +3G(G+H)	-2C(G+H) - 3GH	-2EG	
2	2AH(H+I)	2C(2H+I)(G+H) +3GH(H+I)	-2E(2H+G)(H+I) -H(2E+3I)(G+H)	
3	$\frac{1}{a}$	$\frac{1}{c}$	$\frac{1}{e}$	$-\frac{1}{2}$

Table 19



	M_1	M_3	M_5	M_7	ΣW_L
1	2A(2I+J) +3I(I+J)	-2C (I+J) -3IJ	-2EI		
2	2AJ(J+K)	2C(2J+K)(I+J) +3IJ(J+K)	-2E(2J+I)(J+K) -3JK(I+J)	-2GJ(I+J)	
3		4CK	6E(J+K)+6JK	-4G(2K+J) (J+K)(3K+2G)	
4	$\frac{2}{a}$	$\frac{2}{c}$	$\frac{2}{e}$	$\frac{1}{g}$	-1

Table 20



	M_1	M_3	M_5	M_7	M_2	M_4	M_6	ΣW_L	ΣW_U
1	A	-C			-B	D			
2		C	-E			-D	F		
3	2A(2I+J) +3I(I+J)	-2C(I+J) -3IJ	-2EI		3I(I+J)	-3IJ			
4	2AJ(J+K)	2C(2J+K) × (I+J) +3IJ(J+K)	-2E(2J+I) × (J+K) -3KJ(I+J)	-2GJ(I+J)		3IJ(J+K)	-3KJ(I+J)		
5		2CK	2E(J+K) +3JK	-2G(2K+J) -3K(J+K)			3KJ		
6	$\frac{1}{a}$	$\frac{1}{c}$	$\frac{1}{e}$	$\frac{1}{g}$				-1	
7					$\frac{1}{b}$	$\frac{1}{d}$	$\frac{1}{f}$		-1

$$GM_7 = -\frac{3}{2}m_7 \qquad JM_{12} = -(2m_5 + m_3 - 3\varphi_1)$$

$$KM_{13} = -(2m_5 + m_7 - 3\varphi_1)$$

$$KM_{14} = -(2m_7 + m_5 - 3\varphi_1)$$

Arrange the moment equations into tabulated form and eliminate "m" and "φ" by the same procedure as before.

If we compare equations (1-13) in **Table 15** with equations (15-32) in **Table 7** they are the same except those for the column moments above the floor line. Therefore, the equations a building with any number of bays may be written by inspection:

In a similar manner the equations for a structure with less columns above the floor line than floor below may be established.

MOMENT IN BEAM.

After obtaining the column moments the moments in the beams will be determined by the following method:

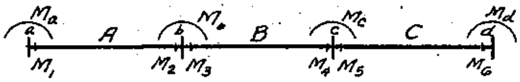


Fig. 13

Consider a series of beams on the same floor as a continuous beam with different member ratios in several sections such as A, B, and C as shown in **Fig. 13** and apply moments at each joint M_a, M_b, M_c and M_d which correspond to the sum of the column moments above and below the floor as the external moments. This, then, can be solved by establishing a set of Influence Equations. From Moment Equations:

$$AM_1 = -(2m_1 + m_2) \qquad BM_4 = -(2m_4 + m_3)$$

$$AM_2 = -(2m_2 + m_1) \qquad CM_5 = -(2m_5 + m_6)$$

$$BM_3 = -(2m_3 + m_4) \qquad CM_6 = -(2m_6 + m_5)$$

From the angular relations;

$$m_2 - \varphi_0 = m_3 - \varphi_0 \qquad m_4 - \varphi_0 = m_5 - \varphi_0$$

$$\varphi_0 = 0 \qquad \therefore \; m_2 = m_3,\; m_4 = m_5$$

Substituting these relations into moment equations we have,

$$AM_1 = -(2m_1 + m_2) \qquad BM_4 = -(2m_4 + m_2)$$

$$AM_2 = -(2m_2 + m_1) \qquad CM_5 = -(2m_4 + m_6)$$

$$BM_3 = -(2m_2 + m_4) \qquad CM_6 = -(2m_6 + m_4)$$

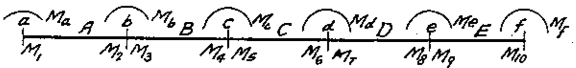
Arranging the moment equations into tabulated form for the elimination of angles "m" and "φ", we have.

Table 21

	M_1	M_2	M_3	M_4	M_5	M_6	m_1	m_2	m_4	m_6	M_a	M_b	M_c	M_d
1	A						2	1						
2		A					1	2						
3			B					2	1					
4				B				1	2					
5					C				2	1				
6						C			1	2				
7	A	-2A	2B	-2B			Eliminating							
8			B	-2B	2C	-C	"m"							
9	1						Apply ΣM=0				1			
10		1	1				around					1		
11				1	1		each joint						1	
12						1								1
13			2(A+B)	-B			Eliminating				-A	2A		
14			B	-2(B+C)			$M_1 M_2 M_5 M_6$						-2C	C

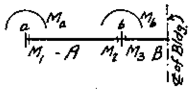
In the same way the equations for a structure with any number of spans will be established:

Table 22



		M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}	M_a	M_b	M_c	M_d	M_e	M_f
Elimination	1	A	-2A	2B	-B												
	2			B	-2B	2C	-C										
	3					C	-2C	2D	-D								
	4							D	-2D	2E	-E						
ΣM=0 at each jt.	5	1										1					
	6		1	1									1				
	7				1	1								1			
	8						1	1							1		
	9								1	1						1	
	10										1						1
Elim. $M_1 \cdots M_{10}$	11			2(A+B)		B						-A	2A	B			
	12			B		2(B+C)	-C							2B			
	13					-C	2(C+D)		D						2D		
	14						D		2(D+E)						D	2E	-E

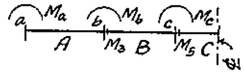
Table 23



M_3	M_a	M_b
2A+B	-A	2A

$M_2 = M_b - M_3$ or $M_2 = -\frac{AM_a+BM_b}{2A+B}$

Table 24



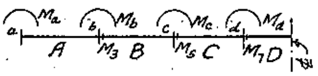
	M_3	M_5	M_a	M_b	M_c
1	2(A+B)	B	-A	2A	B
2	B	2B+C			2B

The equations for symmetrical structures will be much simplified. For instance in equations (13) and (14) in **Table 21** we have,

$M_3 = M_4$, $M_a = M_d$, $M_b = M_c$, $A = C$

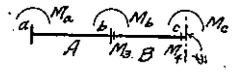
Therefore they will be simplified as **Table 23**. By the same manner the equations for more multiple bays will be established as shown in **Tables 24～28.**

Table 25



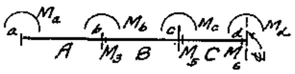
	M_3	M_5	M_7	M_a	M_b	M_c	M_d
1	2(A+B)	B		-A	2A	B	
2	B	2(B+C)	C			2B	C
3		C	2C+D				2C

Table 26



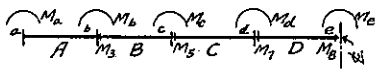
	M_3	M_4	M_a	M_b	M_c
1	2(A+B)	-B	-A	2A	
2		2			1

Table 27



	M_3	M_5	M_6	M_a	M_b	M_c	M_d
1	2(A+B)	B		-A	2A	B	
2	B	2(B+C)	-C			2B	
3			2				1

Table 28



	M_3	M_5	M_7	M_8	M_a	M_b	M_c	M_d	M_e
1	2(A+B)	B			-A	2A	B		
2	B	2(B+C)	-C				2B		
3		C	2(C+D)	-E				2C	
4				2					1

SHEARS IN BEAMS AND COLUMNS.

The magnitude of in a beam or in a column are the same throughout the member under consideration because there is no intervening load applied on it. The magnitude of these shears may be obtained by taking the moment about one end of the member.

$$V = -\left(\frac{M_R + M_L}{l}\right)$$

Where V is the shear in a member.

M_R and M_L are the moments at each end of the member under cosideration.

l is the length of the member.

VERTICAL LOAD IN COLUMNS DUE TO HORIZONTAL LOADS.

(1) **For Exterior Columns.**

Refering to **Fig. 9** pass the sections around joints *a* and *b*.

At joint *a* we have shear, V_1 of beam *ac*, which will be the vertic·l load on the column due to wind. In the colum V_1' is the downward force as the reaction.

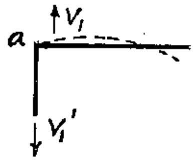


Fig. 14

At joint *b* we have V_1' as the upward force in the upper column and shear in the beam, *bd*, V_2 is also upward force.

Therefore we have downward force $V_2' = V_1' + V_2 = V_1 + V_2$ in the lower column as the reaction.

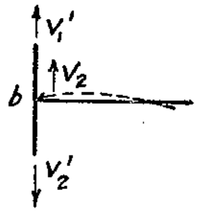


Fig. 15

(2) **For Interior Columns.**

Pass the sections around the joints *c* and *d* same as before.

At joint *c* we have shears, V_1, downward, from beam *ac* and V_3 upward, from beam *ce*. The algebraic sum of these shears is the vertical load on the column below.

$$V_3' = V_3 - V_1$$

Whre V_3' is the dowuward force as the reaction.

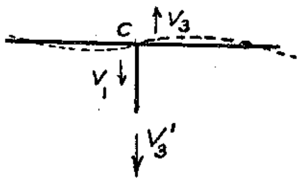


Fig. 16

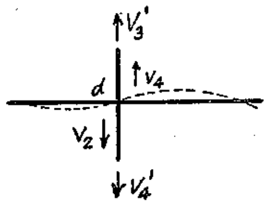


Fig. 17

At joint *d* we have V_3' as the upward force in the upper column and shears V_2, downward, from beam *bd* and V_4, upward, from beam *df*. The algebraic sum of these shears and force is the vertical load on the column below.

$$V_4' = V_3' + V_4 - V_2 = V_3 - V_1 + V_4 - V_2$$

Where V_4' is the downward force as the reaction.

From the above explanation it can be concluded that the vertical load due to horizontal loads at any section in a column is the sum of the shears of all beams joining to the column above the section being considered.

The direction of the vertical load is reverse on the leeward of the center line of the building.

CORRECTION OF MOMENTS IN MEMBERS.

So far the analysis of the problem has been carried under the assumption that the location of the contraflexure point of columns is known, but actually is unknown. Therefore the next problem is to locate the proper location of contraflexure point of colums. But it is laborious to locate exact points. Therefore we assume the contraflexure point as being in the mid-height of columns. Then we compute the column and beam moments by the equations already illustrated. But these moments are not true moments therefore a certain correction must be made by the following method.

Take part of the building as shown in **Fig. 18.** From the deflection moment equations,

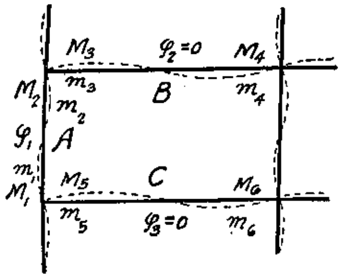


Fig. 18

$$BM_3=-(2m_3+m_4) \quad \cdots\cdots(1)$$

$$BM_4=-(2m_4+m_3) \quad \cdots\cdots(2)$$

2(1)−(2) $$2BM_3-BM_4=-3m_3$$

or $$-m_3=\frac{B(2M_3-M_4)}{3} \quad \cdots\cdots(3)$$

$$CM_5=-(2m_5+m_6) \quad \cdots\cdots(4)$$

$$CM_6=-(2m_6+m_5) \quad \cdots\cdots(5)$$

2(4)−(5) $$-m_5=\frac{C(2M_5-M_6)}{3} \quad \cdots\cdots(6)$$

$$AM_1=-(2m_1+m_2) \quad \cdots\cdots(7)$$

$$AM_2=-(2m_2+m_1) \quad \cdots\cdots(8)$$

(8)+(9) $$\frac{A(M_1+M_3)}{3}=-(m_1+m_2) \quad \cdots\cdots(9)$$

From the angular relations,

$$m_2-\varphi_1=m_3-\varphi_2, \qquad m_1-\varphi_1=m_5-\varphi_3$$

Since φ_2 and φ_3 are zero therefore

$$m_2-\varphi_1=m_3\cdots(10), \qquad m_1-\varphi_1=m_5\cdots(11), \qquad (10)-(11) \quad -(m_1-m_2)=m_3-m_5\cdots(12)$$

Substituting (3) and (6) into (12)

$$-(m_1-m_2)=\frac{C(2M_5-M_6)}{3}-\frac{B(2M_3-M_4)}{3} \quad \cdots\cdots(13)$$

From (9) $$-(m_1+m_2)=\frac{A(M_1+M_2)}{3} \quad \cdots\cdots(14)$$

[(13)+(14)]÷2 $$-m_1=\frac{C(2M_5-M_6)}{6}-\frac{B(2M_3-M_4)}{6}+\frac{A(M_1+M_2)}{6} \quad \cdots\cdots(15)$$

[(14)−(13)]÷2 $$-m_2=\frac{-C(2M_5-M_6)}{6}+\frac{B(2M_3-M_4)}{6}+\frac{A(M_1+M_2)}{6} \quad \cdots\cdots(16)$$

Let M_1^c and M_2^c are the corrected moments.

From the deflection moment equations,

$$AM_1^c=-(2m_1+m_2) \quad \cdots\cdots(17)$$

$$AM_2^c=-(2m_2+m_1) \quad \cdots\cdots(18)$$

Substituting (15) and (16) into (17) and (18) we have,

$$M_1^c=\frac{(M_1+M_2)}{2}-\frac{1}{6A}[B(2M_3-M_4)-C(2M_5-M_6)] \quad \cdots\cdots(19)$$

$$M_2^c=\frac{(M_1+M_2)}{2}+\frac{1}{6A}[B(2M_3-M_4)-C(2M_5-M_6)] \quad \cdots\cdots(20)$$

This relation is true for any bay.

Where M_1, M_2, M_3, M_4, M_5 and M_6 are the known quantities, having been computed by assuming contraflexure points as being in the mid-hight of columns, these values are slightly different from true values.

If M_1 and M_2 were true then $M_1=M_1^c$ and $M_2=M_2^c$ but M_1 and M_2 are not equal to M_1^c and M_2^c respectively, because the contraflexure points in columns are not true. But by comparing M_1 with M_1^c and M_2 with M_2^c we find how closely M_1 and M_2 come to true values. The above equations will give true and correct values by one correction if M_1, M_2, M_3, M_4, M_5 and M_6 were true. But they are different from the true values therefore the equations will not give true values.

The chracter of these equations using untrue moments gives opposite values from those obtained previously. Therefore the average of corrected and non corrected values will be the nearest to the true values.

After finding the closest values of moments the more correct contraflexure points in columns are located by following equations.

$$M_a = Hy, \qquad M_b = H(h-y)$$

$$M_a + M_b = Hh, \qquad \frac{M_a}{M_a+M_b} = \frac{Hy}{Hh}, \qquad y = \frac{M_a}{M_a+M_b}\cdot h$$

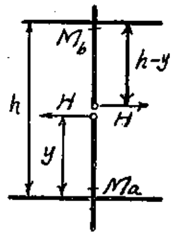


Fig. 19

Table 29

	Story Number	Section of Member			Moment of Iner. (I) in.⁴	Length of Mem. l in.	Member Ratio l/I
		Web Plate	4 Angles	Cover Plate			
Col. A.	1	17×7/8	8×8×7/8	2-18×1¼	6816	264	.03875
	2	DO.	DO.	DO.	6816	192	.02818
	3 and 4	DO.	DO.	2-18×1	5946	168	.02823
	5 & 6	DO.	DO.	2-18×11/16	4926	168	.03411
	7 & 8	DO.	DO.	2-18×7/16	4132	144	.03485
	9 & 10	17×15/16	8×8×15/16		3036	144	.04745
	11 & 12	17×7/8	8×8×13/16		2707	144	.05322
	13 & 14	17×3/4	8×6×13/16		2634	144	.05470
	15 & 16	17×½	8×6×5/8		2055	144	.07008
	17 & above	DO.	8×6×9/16		1891	144	.07619
Col. B.	1	17×7/8	8×8×7/8	2-18×1¼	6816	264	.03875
	2	DO.	DO.	DO.	6816	192	.02818
	3 & 4	DO.	DO.	2-18×1	5946	168	.02823
	5 & 6	DO.	DO.	2-18×3/4	5106	168	.03288
	7 & 8	DO.	DO.	2-18×½	4325	144	.03330
	9 & 10	DO.	DO.	2-18×5/16	3758	144	.03832
	11 & 12	DO.	DO.		2866	144	.05025
	13 & 14	DO.	8×6×13/16		2684	144	.05368
	15 & 16	17×5/8	8×6×5/8		2106	144	.06840
	17 & above	DO.	8×6×9/16		1896	144	.07596
Beams in Bay a	1	43×3/8	6×3½×3/8		8058	264	.03278
	2 & 3	36×3/8	DO.		5641	264	.04680
	4 & 5	DO.	5×3½×3/8		5161	264	.05110
	6	30×3/8	6×3½×3/8		3717	264	.07110
	7	DO.	5×3½×3/8		3387	264	.07800
	8 & above	24×3/8	DO.		2025	264	.13037
Beams in Bay b	1	42×3/8	6×3½×3/8		8058	216	.02681
	2	36×3/8	6×3½×7/16		6303	216	.03426
	3 & 4	DO.	6×3½×3/8		5641	216	.03829
	5	DO.	5×3½×3/8		5161	216	.04185
	6	30×3/8	6×3½×3/8		3717	216	.05812
	7	DO.	5×3½×3/8		3387	216	.06375
	8 & above	24×3/8	DO.		2025	216	.10666

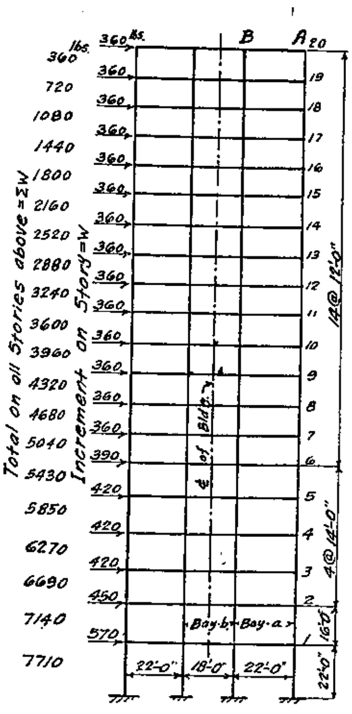


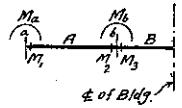
Fig. 20

Table 30

Fl.	A	C	B	D	E	F	2A(2E+F) +3E(E+F)	2C(2E+F) +3EF	3E(E+F)	3EF	1/a = 1/c	1/b = 1/d
13	.02735	.02684	.02735	.02684	.13037	.10666	.11277	.06143	.09269	.04171	.01389	.01389
12	.02661	.02513	.02735	.02684	.13037	.10666	.11224	.06017	.09269	.04171	.01389	.01389
11	.02661	.02513	.02661	.02513	.13037	.10666	.11224	.06017	.09269	.04171	.01389	.01389
10	.02373	.01916	.02661	.02513	.13037	.10666	.11011	.05569	.09269	.04171	.01389	.01389
9	.02373	.01916	.02373	.01916	.13037	.10666	.11011	.05569	.09269	.04171	.01389	.01389
8	.01743	.01665	.02373	.01916	.13037	.10666	.10549	.05395	.09269	.04171	.01389	.01389
7	.01743	.01665	.01743	.01665	.07800	.06375	.04084	.02225	.03318	.01492	.01389	.01389
6	.01706	.01644	.01743	.01665	.07110	.05812	.03442	.01900	.02758	.01242	.01191	.01389
5	.01706	.01644	.01706	.01644	.05110	.04185	.01918	.01116	.01426	.00642	.01191	.01191
4	.01412	.01412	.01706	.01644	.05110	.03829	.01768	.00984	.01371	.00587	.01191	.01191
3	.01412	.01412	.01412	.01412	.04680	.03829	.01567	.00910	.01195	.00538	.01191	.01191
2	.01409	.01409	.01412	.01412	.04680	.03426	.01499	.00842	.01138	.00481	.01042	.01191
1	.01938	.01938	.01409	.01409	.03278	.02681	.00944	.00622	.00586	.00264	.00758	.01042

Table 33

Beam Moments



Fl.	(1) A	(2) B	(3) $M_a = -M_1$	(4) M_b	(5) $M_a \frac{A}{2A+B}$	(6) $M_b \frac{B}{2A+B}$	(7) (5)+(6) = M_2	(8) $M_b - M_2 = M_3$
13	.13037	.10666	64.46	129.95	22.90	37.70	60.60	69.35
12	.13037	.10666	73.33	146.87	26.02	42.61	68.63	78.24
11	.13037	.10666	81.79	164.44	29.03	47.70	76.73	87.71
10	.13037	.10666	88.44	183.68	31.42	53.30	84.72	98.96
9	.13037	.10666	96.58	197.12	34.27	57.20	91.47	105.65
8	.13037	.10666	105.97	196.65	37.60	57.10	94.70	102.00
7	.07800	.06375	116.96	232.79	41.50	67.50	109.00	123.79
6	.07110	.05812	137.81	271.55	48.92	78.80	127.72	143.83
5	.05110	.04185	163.38	310.27	57.90	90.00	147.90	162.37
4	.05110	.03829	170.17	326.93	61.90	89.15	151.05	175.88
3	.04680	.03829	184.15	360.13	65.55	104.50	170.05	190.08
2	.04680	.03426	206.06	417.74	75.50	111.80	187.30	230.44
1	.03278	.02681	303.53	548.87	107.73	159.25	266.98	281.87

Table 31

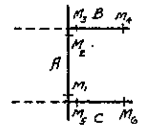
Computation Example 10th Fl. Multiply 100,000 by Coeff.							
	Eq.	M_1	M_3	M_2	M_4	$\Sigma W_L = 3960$	$\Sigma W_u = 3600.$
From Eqs.	1	2373	-1916	-2661	2513		
in Table	2	11011	-5569	9269	-4171		
No. 11	3	1389	-1389			-50000	
	4			1389	1389		-50000
(1)÷2513	5	0.9440	-0.7625	-.10590	1		
(2)÷4171	6	2.6410	-1.3350	2.2222	-1		
(5)+(6)	7	3.5850	-2.0975	1.1632	0		
(4)÷1389	8			1	1		-36.0000
(6)+(8)	9	2.6410	-1.3350	3.2222	0		-36.0000
(7)÷1.1632	10	3.0810	-1.8035	1			
(9)÷3.2222	11	0.8200	-0.4142	1			-11.175
(10)-(11)	12	2.2610	-1.3893	0			11.175
(12)÷1.3893	13	1.6280	-1.				8.044
(3)÷1389	14	1	1			-36.0000	
(13)+(14)	15	2.6280	0			-36.0000	8.044
(15)÷2.6280	16	1				-13.7000	3.061
(14)-(16)	17		1			-22.3000	-3.061
(16)x3.0810	18	3.0810				-42.420	9.430
(17)x1.8035	19		1.8035			-40.220	-5.522.
(18)-(19)	20	3.0810	-1.8035			- 2.200	14.952
(10)-(20)	21	0	0	1		2.200	-14.952
(8)-(21)	22				1	-2.200	-21.048
From(16)	$M_1 = 3960 \times 13.7 - 3600 \times 3.061 = 43,300$						
" (17)	$M_3 = 3960 \times 22.3 + 3600 \times 3.061 = 99,220$						
" (21)	$M_2 = 3960 \times -2.20 + 3600 \times 14.952 = 45,140$						
" (22)	$M_4 = 3960 \times 2.20 + 3600 \times 21.048 = 84,460$						

Table 32

Fl.	Mom't in Ex. Col. below Fl. (M_2) in k.f.	Mom't in In. Col. below Fl. (M_3) in k.f.	Mom't in Ex. Col. above Fl. (M_2) in k.f.	Mom't in In. Col. above Fl. (M_3) in k.f.
13	35.74	67.95	28.72	62.00
12	39.14	77.36	34.19	69.51
11	44.10	85.50	37.69	78.94
10	43.30	99.22	45.14	84.46
9	51.19	104.31	45.39	92.81
8	56.43	90.69	49.54	105.96
7	61.72	119.58	55.24	113.21
6	79.95	147.95	57.86	123.60
5	85.91	159.79	77.47	150.48
4	87.12	176.08	83.05	150.85
3	99.86	181.12	84.29	179.01
2	120.14	222.56	85.92	195.18
1	201.95	307.75	101.58	241.12
0			201.95	307.75

Table 34

Correction of Column Moments.



	Fl.	$2M_5$	M_6	C	$2M_3$	M_4	B	6A	$\frac{M_1+M_2}{2}$	M_2^c	M_1^c	M_1 without Corr.	M_1 Average	M_2 without Corr.	M_2 Average
Exterior Column	13	146.66	68.63	.13037	128.92	60.60	.13037	.3282	34.97	38.78	31.16	34.19	32.68	35.74	37.26
	12	163.58	76.73	.13037	146.66	68.63	.13037	.3194	38.42	42.05	34.79	37.69	36.24	39.14	40.59
	11	176.88	84.72	.13037	163.58	76.73	.13037	.3194	44.62	46.85	42.39	45.14	43.77	44.10	45.48
	10	193.16	91.47	.13037	176.88	84.72	.13037	.2847	44.35	48.63	40.07	45.39	42.73	43.30	45.97
	9	211.94	94.70	.13037	193.16	91.47	.13037	.2847	50.37	57.50	43.24	49.54	46.39	51.19	54.35
	8	233.92	109.00	.07800	211.94	94.70	.13037	.2091	55.84	29.36	82.32	55.24	68.78	56.43	42.91
	7	275.62	127.72	.07110	233.92	109.00	.07800	.2091	59.79	63.47	56.11	57.86	56.98	61.72	62.59
	6	326.76	147.90	.05110	275.62	127.72	.07110	.2047	78.70	72.16	85.24	77.41	81.36	79.95	76.06
	5	340.34	151.05	.05110	326.76	147.90	.05110	.2047	84.48	87.22	81.74	83.05	82.39	85.91	86.56
	4	368.30	170.05	.04680	340.34	151.05	.05110	.1694	85.70	83.16	88.24	84.29	87.27	87.12	85.14
	3	412.12	187.30	.04680	368.30	170.05	.04680	.1694	92.89	100.21	85.57	85.92	85.75	99.86	100.04
	2	607.06	266.98	.03278	412.12	187.30	.04680	.1690	110.86	95.25	125.47	101.58	113.52	120.14	107.65
	1	0	0	0	607.06	266.98	.03278	.2325	201.95	168.10	235.80	201.95	228.88	201.95	185.03
Interior Column	13	156.48	78.24	.10666	138.70	69.35	.10666	.3222	68.73	71.68	65.78	69.51	67.64	67.95	69.8[illegible]
	12	175.42	87.71	.10666	156.48	78.24	.10666	.3016	78.15	81.50	74.80	78.94	76.84	77.36	79.43
	11	197.92	98.96	.10666	175.42	87.71	.10666	.3016	84.98	88.96	81.00	84.46	82.73	85.50	87.23
	10	211.30	105.65	.10666	197.92	98.96	.10666	.2300	96.02	99.06	92.98	92.81	92.90	99.22	99.14
	9	204.00	102.00	.10666	211.30	105.65	.10666	.2300	105.14	103.62	106.66	105.96	106.31	104.31	103.96
	8	247.60	123.80	.06375	204.00	102.00	.10666	.2000	101.95	86.95	116.95	113.21	115.08	90.69	88.82
	7	287.66	143.83	.05812	247.60	123.80	.06375	.2000	121.59	123.94	119.24	123.60	121.42	119.58	121.76
	6	325.74	162.37	.04185	287.66	143.83	.05812	.1974	149.22	141.24	157.20	150.48	153.84	147.95	144.65
	5	351.76	175.88	.03829	325.74	162.37	.04185	.1974	155.32	155.60	155.04	150.85	152.95	159.79	157.65
	4	380.16	190.08	.03829	351.76	175.88	.03829	.1694	177.55	180.98	174.12	179.01	176.51	176.08	178.49
	3	460.88	230.44	.03426	380.16	190.08	.03829	.1694	188.15	191.58	184.72	195.18	189.95	181.12	186.35
	2	563.74	281.87	.02681	460.88	230.44	.03426	.1691	251.84	229.89	233.79	241.12	237.46	222.56	226.23
	1	0	0	0	563.74	281.87	.02681	.2323	307.50	275.60	340.00	307.75	323.87	307.75	291.38

And the new moments are computed by the equations illustrated on previous pages with new contraflexure points. These new moments will be more accurate than any previous ones.

But for practical use the average values is sufficient without making new computations.

NUMERICAL EXAMPLE.

For illustrating this principle the same example which was used by Messrs. M. W. Wilson and G. A. Maney in Illinois University bulletin No. 80, published 1915, will be used.

Determine the moments of members in a symmetrical Three-Span Twenty Story Bent as shown in **Fig. 20**. The bent resists a horizontal wind load of 30-lbs per sq. ft. on a vertical strip one foot wide.

For the illustration of the principle the computation of moments of members will be carried out upto 12th floor of the building. Use **Table 11** and **23**.

The accuracy of this method was compared with other methods in the last

Table 35

Comparison of the Methods with the Slope-Deflection Method.

		Moments at Top of Columns (in. K.P.)									Moments at Bottom of Columns (in. K.P.)								
	Story	Slope-Deflec. Method 100%	Writer's Method		Wilson & Maney Proposed M.		Ross Method		Fleming 1st Method		Slope-Deflec. Method 100%	Writer's Method		Wilson & Maney proposed M.		Ross Method		Fleming 1st. Method	
				Diff. in %		Diff. in %		Diff. in %		Diff. in %			Diff. in %		Diff. in %		Diff. in %		Diff. in %
Exterior Column	12	41.4	40.6	-2	38.7	-6	58.7	-6	38.2	-8	34.1	36.2	+6	38.7	+14	38.7	+14	38.2	+12
	11	47.5	45.5	-4	43.0	-9	43.0	-10	41.9	-12	41.5	43.8	+5	43.0	+4	42.6	+3	41.9	+1
	10	46.5	46.0	-1	45.6	-2	46.6	0	46.0	-1	42.4	42.7	+1	45.6	+8	46.3	+9	46.0	+8
	9	52.6	54.4	+3	49.8	-5	50.2	-4	51.4	-2	51.9	46.6	-11	49.8	-4	50.5	-2	51.4	-1
	8	44.4	42.9	-3	54.7	+25	55.0	+24	55.9	+26	67.7	68.8	+2	54.7	-19	56.2	-17	55.9	-7
	7	57.6	62.6	+9	60.3	+5	60.6	+5	59.2	+2	60.0	57.0	-5	60.3	+1	60.8	+1	59.2	-1
	6	75.0	76.1	+1	76.8	+2	76.7	+2	74.3	-1	83.2	81.4	-2	76.8	-8	78.4	-6	74.2	-10
	5	84.5	86.6	+2	85.6	+1	84.2	0	80.9	-4	82.8	82.4	-1	85.6	+3	81.8	-1	80.9	-2
	4	87.8	85.1	-3	87.1	-1	87.8	0	87.1	-1	87.8	87.3	-1	87.1	-1	90.3	+3	87.1	-1
	3	100.3	100.0	-1	96.8	-3	96.2	-3	91.5	-9	90.7	85.8	-4	96.8	+7	93.6	+3	91.5	+1
	2	113.0	107.7	-4	113.8	+1	114.0	+1	111.0	-2	107.5	113.5	-5	113.8	+6	123.5	+15	111.0	+3
	1	178.8	185.0	+3	191.5	+7	167.0	-6	166.9	-6	272.0	228.9	-15	191.5	-30	264.0	-3	166.9	-39
Interior Column	12	81.5	79.4	-2	77.9	-4	77.9	-4	79.1	-3	75.1	76.9	+2	77.9	+4	78.0	+4	79.1	+5
	11	89.0	87.2	-2	86.4	-3	86.5	-3	88.1	-1	81.2	82.7	+1	86.4	+7	97.2	+7	88.1	+9
	10	102.2	99.1	-3	96.3	-6	95.8	-6	96.8	-5	94.2	92.9	-1	96.3	+2	96.1	+2	96.8	+3
	9	104.2	104.0	0	105.5	+1	105.1	+1	104.5	0	103.3	106.3	+3	105.5	+2	104.7	+1	104.5	+1
	8	97.2	88.8	-9	114.0	+17	113.6	+17	113.0	+16	127.5	115.1	-10	114.0	-10	112.1	-12	113.0	-11
	7	122.0	121.8	0	120.7	-1	121.0	-1	122.0	0	124.0	121.4	-3	120.7	-3	120.6	-3	122.0	-2
	6	143.3	144.6	+1	151.8	+6	151.5	+6	153.0	+7	155.7	153.8	-1	151.8	-3	149.0	-4	153.0	-2
	5	162.3	157.6	-3	160.0	-2	161.3	-1	163.1	+1	162.0	153.0	-6	160.0	-1	163.5	+1	163.1	+1
	4	176.1	178.5	+1	175.0	-1	175.8	0	175.5	0	173.5	176.5	+2	175.0	+1	173.4	0	175.3	+1
	3	187.5	186.4	-1	184.1	-2	184.7	-1	187.6	0	182.0	190.0	+4	184.1	+1	187.5	+3	187.6	+3
	2	226.5	226.2	0	227.5	0	228.8	+1	228.6	+1	239.0	237.5	-1	227.5	-5	219.5	-8	222.6	-4
	1	251.0	291.4	+16	319.0	+27	262.5	÷4	343.2	+37	308.2	323.9	+5	319.0	+4	326.0	+6	343.2	+11

tables. In regards to accuracy there are several methods which have not taken the member ratio into consideration; hence the result of these methods is quite erroneous. The writer believe that the lack of consideration of member ratio is the cause of error and it is dangerous to use these methods because of the following reason.

Suppose we have members as

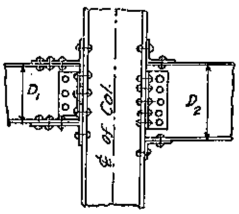


Fig. 21

shown in **Fig. 21**. Sometimes moments of both sides of the column are the same, sometimes the left hand side of the column is greater than the right hand side if the analysis is done by a method which does not cosider the member ratio of the building. Then from the standpoint of detailing the connection against wind moments it will be noticed that the left hand side connection is much stronger than the right hand side, which is utterly unbalanced connection. The number of rivets required on the flanges of the beam will be determined by the magnitude of moments and the depth of the beam assuming that the web connection of the beam take care of the vertical and horizontal loads in the beam.

Table 36

		End Moments of Beams (in. K.P.)								
	Story	Slope-Deflec. Method 100%	Writer's Method		Wilson & Maney Proposed Md.		Ross Method		Fleming 1st Method	
				Diff. in %		Diff. in %		Diff. in %		Diff. in %
At Left End in Exterior Bay	12	73.4	73.3	0	73.1	0	73.2	0	72.3	-1
	11	81.9	81.8	0	81.7	0	81.7	0	80.3	-2
	10	89.6	88.4	-1	88.6	-1	89.2	-1	87.1	-3
	9	95.0	96.6	+2	95.4	0	96.8	+2	96.8	+2
	8	95.8	106.0	+10	104.5	+9	105.5	+10	108.7	+13
	7	125.9	117.0	-7	115.0	-8	117.0	-7	114.5	-9
	6	135.0	137.8	+2	137.1	+1	137.5	+2	133.2	-1
	5	167.7	163.4	-2	162.4	-3	162.5	-3	154.8	-8
	4	170.5	170.2	0	172.7	+1	169.6	0	169.2	0
	3	187.1	184.2	-2	183.9	-2	186.5	0	178.2	-5
	2	203.0	206.1	+2	210.6	+4	215.0	+2	199.8	-1
	1	287.8	303.5	+9	305.3	+6	304.0	+6	279.3	-3
At Right End in Exterior Bay	12	68.6	68.6	0	68.5	0	68.8	0	72.3	+6
	11	76.6	76.7	0	77.0	0	76.8	0	80.3	+5
	10	85.0	84.7	0	85.7	+1	85.0	0	87.1	+2
	9	91.1	91.5	0	95.8	+5	92.8	+2	96.8	+12
	8	92.0	94.7	+1	103.5	+13	100.9	+10	108.7	+18
	7	117.0	109.0	-7	110.9	-5	109.2	-7	114.5	-2
	6	125.6	127.7	+2	128.5	+3	122.7	+2	133.2	+12
	5	152.0	147.9	-3	151.8	0	148.0	-3	154.8	+2
	4	153.6	151.1	-2	152.0	-1	153.6	0	169.2	+10
	3	171.7	170.1	0	173.1	+1	170.3	-1	178.2	+9
	2	184.0	187.3	+2	185.6	+1	187.6	+2	199.8	+8
	1	245.0	216.9	+9	274.5	+12	267.0	+9	279.3	+14
At Left End in Interior Bay	12	78.3	78.2	0	78.1	0	78.4	0	75.9	-3
	11	87.5	87.7	0	87.3	0	87.7	0	85.9	-2
	10	98.5	98.9	0	97.0	-1	98.2	0	97.5	-1
	9	106.8	105.7	-1	106.0	0	108.5	+2	104.3	-2
	8	107.8	102.0	-6	116.0	+8	117.5	+9	106.2	-4
	7	132.5	123.8	-6	123.8	-7	123.6	-7	121.6	-8
	6	141.5	143.8	+2	144.0	+2	144.1	0	140.8	0
	5	166.0	162.4	-2	160.0	-3	164.4	-2	164.4	-1
	4	184.4	175.9	-4	183.0	-1	185.5	+1	172.1	-6
	3	189.6	190.1	0	186.0	-2	187.6	-1	187.9	-1
	2	226.0	230.4	+2	226.0	0	228.5	+1	211.2	-7
	1	242.5	281.9	+14	272.0	+10	278.0	+12	295.0	+19

In **Table 35** and **36** Ross' Method was published in Am. Soc. of Civil Engineers proceedings May 1928 Part I.

Fleming's two other methods results are very inaccurate therefore are not shown in the table.

$$N=\frac{M}{DS}$$

Where N=number of rivets required.

M=end moment of the beam.
D=depth of the beam.
S=one rivet value.

If we take the member ratio consideration then these discrapancies do not happen.

SUGGESTION FOR PRACTICAL DESIGN.

When making a preliminary design it is impossible to use this method before determining the sizes of members therefore as the approximate method the writer suggests the following method.

(1) Distribute the total horizontal loads among columns as the shears in columns. The ratio of distribution of horizontal loads among columns will be as follows:

$$H_{EX}=\frac{\Sigma W}{2(n-1)}\cdots\cdots\text{Shear in Exterior columns.}$$

$$H_{IN}=\frac{\Sigma W}{(n-1)}\cdots\cdots\text{Shear in Interior columns.}$$

Where ΣW=Total horizontal loads above the floor being considered.
n=number of columns on the floor.

(2) Assume the contraflexure point as being in the mid-height of columns.

(3) Find the column moments of the floor above and below.

(4) Find the beam moments by the same procedure illustrated in "Moment in Beams" assuming that the moment of inertia of beams on the floor is constant, which means that the member ratio of beams is proportional to the span length of beams.

Illustrative Problem. Find the moments of columns and beams on 10-th floor in the previous numerical example.

10-th Fl. $\begin{cases} H_{EX}=\dfrac{3\,600}{2(4-1)}=600\text{ lbs.} \\ H_{IN}=\dfrac{3\,600}{4-1}=1\,200\text{ lbs.} \end{cases}$ 9-th Fl. $\begin{cases} H_{EX}=\dfrac{4\,220}{2(4-1)}=720\text{ lbs.} \\ H_{IN}=\dfrac{4\,220}{4-1}=1\,440\text{ lbs.} \end{cases}$

10-th Fl. Col. Moments $\begin{cases} 600\times\dfrac{12}{2}\times 12=43\,200\text{ in. lbs.}\cdots\cdots\text{Exterior Col.} \\ 1\,200\times\dfrac{12}{2}\times 12=86\,400\text{ in. lbs.}\cdots\cdots\text{Interior Col.} \end{cases}$

9-th Fl. Col. Moments. $\begin{cases} 720\times\dfrac{12}{2}\times 12=51\,840\text{ in. lbs.}\cdots\cdots\text{Exterior Col.} \\ 1\,440\times\dfrac{12}{2}\times 12=103\,680\text{ in. lbs.}\cdots\cdots\text{Interior Col.} \end{cases}$

From **Table 23.**

$$M_2=-\frac{AM_a+BM_b}{2A+B}$$

$$M_1=-M_a=-(43\,200+51\,840)=-95\,040\text{ in. lbs.}$$

$$M_b=86\,400+103\,680=190\,080\text{ in. lbs.}$$

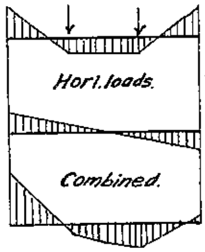
$$M_2=-\frac{22\times 95\,040+18\times 190\,080}{2\times 22+18}=-88\,850\text{ in. lbs.}$$

$M_3 = -(M_b + M_2) = -(190\,080 - 88\,850) = -91\,230$ in. lbs.

COMBINED MOMENTS.

To determine the sections of columns and girders, the axial loads and moment due to the vertical and horizontal loads must be combined.

Diagram 1.



The combined moments for any floor beam can be obtained from the moment diagrams.

Some building codes for steel structures assume that both ends of beam are free to rotate in order to simplify the analysis of the building, in cases where the wind loads may be neglected.

But when the wind load is taken into consideration both ends of the floor beam must be treated as being rigidly connected to the columns. It is not logical to assume that both ends of a beam are free to rotate for vertical loads and rigidly connected for horizontal wind loads. The assumption concerning the condition of both ends of a beam must be either one cannot be both, since the actual condition can only be one. If we assume that both ends are free to rotate for the vertical loads then there would be no end moments due to wind loads. Therefore it is more logical to assume that both ends of a beam rigidly connected to the columns. This may be clearly observed from the results of the experiments recorded in "Test of the Rigidity of Riveted Joints of Steel Structures" published in bulletin No. 104 University of Illinois, when we compare these with the kind of connections used in mordern tall building bnidings such as Empire State Building, Chysler Building, Hotel New Yorker in New York City, Liberty Building in Philadelphia. In the last two buidings the writer was personally engaged in doing detail designing of wind connection by the American Bridge Company.

The connection of beams and columns were made as shown in **Fig. 22.** Top and bottom connection lugs are cut out of beam and take care of the moment at joint. Web connection of the beam take care of vertical and horizontal reactions of the beam due to the vertical loads and horizontal load.

For reinforced concrete building, undoubtedly, it is always assumed that the connections are rigid although we have construction joint at the foot of the column.

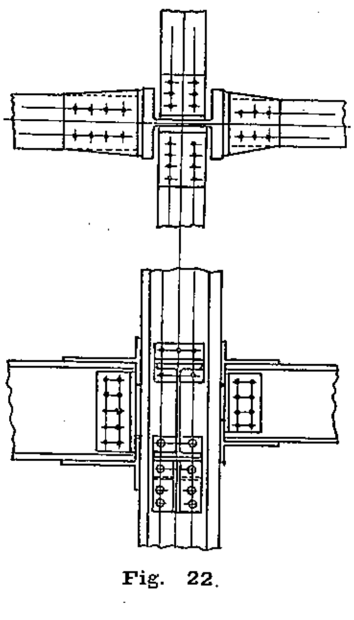


Fig. 22.