論 說 報 告

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THEORY OF INFLUENCE EQUATIONS.

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Synopsis.

In 1932 the writer presented in a very brief way the Theory of Influence Equations in this journal under the title of "The Derivation of Influence Equations of Statically Indeterminate Structures." Since that presentation of the principle was rather too general to insure a complete understanding of it and its practical value in application to curved structures. In the following pages the writer wishes to present the matter more in detail in order to clear any doubt about this theory and to illustrate with specific examples the degree of accuracy with which the principle can be applied.

INTRODUCTION.

Before going into the discussion the writer wants to express his appreciation for the sincere and inspiring discussion and comments made by Mr. Makizi Shono.

The main emphasis of the Influence Equation Theory lies in the close harmony between it and its practical application in the solution to statically indeterminate structures within a reasonable limit. The usefulness of the Influence Equation Theory depends upon its flexibility in practical use, its high degree of accuracy, and the ease with which it can be applied to the solution of practical problems.

As an illustration of the accuracy of the principle the writer took the same fixed arch which was built at the Experimental Station at the University of Illinois and on which a series of tests were made recently by Prof. W. M. Wilson who kindly contributed the experimental data to the writer.

Here the writer would like to express his deep appreciation for the Prof. Wilson's cooperation.

For the other illustration the writer took the ring which was also built at the Experimental Station at the University of Illinois and upon which a series of tests were made in 1908 by Prof. Talbot. The experimental results are given in bulletin No. 22 published by the same institution in 1908.

The writer has solved the problem of these structures with his Influence Equation Theory and made tables and diagrams which check with results obtained by experimental and theoretical methods. These tables and diagrams show the accuracy of the theory and prove conclusively that there is no danger in the application of this principle to practical problems.

DATA.

In the test a concentrated load, 2 000 lbs., was applied at point 10, 12, 14 and 16 of a reinforced concrete fixed arch whose properties are given in **Fig. 1** and **Table 1.** Modulus of elasticity of steel and concrete are as follows:

Steel $E_s = 29\,000\,000$ lbs. per sq. in.

Concrete $E_c = 3\,300\,000$ lbs. per sq. in.

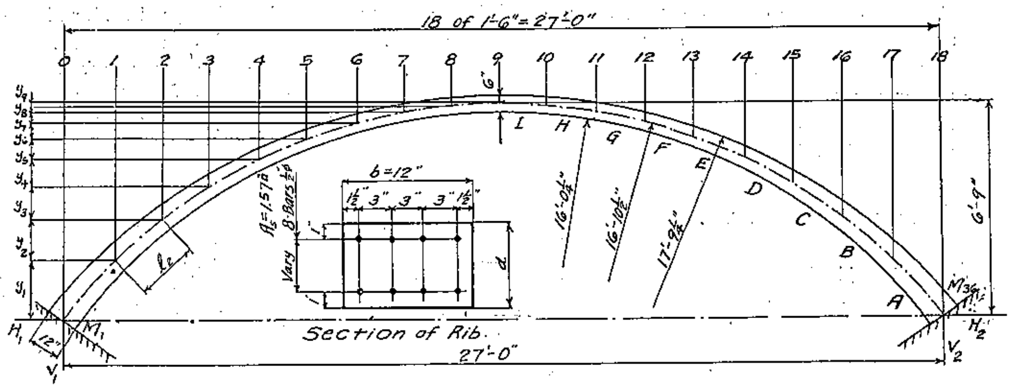

Fig. 1

APPLICATION OF THEORY.

Before going into the computation let it be understood that the arch was divided into 18 equal parts as shown in **Fig. 1.**

As was explained and illustrated in the writer's previous article published in the journal September 1932 that first of all it is necessary to establish Influence Equation by inspection as shown in **Table 2.** For establishing the equations it is necessary to refer to **Table 5** and **6** in the September 1932 journal.

Table 1

	x	y	ℓ	d	b	$(n-1)A_s$	Average I	$\frac{\ell}{I}$
1	18"	20.873"	27.562	11.119"	12"	$n=\frac{E_s}{E_c}=8.8$ $n-1=7.8$ $(n-1)A_s=7.8\times1.57=12.25\ \square''$	1831.98	A=.015045
2	"	16.152	24.185	10.346			1475.87	B=.016386
3	"	12.770	22.070	9.640			1198.31	C=.018417
4	"	10.106	20.643	8.980			973.97	D=.021215
5	"	7.866	19.644	8.352			789.76	E=.024872
6	"	5.900	18.942	7.747			636.14	F=.029776
7	"	4.106	18.462	7.158			507.18	G=.036400
8	"	2.425	18.162	6.576			398.36	H=.045538
9	"	0.802	18.018	6.000			306.75	I=.058738

In **Table 2** we have 21 unknowns and 21 conditional equations. It looks like a quite laborious piece of work to solve 21 simultaneous equations. However this number can be reduced to 3 equations for 3 unknowns at the left end, M_1, H_1 and V_1.

Replace the coefficients of moments by X, Y and Z terms as shown in **Table 3** then proceed the operation of elimination as shown in equations (22) to (39) for X and the same for Y and Z. Then sum up equation (22) to (39) and subtract equation

(1) for X and repeat for Y and Z then we have 3 equations as shown in **Table 4.**

Equations (40), (41) and (42) are the Influence Equations for the left and unknowns M_1, H_1 and V_1. These 3 equations can be written by inspection for any

Table 2

	M_1	M_2	M_4	M_6	M_8	M_{10}	M_{12}	M_{14}	M_{16}	M_{18}	M_{20}	M_{22}	M_{24}	M_{26}	M_{28}	M_{30}	M_{32}	M_{34}	M_{36}	H_1	V_1	A.S. GE	Load at Pt.			
																							10	12	14	16
1	$-A$	$A+B$	$B+C$	$C+D$	$D+E$	$E+F$	$F+G$	$G+H$	$H+I$	$2I$	$H+I$	$G+H$	$F+G$	$E+F$	$D+E$	$C+D$	$B+C$	$A+B$	A							
2	$-A\phi_1$	$A\phi_2+B\phi_3$	$B\phi_4+C\phi_5$	$C\phi_6+D\phi_7$	$D\phi_8+E\phi_9$	$E\phi_{10}+F\phi_{11}$	$F\phi_{12}+G\phi_{13}$	$G\phi_{14}+H\phi_{15}$	$H\phi_{16}+I\phi_{17}$	$2I\phi_{18}$	$H\phi_{16}+I\phi_{17}$	$G\phi_{14}+H\phi_{15}$	$F\phi_{12}+G\phi_{13}$	$E\phi_{10}+F\phi_{11}$	$D\phi_8+E\phi_9$	$C\phi_6+D\phi_7$	$B\phi_4+C\phi_5$	$A\phi_2+B\phi_3$	$A\phi_1$			$+\Delta_H$				
3	$-A\theta_1$	$A\theta_2+B\theta_3$	$B\theta_4+C\theta_5$	$C\theta_6+D\theta_7$	$D\theta_8+E\theta_9$	$E\theta_{10}+F\theta_{11}$	$F\theta_{12}+G\theta_{13}$	$G\theta_{14}+H\theta_{15}$	$H\theta_{16}+I\theta_{17}$	$I\theta_{18}+I\theta_{19}$	$I\theta_{20}+H\theta_{21}$	$H\theta_{22}+G\theta_{23}$	$G\theta_{24}+F\theta_{25}$	$F\theta_{26}+E\theta_{27}$	$E\theta_{28}+D\theta_{29}$	$D\theta_{30}+C\theta_{31}$	$C\theta_{32}+B\theta_{33}$	$B\theta_{34}+A\theta_{35}$	$A\theta_{36}$							
4																		-1	1	$-y_1$	x		x	x	x	x
5																	-1	1		$-y_2$	x		x	x	x	x
6																-1	1			$-y_3$	x		x	x	x	
7															-1	1				$-y_4$	x		x	x	x	
8														-1	1					$-y_5$	x		x	x		
9													-1	1						$-y_6$	x		x	x		
10												-1	1							$-y_7$	x		x			
11											-1	1								$-y_8$	x		x			
12										-1	1									$-y_9$	x					
13									-1	1										y_9	x					
14								-1	1											y_8	x					
15							-1	1												y_7	x					
16						-1	1													y_6	x					
17					-1	1														y_5	x					
18				-1	1															y_4	x					
19			-1	1																y_3	x					
20		-1	1																	y_2	x					
21	+1	1																		y_1	x					

Table 3

		M_1	M_2	M_4	M_6	M_8	M_{10}	M_{12}	M_{14}	M_{16}	M_{18}	M_{20}	M_{22}	M_{24}	M_{26}	M_{28}	M_{30}	M_{32}	M_{34}	M_{36}	H_1	V_1	A.S. GE	Load at point			
																								10	12	14	16
	1	$-X_s$	X_r	X_q	X_p	X_o	X_n	X_m	X_l	X_k	X_j	X_i	X_h	X_g	X_f	X_e	X_d	X_c	X_b	X_a							
	2	$-Y_s$	Y_r	Y_q	Y_p	Y_o	Y_n	Y_m	Y_l	Y_k	Y_j	Y_i	Y_h	Y_g	Y_f	Y_e	Y_d	Y_c	Y_b	Y_a			$+\Delta_H$				
	3	$-Z_s$	Z_r	Z_q	Z_p	Z_o	Z_n	Z_m	Z_l	Z_k	Z_j	Z_i	Z_h	Z_g	Z_f	Z_e	Z_d	Z_c	Z_b	Z_a							
	4																		-1	1	$-y_1$	x		x	x	x	x
	5																	-1	1		$-y_2$	x		x	x	x	x
	6																-1	1			$-y_3$	x		x	x	x	
	7															-1	1				$-y_4$	x		x	x	x	
	8														-1	1					$-y_5$	x		x	x		
	9													-1	1						$-y_6$	x		x	x		
	10												-1	1							$-y_7$	x		x			
	11											-1	1								$-y_8$	x		x			
	12										-1	1									$-y_9$	x					
	13									-1	1										y_9	x					
	14								-1	1											y_8	x					
	15							-1	1												y_7	x					
	16						-1	1													y_6	x					
	17					-1	1														y_5	x					
	18				-1	1															y_4	x					
	19			-1	1																y_3	x					
	20		-1	1																	y_2	x					
	21	+1	1																		y_1	x					
(4) X_a	22																		$-X_a$	X_a	$-y_1 X_a$	$x X_a$		$x X_a$	$x X_a$	$x X_a$	$x X_a$
(5) $\Sigma_a^b X$	23																	$-\Sigma_a^b X$	$\Sigma_a^b X$		$-y_2 \Sigma_a^b X$	$x \Sigma_a^b X$		$x \Sigma_a^b X$	$x \Sigma_a^b X$	$x \Sigma_a^b X$	$x \Sigma_a^b X$
(6) $\Sigma_a^c X$	24																$-\Sigma_a^c X$	$\Sigma_a^c X$			$-y_3 \Sigma_a^c X$	$x \Sigma_a^c X$		$x \Sigma_a^c X$	$x \Sigma_a^c X$	$x \Sigma_a^c X$	
(7) $\Sigma_a^d X$	25															$-\Sigma_a^d X$	$\Sigma_a^d X$				$-y_4 \Sigma_a^d X$	$x \Sigma_a^d X$		$x \Sigma_a^d X$	$x \Sigma_a^d X$	$x \Sigma_a^d X$	
(8) $\Sigma_a^e X$	26														$-\Sigma_a^e X$	$\Sigma_a^e X$					$-y_5 \Sigma_a^e X$	$x \Sigma_a^e X$		$x \Sigma_a^e X$	$x \Sigma_a^e X$		
(9) $\Sigma_a^f X$	27													$-\Sigma_a^f X$	$\Sigma_a^f X$						$-y_6 \Sigma_a^f X$	$x \Sigma_a^f X$		$x \Sigma_a^f X$	$x \Sigma_a^f X$		
(10) $\Sigma_a^g X$	28												$-\Sigma_a^g X$	$\Sigma_a^g X$							$-y_7 \Sigma_a^g X$	$x \Sigma_a^g X$		$x \Sigma_a^g X$			
(11) $\Sigma_a^h X$	29											$-\Sigma_a^h X$	$\Sigma_a^h X$								$-y_8 \Sigma_a^h X$	$x \Sigma_a^h X$		$x \Sigma_a^h X$			
(12) $\Sigma_a^i X$	30										$-\Sigma_a^i X$	$\Sigma_a^i X$									$-y_9 \Sigma_a^i X$	$x \Sigma_a^i X$					
(13) $\Sigma_a^j X$	31									$-\Sigma_a^j X$	$\Sigma_a^j X$										$y_9 \Sigma_a^j X$	$x \Sigma_a^j X$					
(14) $\Sigma_a^k X$	32								$-\Sigma_a^k X$	$\Sigma_a^k X$											$y_8 \Sigma_a^k X$	$x \Sigma_a^k X$					
(15) $\Sigma_a^l X$	33							$-\Sigma_a^l X$	$\Sigma_a^l X$												$y_7 \Sigma_a^l X$	$x \Sigma_a^l X$					
(16) $\Sigma_a^m X$	34						$-\Sigma_a^m X$	$\Sigma_a^m X$													$y_6 \Sigma_a^m X$	$x \Sigma_a^m X$					
(17) $\Sigma_a^n X$	35					$-\Sigma_a^n X$	$\Sigma_a^n X$														$y_5 \Sigma_a^n X$	$x \Sigma_a^n X$					
(18) $\Sigma_a^o X$	36				$-\Sigma_a^o X$	$\Sigma_a^o X$															$y_4 \Sigma_a^o X$	$x \Sigma_a^o X$					
(19) $\Sigma_a^p X$	37			$-\Sigma_a^p X$	$\Sigma_a^p X$																$y_3 \Sigma_a^p X$	$x \Sigma_a^p X$					
(20) $\Sigma_a^q X$	38		$-\Sigma_a^q X$	$\Sigma_a^q X$																	$y_2 \Sigma_a^q X$	$x \Sigma_a^q X$					
(21) $\Sigma_a^r X$	39	$+\Sigma_a^r X$	$\Sigma_a^r X$																		$y_1 \Sigma_a^r X$	$x \Sigma_a^r X$					

Table 4

		M_1	H_1	V_1	AS / GE	Load at Point			
						10	12	14	16
(22)+(23) + …… +(39) −(1)	40	$\sum_a^s X$	$y_1\Sigma_b^r X+y_2\Sigma_c^q X+y_3\Sigma_d^p X+ y_4\Sigma_e^o X+y_5\Sigma_f^n X+y_6\Sigma_g^m X+ y_7\Sigma_h^l X+y_8\Sigma_i^k+y_9 X_j$	$x(18X_a+17X_b+16X_c+15X_d+14X_e+13X_f +12X_g+11X_h+10X_i+9X_j+8X_k+7X_l+ 6X_m+5X_n+4X_o+3X_p+2X_q+X_r)$		$x(8X_a+7X_b+ 6X_c+5X_d+4X_e +3X_f+2X_g+X_h)$	$x(6X_a+5X_b +4X_c+3X_d +2X_e+X_f)$	$x(4X_a+3X_b +2X_c+X_d)$	$x(2X_a +X_b)$
In the same manner	41	$\sum_a^s Y$	$y_1\Sigma_b^r Y+y_2\Sigma_c^q Y+y_3\Sigma_d^p Y+ y_4\Sigma_e^o Y+y_5\Sigma_f^n Y+y_6\Sigma_g^m Y+$	$x(18Y_a+17Y_b+16Y_c+15Y_d+14Y_e+13Y_f +12Y_g+11Y_h+10Y_i+9Y_j+8Y_k+7Y_l+ 6Y_m+5Y_n+4Y_o+3Y_p+2Y_q+Y_r)$	$-\Delta_H$	$x(8Y_a+7Y_b+ 6Y_c+5Y_d+4Y_e +3Y_f+2Y_g+Y_h)$	$x(6Y_a+5Y_b +4Y_c+3Y_d +2Y_e+Y_f)$	$x(4Y_a+3Y_b +2Y_c+Y_d)$	$x(2Y_a +Y_b)$
In the same manner	42	$\sum_a^s Z$	$y_1\Sigma_b^r Z+y_2\Sigma_c^q Z+y_3\Sigma_d^p Z+ y_4\Sigma_e^o Z+y_5\Sigma_f^n Z+y_6\Sigma_g^m Z+ y_7\Sigma_h^l Z+y_8\Sigma_i^k Z+y_9 Z_j$	$x(18Z_a+17Z_b+16Z_c+15Z_d+14Z_e+13Z_f +12Z_g+11Z_h+10Z_i+9Z_j+8Z_k+7Z_l+ 6Z_m+5Z_n+4Z_o+3Z_p+2Z_q+Z_r)$		$x(8Z_a+7Z_b+ 6Z_c+5Z_d+4Z_e +3Z_f+2Z_g+Z_h)$	$x(6Z_a+5Z_b +4Z_c+3Z_d +2Z_e+Z_f)$	$x(4Z_a+3Z_b +2Z_c+Z_d)$	$x(2Z_a +Z_b)$

number of segments made.

In this numerical example, first of all, we have to obtain the individual value of ϕ, θ, X, Y and Z. The values of ϕ and θ will be determined by general equations given in September 1932 journal.

$$\phi_{(2n-1)}=3\sum_1{}^{(n-1)}y\pm y_n \qquad \phi_{2n}=3\sum_1{}^{(n-1)}y\pm 2y_n$$

$$\theta_{(2n-1)}=3\sum_1{}^{(n-1)}x\pm x_n \qquad \theta_{2n}=3\sum_1{}^{(n-1)}x\pm 2x_n$$

After finding the values of ϕ and θ determine the individual values of X, Y and Z refering to **Table 2** and **3**.

All values of ϕ, θ, X, Y and Z are given in **Table 5** and **6** for this numerical example.

Table 5

ϕ		θ	
1	$y_1 = 20,873$	$\theta_1 = x$	$\theta_{19} = 28x$
2	$2y_1 = 41.746$	$\theta_2 = 2x$	$\theta_{20} = 29x$
3	$3y_1+y_2 = 78.771$	$\theta_3 = 4x$	$\theta_{21} = 31x$
4	$3y_1+2y_2 = 94.923$	$\theta_4 = 5x$	$\theta_{22} = 32x$
5	$3\Sigma_1^2 y+y_3 = 123.845$	$\theta_5 = 7x$	$\theta_{23} = 34x$
6	$3\Sigma_1^2 y+2y_3 = 136.615$	$\theta_6 = 8x$	$\theta_{24} = 35x$
7	$3\Sigma_1^3 y+y_4 = 159.491$	$\theta_7 = 10x$	$\theta_{25} = 37x$
8	$3\Sigma_1^3 y+2y_4 = 169.527$	$\theta_8 = 11x$	$\theta_{26} = 38x$
9	$3\Sigma_1^4 y+y_5 = 187.569$	$\theta_9 = 13x$	$\theta_{27} = 40x$
10	$3\Sigma_1^4 y+2y_5 = 195.435$	$\theta_{10} = 14x$	$\theta_{28} = 41x$
11	$3\Sigma_1^5 y+y_6 = 209.201$	$\theta_{11} = 16x$	$\theta_{29} = 43x$
12	$3\Sigma_1^5 y+2y_6 = 215.101$	$\theta_{12} = 17x$	$\theta_{30} = 44x$
13	$3\Sigma_1^6 y+y_7 = 225.107$	$\theta_{13} = 19x$	$\theta_{31} = 46x$
14	$3\Sigma_1^6 y+2y_7 = 229.213$	$\theta_{14} = 20x$	$\theta_{32} = 47x$
15	$3\Sigma_1^7 y+y_8 = 235.744$	$\theta_{15} = 22x$	$\theta_{33} = 49x$
16	$3\Sigma_1^7 y+2y_8 = 238.169$	$\theta_{16} = 23x$	$\theta_{34} = 50x$
17	$3\Sigma_1^8 y+y_9 = 241.396$	$\theta_{17} = 25x$	$\theta_{35} = 52x$
18	$3\Sigma_1^8 y+2y_9 = 242.198$	$\theta_{18} = 26x$	$\theta_{36} = 53x$

Table 6

	X	Y	Z
a	.015045	.314034	.797385
b	.031431	1.918811	1.601640
c	.034803	3.836261	1.668513
d	.039632	5.899639	1.780642
e	.046087	8.263216	1.931997
f	.054648	11.090028	2.126368
g	.066176	14.598742	2.375712
h	.081988	19.090450	2.696416
i	.104326	25.031691	3.116630
j	.117476	28.452452	3.171852
k	.104326	25.031691	2.516974
l	.081988	19.090450	1.730936
m	.066176	14.598742	1.197792
n	.054648	11.090028	.824624
o	.046087	8.263216	.556701
p	.039632	5.899639	.359486
q	.034803	3.836261	.210849
r	.031431	1.918811	.095634
s	.015045	.314034	.015045

For purposes of simplification we can divide equations (40), (41) and (42) by $x=18''$ before or after the substitution of numerical values into those equations.

Now we substitute the numerical values into equations (40), (41) and (42) then we have 3 equations as shown in (1), (2) and (3) in **Table 7** Solving these 3 equations we have influence values of M_1, H_1 and V_1.

Table 7

	M_1	H_1	V_1	Arch Short. $6E\Delta H/x$	Load at point 10	12	14	16
1	.0592082	3.862007	9.591732	- -	1.309987	.652355	.263657	.061521
2	11.585455	826.9284	1876.8418	−1	183.07059	72.13868	20.58473	2.546879
3	1.598622	104.1469	314.94427		58.05978	30.79885	13.11213	3.196410
4	1			.92443	23.97448	24.01914	15.68642	5.34275
5		1		−.014078	−1.028240	−.779133	−.435221	−.133219
6			1	0	0.40541	0.23559	0.10708	0.02743

In this experimental example a load 2 000 lbs. was applied at point 10, 12, 14 and 16 therefore the influence values of those M_1, H_1 and V_1 will be multiplied by 2 000 lbs. Let it be understood that the equations in the tables are only the left hand side of equations and that the right hand side is equal to zero which is omitted, i.e. in equation (4) **Table 7** when the load is at point 10,

$$M_1+23.9744\,8\times2\,000=0 \qquad V_1+0.4054\,1\times2\,000=0$$
$$M_1=47\,949''\# \qquad V_1=-811\#$$
$$H_1-1.0282\,40\times2\,000=0$$
$$H_1=+2\,056\#$$

CONVENTIONAL SIGNS.

In the auther's previous article his statement concerning conventional signs may have been misunderstood therefore it will be reiterated here in a different way.

The clockwise moment will be assumed as positive and it will be also assumed that all lever arms of moments measured from the force toward the right, or upward, are positive. It follows therefore that forces acting upward or toward the left should be considered as positive, in order to be consistent with the assumed direction for positive moments.

And it is always necessary to note what portion of the member is being considered and that portion under consideration must always be specified and indicated by subscripts, M_1, M_2, M_3 etc.

Now M_1, H_1 and V_1 are the moment and forces at the left end of the member therefore when we consider the reactions at the left springing the signs will be reversed because of $\Sigma M=0,\ \Sigma H=0,\ \Sigma V=0$,

$$M_L=-M_1=+47\,949''\#,\quad H_L=-H_1=-2\,056\#,\quad V_L=-V_1=+811\#$$

in other words M_1 is clockwise, H_1 is acting toward the right and V_1 is acting upward.

ARCH SHORTENING.

In this experiment the effect subject to temperature change was disregarded because there was practically no change in temperature before and after load was applied. Therefore only arch shortening will be considered.

In equations (40), (41) and (42) we have the term $6E\Delta_H/x$ which will answer the question of arch shortening.

$$\Delta_H=\frac{SL}{AE},\quad \frac{6E\Delta_H}{x}=\frac{6E}{x}\cdot\frac{SL}{AE}=\frac{6SL}{Ax}=6nf$$

Where E = Modulus of elasticity.

S = Stress.

A = Cross section area of arch rib under consideration.

L = Span.

x = Horizontal distance of each segment.

n = Number of segments, in this case 18.

f = Average unit stress.

The average unit stress will be tabulated as follows considering five different sections.

Arch Shortening effect,

$M_1=-0.9244\,3\times 6nf=-0.9244\,3\times 6\times 18 \times f=-99.75f$

$H_1=0.0140\,78\times 6nf=1.52f$

$V_1=0$

Table 8

Section	f #/□" Whe Load at Pt. 10	12	14	16
0	12.03	8.82	4.10	0.13
4	18.30	13.46	7.34	2.21
9	24.40	18.50	10.33	3.16
14	19.56	17.00	12.58	7.96
18	13.85	13.42	11.46	10.20
Total	88.14	71.20	45.81	23.66
Aver. f.	17.60	14.25	9.17	4.13

Table 9

		Load at point 10	12	14	16
M_1		47949	48038	31373	10686
	Ar. Short.	1755	1420	917	472
	Total	49704"#	49458"#	32290"#	11158"#
H_1		2056	1558	870	266
	Ar. Short.	26	21	14	7
	Total	2082 #	1579 #	884 #	273 #
V_1		811 #	471 #	214 #	54 #

Table 10

	Experiment 100%	Load at point 10	12	14	16	When half Arch loaded
M_L	Experi't	49596"#	52233"#	35205"#	10915"#	147949"#
	Elastic Theo.	51022	52400	33216	10322	146960
	Diff. in %	3.0	0.4	−5.7	−5.5	−0.7
	Inf. Eq. The.	49704	49458	32290	11158	142610
	Diff. in %	0.4	−5.2	−8.2	1.0	−3.6
H_L	Experi't	2055 #	1567 #	878 #	251 #	4751 #
	Elastic Theo.	2069	1578	852	241	4740
	Diff. in %	0.7	0.8	−3.0	−4.0	−0.2
	Inf. Eq. The.	2082	1579	884	273	4818
	Diff. in %	1.3	0.9	0.8	8.8	1.6
V_L	Experi't	801 #	442 #	184 #	45 #	1472 #
	Elastic The.	803	450	190	44	1487
	Diff. in %	0.3	2.0	3.3	−2.0	1.02
	Inf. Eq. The.	811	471	214	54	1550
	Diff. in %	1.2	6.1	16.0	20.0	5.3
M Pt. 4	Experi't	−15830"#	−9747"#	−4145"#	−885"#	−30605"#
	Elastic The.	−15178	−9800	−4154	−948	−30080
	Diff. in %	−4.1	0.7	0.1	6.9	−2.0
	Inf. Eq. The.	−16496	−11182	−5260	−1312	−33250
	Diff. in %	8.1	15.0	27.0	48.0	8.5
M Crown	Experi't	13903	−3090	−6105	−2126	2582
	Elastic The.	13119	−2518	−5016	−2071	4298
	Diff. in %	−5.7	−18.5	−17.9	−2.6	65.0
	Inf. Eqs. The.	12404	−2192	−4630	−2222	3360
	Diff. in %	−11.0	−29.0	−25.0	5.0	24.0
M Pt. 14	Experi't	−15600	−2167	28955	7215	18403
	Elastic The.	−14678	−800	30036	6972	21530
	Diff. in %	−6.0	−60.0	3.5	−3.3	17.0
	Inf. Eq. The.	−14496	1508	33320	8418	27670
	Diff. in %	−7.0	200.0	14.0	16.7	50.0
M Pt. 18	Experi't	21392	−20388	−49082	−46608	−94686
	Elastic The.	23556	−17930	−49474	−47350	−91198
	Diff. in %	10.5	−12.1	0.8	1.6	−3.7
	Inf. Eq. The.	20901	−16721	−44152	−44010	−83982
	Diff. in %	−2.0	−18.0	−10.0	−5.6	−12.0

FINAL RESULTS.

After finding M_L, H_L and V_L the moment at any point may be obtaind by passing a section and taking the moment of the left hand side of the section.

$$M_L+V_Lx-H_Ly-M_0=0$$

M_0 is the moment due to an external force acting on the left hand side of the section.

CONCLUSION.

If we examine **Table 10** we will find, in some cases, that the difference in percentage is quite large. However, from the practical point of view this is not our main concern but we are interested in what the difference is in the influence values as a whole, which is shown in diagrams **Diagram 1** and **2.** From these diagrams it can be judged whether this Influence Equation Theory is applicable to the solution of practical problem or not.

SECOND ILLUSTRATION. RING.

The usefulness of the Influence Equation Theory can also be seen

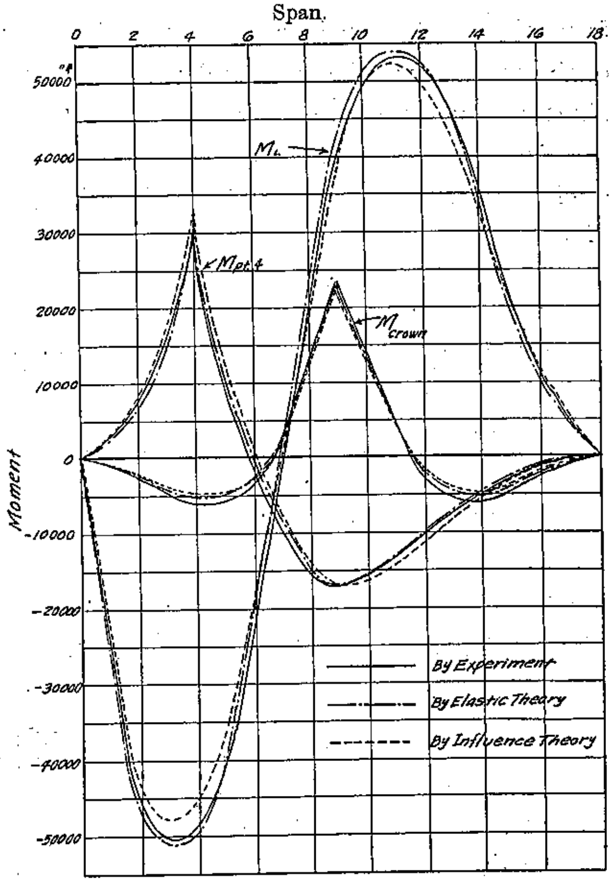

Diagram 1

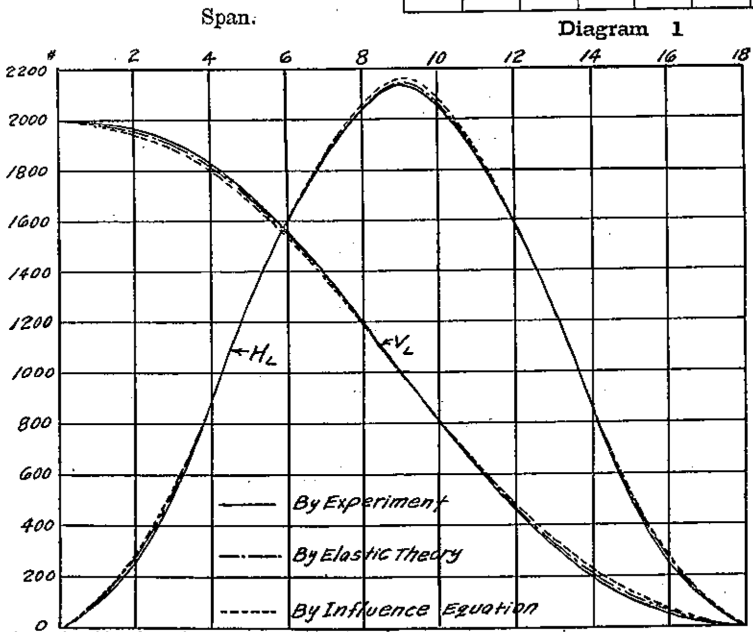

Diagram 2

from the illustration in connection with the analysis of a circular ring. The results of the analysis will be compared with experimental results.

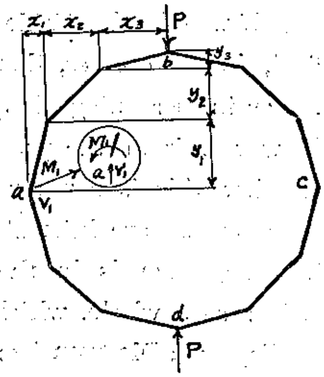

Fig. 2

Find the value of the moment, horizontal and vertical reactions at point a if concentrated loads P are applied on a circular ring having uniform thickness, as shown in **Fig. 2.**

Divide the ring into 12 equal segments then all the member ratio become the same let us say unity or 1.

Establish Influence Equations for this structure by inspection refering to the general equations of ring problems which were given in **Table 10** in September 1932 journal.

In this case the member ratio is the same therefore the Influence Equations will be much simplified and since loads P were applied at the joints therefore the constant terms, α and β, disappear from equations then we have 3 Relational Moment Equations as shown in **Table 11.**

Table 11

	M_1	M_2	M_4	M_6	M_8	M_{10}	M_{12}	M_{14}	M_{16}	M_{18}	M_{20}	M_{22}
1	-2	2	2	2	2	2	2	2	2	2	2	2
2	$-\phi_1-\phi'_{24}$	$\phi_2+\phi_3$	$\phi_4+\phi_5$	$\phi_6+\phi'_7$	$\phi_8+\phi_9$	$\phi_{10}+\phi_{11}$	$\phi_{12}+\phi_{13}$	$\phi_{14}+\phi_{15}$	$\phi_{16}+\phi_{17}$	$\phi_{18}+\phi_{19}$	$\phi_{20}+\phi_{21}$	$\phi_{22}+\phi'_{23}$
3	$-\theta_1-\theta'_{24}$	$\theta_2+\theta_3$	$\theta_4+\theta_5$	$\theta_6+\theta_7$	$\theta_8+\theta_9$	$\theta_{10}+\theta_{11}$	$\theta_{12}+\theta_{13}$	$\theta_{14}+\theta_{15}$	$\theta_{16}+\theta_{17}$	$\theta_{18}+\theta_{19}$	$\theta_{20}+\theta_{21}$	$\theta_{22}+\theta'_{23}$

The values of ϕ and θ are as shown in **Table 12.**

Substituting these values into Relational Moment Equations (2) and (3) and dividing equation (1) by 2 and establishing Influence Equations we have **Table 13.**

Solving these equations we obtain,

$$M_1 = -\frac{1}{12}(5x_1+3x_2+x_3)P$$

$$H_1 = 0$$

$$V_1 = \frac{1}{2}P$$

Table 12

ϕ	θ
$\phi_1 = -\phi_{13} = y_1$	$\theta_1 = \theta'_{24} = x_1$
$\phi_2 = -\phi_{14} = 2y_1$	$\theta_2 = \theta'_{23} = 2x_1$
$\phi_3 = -\phi_{15} = 3y_1+y_2$	$\theta_3 = \theta_{22} = 3x_1+x_2$
$\phi_4 = -\phi_{16} = 3y_1+2y_2$	$\theta_4 = \theta_{21} = 3x_1+2x_2$
$\phi_5 = -\phi_{17} = 3(y_1+y_2)+y_3$	$\theta_5 = \theta_{20} = 3(x_1+x_2)+x_3$
$\phi_6 = -\phi_{18} = 3(y_1+y_2)+2y_3$	$\theta_6 = \theta_{19} = 3(x_1+x_2)+2x_3$
$\phi_7 = -\phi_{19} = 3(y_1+y_2)+2y_3$	$\theta_7 = \theta_{18} = 3(x_1+x_2+x_3)+x_3$
$\phi_8 = -\phi_{20} = 3(y_1+y_2)+y_3$	$\theta_8 = \theta_{17} = 3(x_1+x_2+x_3)+2x_3$
$\phi_9 = -\phi_{21} = 3y_1+2y_2$	$\theta_9 = \theta_{16} = 3(x_1+x_2+2x_3)+x_2$
$\phi_{10} = -\phi_{22} = 3y_1+y_2$	$\theta_{10} = \theta_{15} = 3(x_1+x_2+2x_3)+2x_2$
$\phi_{11} = -\phi'_{23} = 2y_1$	$\theta_{11} = \theta_{14} = 3(x_1+2x_2+2x_3)+x_1$
$\phi_{12} = -\phi'_{24} = y_1$	$\theta_{12} = \theta_{13} = 3(x_1+2x_2+2x_3)+2x_1$

Next divide the ring into 16 equal segments and find H_1, V_1 and M_1 by the same procedure then we have,

$$H_1 = 0, \qquad V_1 = \frac{1}{2}P$$

$$M_1 = -\frac{1}{16}(7x_1+5x_2+3x_3+x_4)P$$

Comparing these two cases it will be noticed that there is similarity in the construc-

Table 13

	M_1	M_2	M_4	M_6	M_8	M_{10}	M_{12}	M_{14}	M_{16}	M_{18}	M_{20}	M_{22}	H_1	V_1	$\frac{M_0}{P}$
4	-1	1	1	1	1	1	1	1	1	1	1	1			
5	0	$5y_1+y_2$	$6y_1+5y_2+y_3$	$6(y_1+y_2)+4y_3$	$6y_1+5y_2+y_3$	$5y_1+y_2$	0	$5y_1+y_2$	$6y_1+5y_2+y_3$	$6(y_1+y_2)+4y_3$	$6y_1+5y_2+y_3$	$5y_1+y_2$			
6	$2x_1$	$5x_1+x_2$	$6x_1+5x_2+x_3$	$6(x_1+x_2+x_3)$	$6x_1+7x_2+11x_3$	$7x_1+11x_2+12x_3$	$10x_1+12x_2+12x_3$	$7x_1+11x_2+12x_3$	$6x_1+7x_2+11x_3$	$6(x_1+x_2+x_3)$	$6x_1+5x_2+x_3$	$5x_1+x_2$			
7											-1	1	y_2	$-x_2$	0
8										-1	1		y_3	$-x_3$	0
9									-1	1			$-y_3$	$-x_3$	x_3
10								-1	1				$-y_2$	$-x_2$	x_2
11							-1	1					$-y_1$	$-x_1$	x_1
12						-1	1						$-y_1$	x_1	$-x_1$
13					-1	1							$-y_2$	x_2	$-x_2$
14				-1	1								$-y_3$	x_3	$-x_3$
15			-1	1									y_3	x_3	0
16		-1	1										y_2	x_2	0
17	$+1$	1											y_1	x_1	0

tion of equations and so the equations may be written in the following general equations.

$$H_1=0, \qquad V_1=\frac{1}{2}P$$

$$M_1=-\frac{P}{n}\Big[\Big(\frac{n}{2}-1\Big)x_1+\Big(\frac{n}{2}-3\Big)x_2+\cdots\cdots+x_{\frac{n}{4}}\Big)\Big]$$

Where n is the number of segments. For example, let us divide the ring into 36 equal segments then

$$M_1=-\frac{P}{36}(17x_1+15x_2+13x_3+11x_4+9x_5+7x_6+5x_7+3x_8+x_9)$$

$$x=r \text{ vers } \psi$$

$$l=2r\sin\frac{\psi}{2}$$

Substituting the values in **Table 14** into above equation we have,

$$M_1=-\frac{P}{36}\times 6.57r=-0.1825Pr$$

As the reaction at a

$$M_a=-M_1=0.1825Pr$$

The moment at point b where load P is applied,

$$M_b=0.3175Pr$$

From the experiments

$$M_a=0.182Pr, \qquad M_b=0.318Pr$$

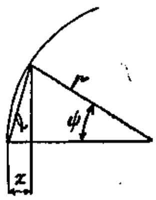

Fig. 3

Table 14

	x
1	$.01519r$
2	$.04512r$
3	$.07366r$
4	$.09999r$
5	$.12325r$
6	$.14279r$
7	$.15798r$
8	$.16837r$
9	$.17365r$

We can readily see from the illustrations given that this theory is not dangerous to use for practical applications.