論 說 報 告

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ON THE INVESTIGATION OF THE STRESS DISTRIBUTION IN A TUNNEL WITH THE AGAR-AGAR MODEL EXPERIMENTS

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Synopsis.

If we assume that the stresses in a circular tunnel can be treated theoretically as those in the gravitating elastic solid body with a horizontal hole, we are able to calculate them by the theory of the plane strains. Comparing these with the experimental results obtained by the agar-agar model, it was ascertained that the theoretical and experimental results coincide with very well. Therefore going one step further, we can obtain by the agar-agar models the stresses in the tunnel too complicated to be solved mathematically. Treating a horse-shoe shaped tunnel by such method, we have the conclusion that the horse-shoe shape is a very favorable figure to sustain the pressure in the gravitating elastic solid body just as a circle in the fluid. An explanation of the phenomenon of splitting of the side walls in the heading of the Simizu Tunnel is given by the same method.

1. Introduction.

It may be one of the most difficult problems in engineering science to estimate the actual stress distributions in a tunnel. If we dare to treat it mathematically, it is of usual to apply the theory of earth pressure, but it is evident that the theory of earth pressure is hardly aplicable to the case of a mountain tunnel running through rock or hard soil. The writer calculated the stress distribution in a circular tunnel without lining assuming the mountain as the heavy elastic body and compared them with those obtained from the deformation of the agar-agar model. They coincide fairly well. Though agar-agar seems to have a larger Poisson's ratio than the ordinary rocks which affects the stress distribution, it is highly probable that the agar-agar model experiments will serve to obtain the stress distributions in the tunnels of the more complicated shapes which are too difficult to be solved mathematically.

2. Elastic properties of agar-agar.

The specific gravity and the elastic constants of agar-agar vary with water contents &c. We mix usually 97% of water and 3% of dry agar-agar in weight and its specific gravity is 1.003. Cutting off a test piece as shown in **Fig. 1** from this, we tested the tensile strength of it. To reject the errors due to friction we pulled the test piece in oil whose specific gravity is about unity and measured the longitudinal elongation and the lateral contraction by the travelling microscope. One of these results is given in **Fig. 1.** From this, we see that agar-agar has comparatively good elastic property up to the breaking point, and its Poisson's ratio lies somewhere about between 1/2 and 1/4. The measuring apparatus is shown in **Fig. 2.**

3. Stresses in undisturbed mountain.

In the case of a sufficiently long tunnel through rock or hard soil we might treat the problem as the *plane strains* of the elastic body with the body force, i.e., we have the following equations as the fundamental relation :

$$\frac{\partial \widehat{xx}}{\partial x}+\frac{\partial \widehat{xy}}{\partial y}=\rho g, \qquad \frac{\partial \widehat{yy}}{\partial y}+\frac{\partial \widehat{xy}}{\partial x}=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where ρ is the density of elastic solid, i.e. ρg is its unit weight and the x-axis is taken positive in the upward direction ; the position of the origin being x_0 from the free surface.

The stresses in the undisturbed heavy elastic body are

$$\left.\begin{aligned} &\widehat{xx}=-\rho g(x-x_0), \qquad \widehat{yy}=\widehat{zz}=-\frac{\sigma}{1-\sigma}\rho g(x-x_0) \\ &\widehat{xy}=\widehat{zx}=\widehat{yz}=0 \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots(2)$$

where σ is the Poisson's ratio.

This was verified by the agar-agar experiment. Hot agar-agar was cast in the rectangular box having the size $70\times50\times8$ cm. When it is solidified, the box is erected so as its largest side stand vertically. If we prepare beforehand the surface of agar-agar with cross lines drawn with the hecto-graph ink, we can take the prints of it whenever we need. Comparing the prints taken before and after erection we obtain the deformations due to the body force (the gravity) acting in agar-agar.

If we measure the vertical strains at various depth and plot them in **Fig. 3**, we see that they lie on a straight line. If we deduce the vertical strains from the equations (2), we have

$$\frac{\partial u}{\partial x}=\frac{1}{E}\left\{(1+2\sigma)\widehat{xx}-(\widehat{yy}+\widehat{zz})\right\}=\frac{1+\sigma}{1-\sigma}\ \frac{2\sigma-1}{E}\rho g(x_0-x) \quad \ldots\ldots(3)$$

therefore the vertical strain is proportional to the depth from the free surface.

4. Stresses in a circular tunnel.

If a circular hole with a radius a is pierced horizontally at the depth of x_0 from the free surface the stresses are obtained by the following equations.

$$\left.\begin{aligned}\widehat{rr}&=k_1W+k_2V\\ \widehat{\theta\theta}&=l_1W+l_2V\\ \widehat{r\theta}&=m_1W+m_2V\end{aligned}\right\}\quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

where the origin of the polar coordinates is taken at the center of the hole and the initial line on the x-axis.

$W=\rho g x_0$, i.e., weight of unit column of the elastic body with the height equal to the depth of the centre of the hole from the free surface.

$V=\rho g a$, i.e., weight of unit column of the elastic body with the height equal to the radius of the hole.

k_1, k_2, l_1, l_2, m_1 and m_2 computated for the Poisson's ratio $\sigma=1/5$ (rock) are shown in my paper : On the Stresses Around a Horizontal Circular Hole in Gravitating Elastic Solid.[1]

The most important stress concerning the stability of the circular tunnel is the circumferential stress along the periphery of the hole :

$$\left.\begin{aligned}&\widehat{\theta\theta}=l_1W+l_2V\\ \text{where}\quad &l_1=-\left(1+\frac{\sigma}{1-\sigma}\right)+2\left(1-\frac{\sigma}{1-\sigma}\right)\cos 2\theta\\ &l_2=\frac{\sigma}{1-\sigma}\cos\theta-\frac{1}{4}\left(1-\frac{\sigma}{1-\sigma}\right)\cos 3\theta\ .\end{aligned}\right\}\quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

If the depth of the hole is sufficiently large compared with its diameter

(1) Journal of Civil Engineering Society, Japan. Vol. XV. No. 4. (1929).

we might neglect the second term of the right hand member to obtain the stresses near the hole :

$$\widehat{\theta\theta}=l_1\rho g x_0 \qquad (6)$$

In the agar-agar block in the erected state as already mentioned a horizontal circular hole with a diameter 7.5 cm. was pierced at the depth of 50 cm. from the top surface. This is shown in **Fig. 4.** Comparing the prints taken in the erected and layed down states (**Fig. 5** & **6**), we obtained the circumferential strains as shown in **Fig. 7.** The chain line of the inner circle in this figure is $\widehat{\theta\theta}$ computed by (6) with $\sigma=\frac{1}{2.3}$. This coincides very well with the experimental result obtained from the circumferential strains which is shown in the full line in the same figure.

The equation (6) shows that the circumferential stress $\widehat{\theta\theta}$ is independent of the diameter of the hole. This was verified by comparing the circumferential stresses around the two holes with the diameters 7.5 cm. and 5.0 cm. respectively pierced at the equal depth. This is shown in **Fig. 8,9** and **10.**

The equation (6) shows that the distribution of the circumferential stresses differs considerably by the materials having the different Poisson's ratio. This is shown in **Fig. 11.** Usually the mountain rocks seem to have a little smaller Poisson's ratio than that of agar-agar. Accordingly we can not say that our experimental results will not involve some errors therefrom.

5. Peripheral stresses in tunnels which are not circular.

If we assume, for the present, the Poisson's ratio of agar-agar and the mountain rock are of the same value, l_1 in (6) does not charge in two materials. We, therefore, obtain the circular circumferential stress in rock with the density $(\rho)_r$ and a hole placed at the depth $(x_0)_r$ from the top surface by the following equation:

$$(\widehat{\theta\theta})_r=\frac{(\rho)_r(x_0)_r}{(\rho)_a(x_0)_a}(\widehat{\theta\theta})_a \qquad (7)$$

where $(\widehat{\theta\theta})_a$ is the circular circumferential stress in agar-agar with the density $(\rho)_a$ and a hole placed at the depth $(x_0)_a$ from the top surface.

If we introduce the circular circumferential strain $(\theta\theta)_a$ in agar-agar in (7) we have

$$(\widehat{\theta\theta})_r = \alpha(\theta\theta)_a \quad \cdots\cdots (8)$$

where
$$\alpha = \frac{(\rho)_r(x_0)_r}{(\rho)_a(x_0)_a} E_a \quad \cdots\cdots (9)$$

The equation (8) is more practical to determine the circular circumferential stresses in tunnels by the agar-agar model experiment, as we always measure the strains instead of stresses in agar-agar model.

This relation might also be applied to the determination of the peripheral stresses in tunnels which are not circular.

To determine the peripheral stresses in tunnels which are not circular and consequently too complicated to be solved mathematically, we have no other way than to resort to the experimental method. For the first example we cut off a horse-shoe shaped hole with the height 10 cm. and the largest breadth 8 cm. at the depth of 50 cm. from the top surface of the agar-agar model, and measured the peripheral strains. This is shown in **Fig. 12, 13** and **14**. **Fig. 14** shows that the peripheral strains or stress distribution in a horse-shoe shaped tunnel is nearly equal along the periphery except for the corners of the side walls and the bottom. We may, therefore, conclude that a horse-shoe shape is a nearly ideal one for the mountain tunnel just as a circle is for the subaqueous tunnel. We have better distribution if we give the invert arch to the bottom and round off its connection with the side walls.

For the second example, we tried to obtain an explanation of the phenomenon of the splitting of the side walls taken place in the heading of the *Simizu Zuido*, the longest tunnel in Japan. The heading is of a rectangular form having the size of 5.5×2.75 m., the smaller dimension being the height. Where the splitting of the side walls occured is situated under the depth of 1 212 m. from the top surface. The rock is very hard and homogeneous diorite. Splitting took place only at the vertical walls. Sometimes large pieces having more than a square meter surface with the thickness 30 cm. were splitted with the bursting speed. The similar phenomenon occurs when we test the crushing strength of hard brittle materials such like cast iron and concrete.

The equation (8) can be used to deduce the peripheral stresses in the heading of the *Simizu* tunnel from the peripheral strains measured by the agar-agar model with a hole of the similar shape, provided the coefficient α is known.

The equation (9) will serve to obtain α, but it is more direct to compare the calculated values of the peripheral stresses in the circular tunnel in rock with the peripheral strains measured by the agar-agar model with a circular hole. Preparing two blocks of agar-agar with the same material and the same size $70\times50\times8$ cm., we cut off at the same depth of 50 cm. from the top surface a circular hole with the diameter of 7.5 cm. and a rectangular hole with the sides of 9×4.5 cm. (the smaller dimension being vertical), respectively.

Putting $x_0=1\,212$ m. and $\rho=2.764$ (density of the diorite actually measured at the *Simizu* tunnel) in the equation (6) we obtained the theoretical peripheral stresses in a circular tunnel which is imagined to be excavated at the same site with the existing *Simizu-Zuido*. Comparing these with the measured peripheral strains of the above mentioned agar-agar model with a circular hole (**Fig. 15**), we obtained $\alpha=55.6\times10^2$.

Now from the peripheral strains measured by the agar-agar model with a rectangular hole (**Fig. 16**) we obtain the peripheral stresses in the heading of the *Simizu* tunnel by the equation (8). These results are shown in **Fig. 17**. The largest peripheral stresses in a rectangular tunnel reach as high as 1 670 kg./cm^2. as are shown in the figure which are very near the actually measured crushing strength of the diorite 1 760 kg./cm^2.

In conclusion, the writer expresses with cordial thanks the indebtedness of the above experiments to Mssrs. Hatta and Kubota, the Research Bureau of the Japanese Government Railway.

Jan. 1930.

Tokyo, Japan.

Fig. 1

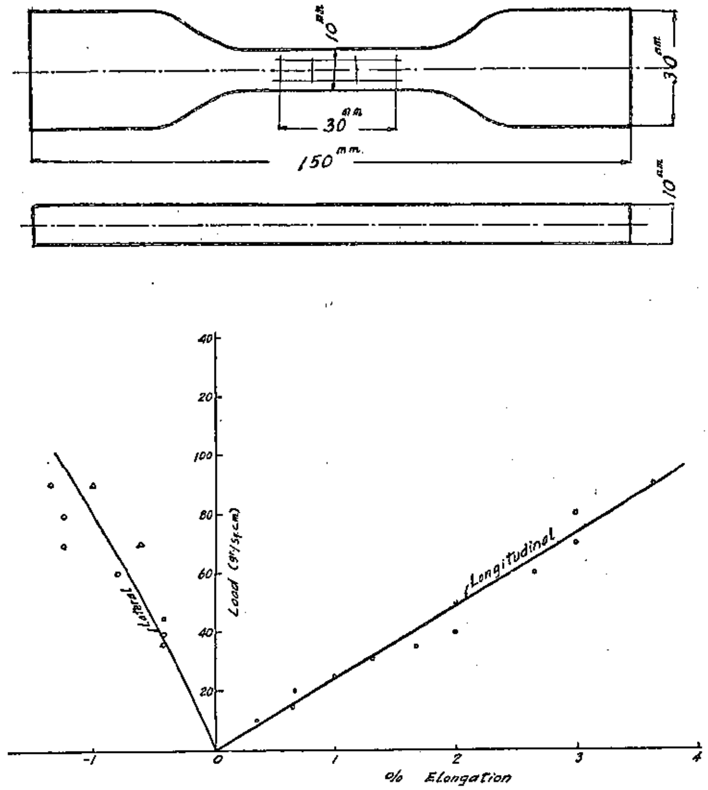
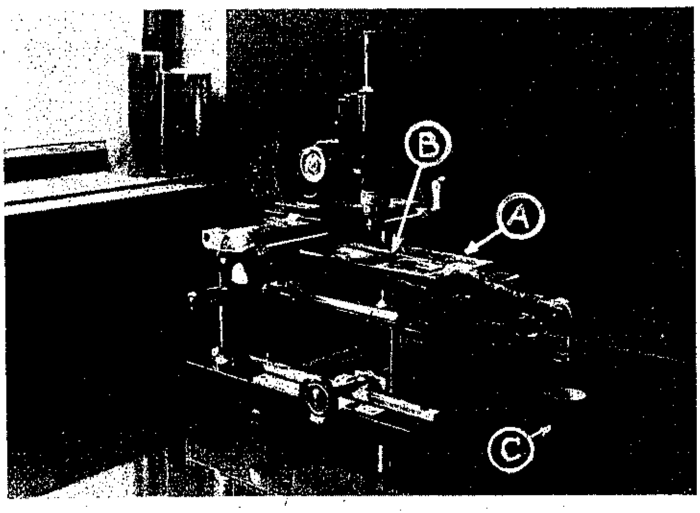


Fig. 2



A : Vessel containing oil
B : Test piece of agar-agar
C : Loading weight.

(土木學會誌第十六卷第三號附圖)

Fig. 3

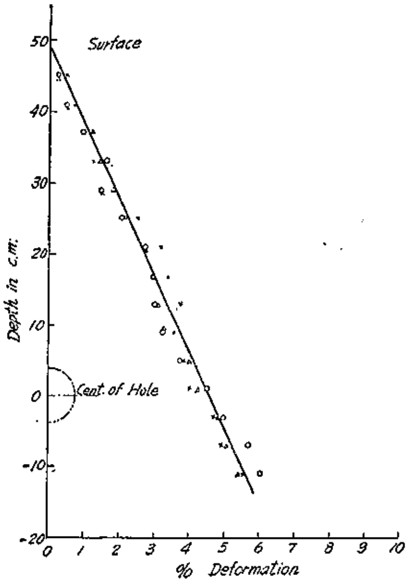

Surface
Depth in c.m.
Cent. of Hole
% Deformation
50
40
30
20
10
0
-10
-20
0
1
2
3
4
5
6
7
8
9
10

Fig. 4

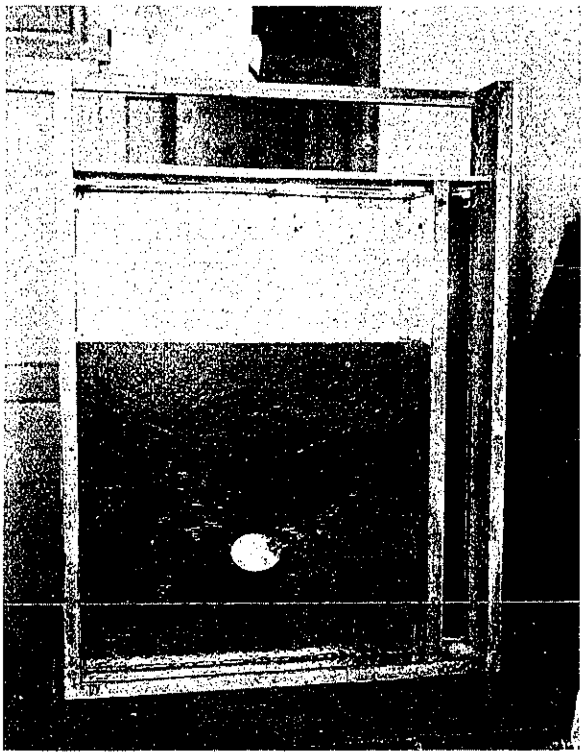


Fig. 5

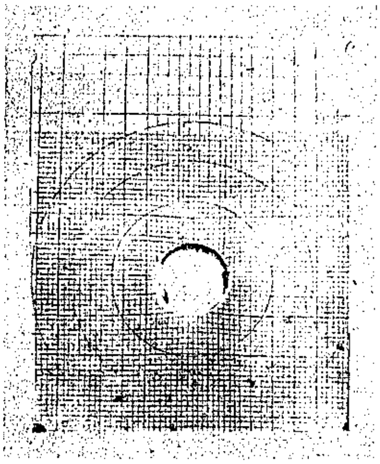


Fig. 6

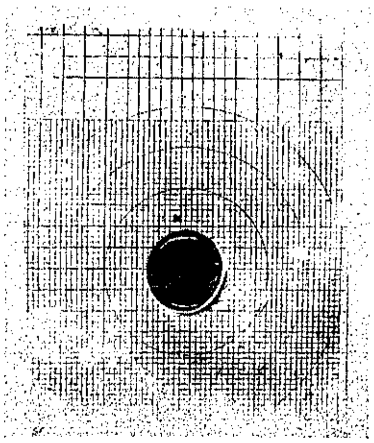


Fig. 7

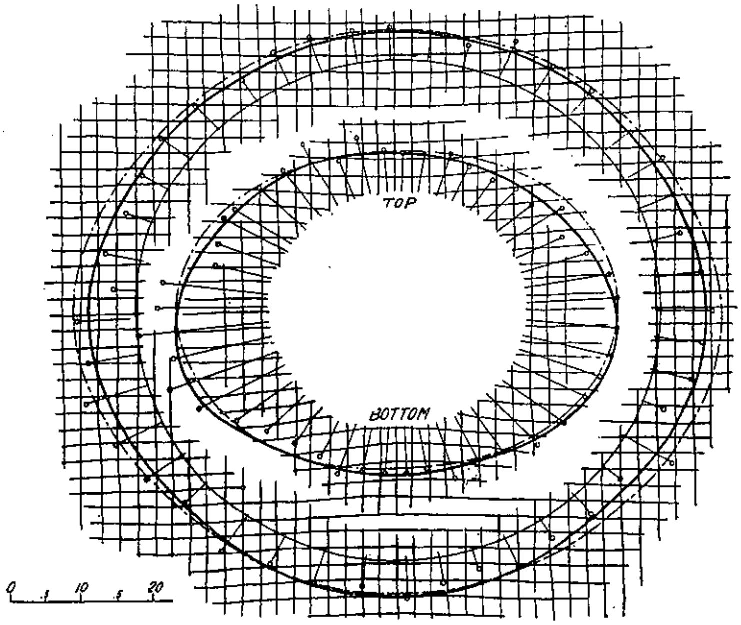


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Fig. 8

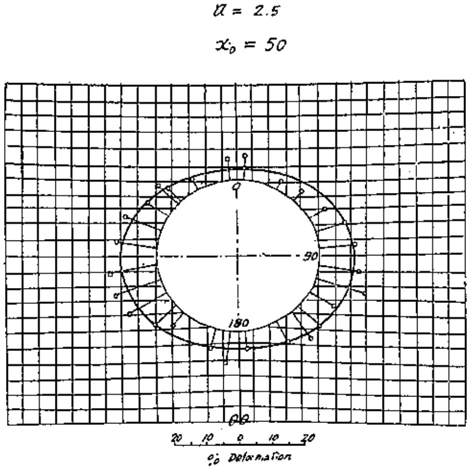

Fig. 9

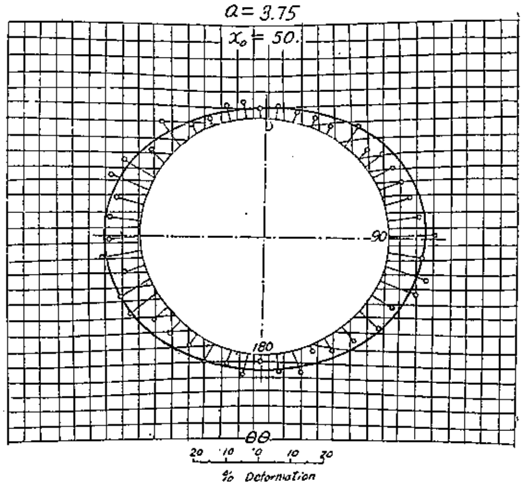

Fig. 10

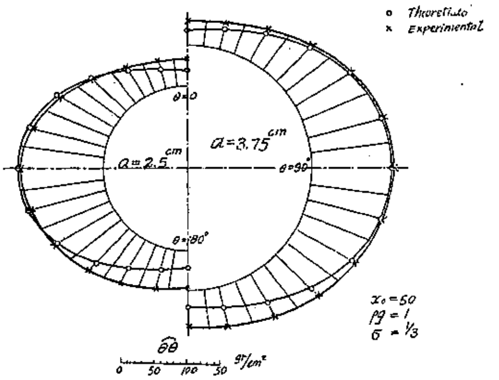

Fig. 11

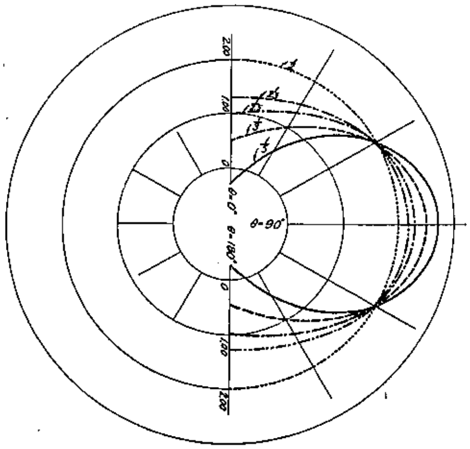

Fig. 12

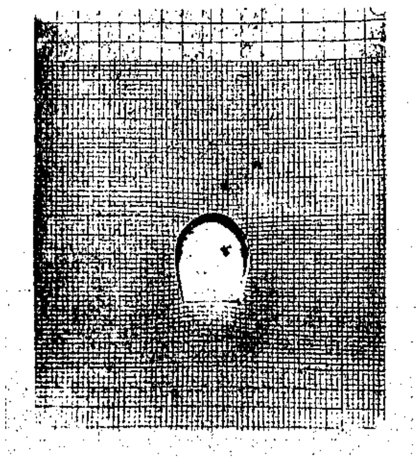

Fig. 13

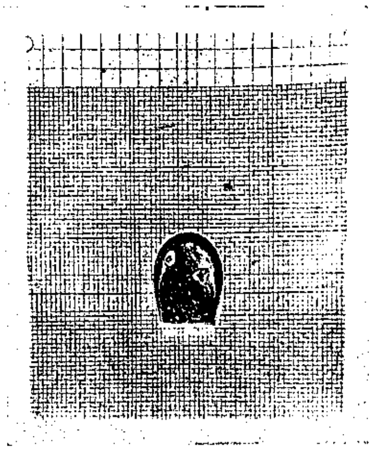

Fig. 14

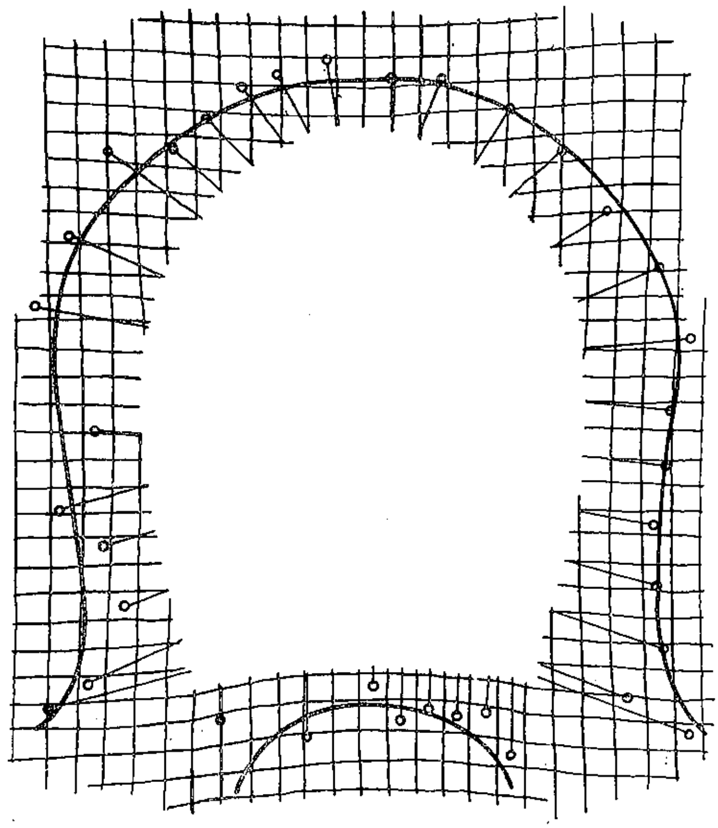

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Fig. 15

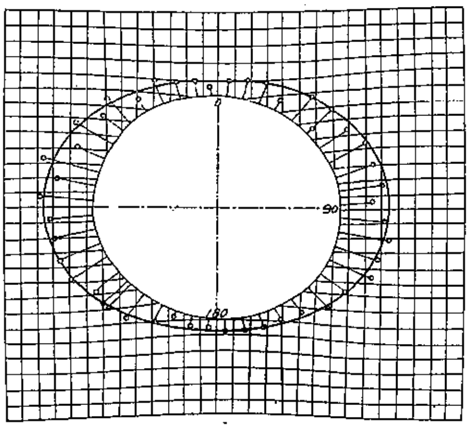

Fig. 16

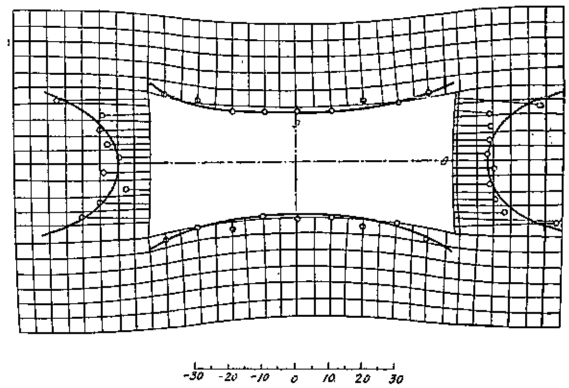

Fig. 17