論 說 報 告

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ASSUMPTION OF THE EXTERNAL FORCES FOR ANALYSIS OF THE STRESSES IN SUBAQUEOUS TUNNELS

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Synopsis

This paper is a supplementary remark to "Stresses in Subaqueous Tunnels Built in the Water-Bearing Soil" written by the author (Vol. XV, No. 1 of this journal), to make the fundamental assumption made in that paper more clearly understood by the readers.

In order to get rational solution of the stresses in subaqueous tunnels, it is necessary to know all forces acting on the section.

Vertical forces can be reasonably assumed by the Law of Gravitation, but as to the lateral forces acting on the tunnel, there is no recognized rule to determine its magnitude.

Some engineers assume lateral pressure from surrounding soil by Rankine's "Theory of Conjugate Stresses". However, this theory is based on the Law of Friction, and is true only when such law will hold, as in the case of earth in dry state, but it will not be correct to apply to the earth saturated with water. Moreover, it is difficult to know what Angle of Repose is to be taken for the particular soil, and supposing this was assumed someway, there remains still another question, what value of pressure to assign it, within two extreme limits to be given by that theory, and therefore lateral pressure thus determined will be nothing but an arbitrarily assigned value of the engineer.

On the other hand, if we assume the surrounding soil to be cohesionless particles permeated with water, and treat earth and water separately, in other words, by assuming earth and water to act independently, under the Law of Gravitation and Fluid respectively, the uncertainty about external forces will be removed, except for the reaction of earth against vertical loading.

Prof. Steiner, in his treatise on circular tube, assumed the reaction to the vertical forces to be also vertical, and of uniform intensity on its projected diameter, thus entirely neglecting its horizontal component from the consideration.

However, since we can not press the section, with curved or inclined bottom surface, vertically without causing lateral pressure, his assumption is not right, and should be corrected, in some way, to take horizontal component of reaction into the consideration.

In this respect, assuming the reaction to the vertical forces to act uniformly and normal to its bottom surface, and determining its intensity that sum of all vertical components of reactions to be equal to total vertical forces, seems to be simplest way of correction, and that such assumption is a reasonable one will be explained below.

Let us first assume vertical component of reaction to be uniformly distributed horizontally.

Dividing bottom surface of the section into equal elementary lengths, and multiplying its horizontal projection with the unit intensity, we get vertical reactions on these elementary divisions as shown in **Fig. 1**, greatest value being attained at the invert B and gradually decreasing upwards to zero at the point A.

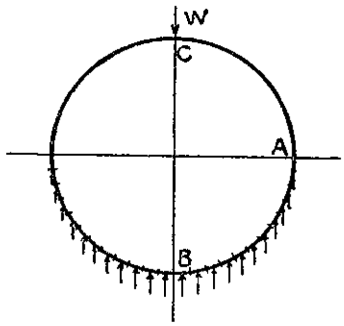

Fig. 1.

Next, let us consider of the lateral component of reaction.

Case I. If the section is a solid or a ring having sufficient stiffness, that it will undergo practically no deformation, lateral pressure due to the vertical loading will be called out by the wedge action of the bottom half circle.

In **Fig. 2**, suppose the bottom surface of the section to consist of indefinitely small polygonal surfaces.

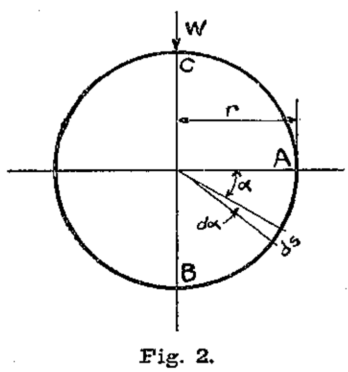

Fig. 2.

Taking unit length of the tube, consider its elementary length ds, which subtends central angle $d\alpha$ and whose normal makes an angle α with the horizontal diameter.

Let

W = Total vertical load,

$2r$ = Diameter of the tube,

then, intensity of vertical reaction is

$$p=\frac{W}{2r}.$$

We have also,

$$ds=r.\,d\alpha.$$

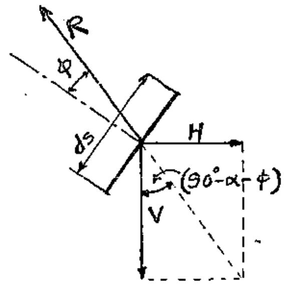

Fig. 3.

In **Fig. 3,** let V and H be the vertical and horizontal forces respectively, acting on the elementary area ds.

Then, the resultant R on this elementary area will act on the direction making an angle ϕ, equal to the angle of friction, with the normal to ds, and we have

$$V=p.\,ds.\,\mathrm{Sin}\,\alpha=\frac{W}{2}.\,d\alpha.\,\mathrm{Sin}\,\alpha$$

$$R=\frac{V}{\mathrm{Cos}\,(90^\circ-\alpha-\phi)}=\frac{V}{\mathrm{Sin}\,(\alpha+\phi)}$$

$$=\frac{H}{\mathrm{Sin}\,(90^\circ-\alpha-\phi)}=\frac{H}{\mathrm{Cos}\,(\alpha+\phi)}$$

$$\therefore \qquad R=\frac{W}{2}.\,d\alpha.\frac{\mathrm{Sin}\,\alpha}{\mathrm{Sin}\,(\alpha+\phi)}$$

$$H=\frac{W}{2}.\,d\alpha.\,\mathrm{Sin}\,\alpha.\,\mathrm{Cot}\,(\alpha+\phi)$$

If there were no friction, ϕ would be zero and above equations become

$$H=\frac{W}{2}.\,d\alpha.\,\mathrm{Cos}\,\alpha$$

$$R=\frac{W}{2}.\,d\alpha$$

Thus, if friction is neglected, horizontal reaction will have its greatest value at the point A, with equal intensity as the vertical reaction at B, and gradually it decreases to zero at the point B, as shown in **Fig. 4.**

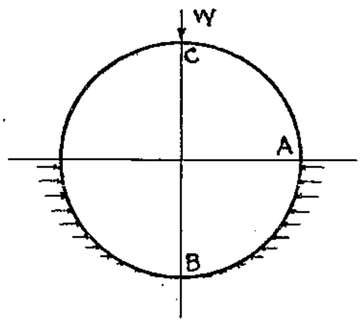

Fig. 4.

Resultant of these two components will make uniform and normal reaction on the lower half circle as shown in **Fig. 5.**

Case II. When the section is not stiff enough to resist deformation, deflection as shown in **Fig. 6** will take place under the vertical loading.

This lateral displacement will cause horizontal reaction, which in combination with its vertical component, will give resultant reaction as shown in **Fig. 7,** and it will be noticed that in addition to the reactions on the bottom surface, there will act some horizontal reactions on its upper half circle.

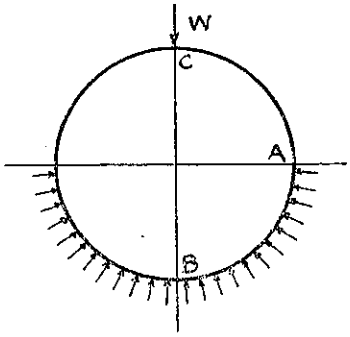

Fig. 5.

Usually, this elastic deflection of the circular tube will be very small compared to its diameter, and the deformation of the original shape will be very slight, and therefore reaction on the bottom half circle of the Case I will also obtain, even if elastic deflection will take place.

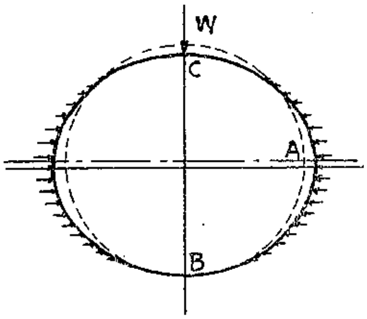

Fig. 6.

Hence if we neglect horizontal reactions on the upper half circle, reaction of the Case II will be reduced to that of Case I.

Neglecting horizontal reactions on the upper half circle means increasing positive moment at the crown and invert points, where stresses are usually greatest, and will give an error on safe side.

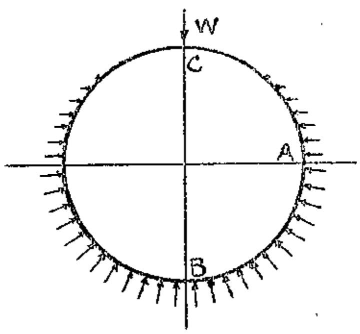

Fig. 7.

As the circular tunnel is usually built of cast iron segments and lined afterwards with thick concrete, deformation of finished section will be practically nil, and its reaction will generally correspond to Case I.

When cast iron segments are not lined with concrete, ring stiffness is not sufficient, and the reaction corresponding Case II will occur. In such case, however, neglecting horizontal reaction on the upper half circle in calculation and erring on safe side, will give rather desirable effect practically, for, during construction cast iron lining is subjected to the thrust from hydraulic rams, and require extra strength to stand the pressure and shocks.

The foregoings are reasons that led the writer to make the assumptions as stated before. If these assumptions differ more or less from actual conditions, they are believed to be sufficient for the purpose of deriving formulas, which was given in my paper as the best working approximation, for, in the design of a subaqueous tunnel, external loadings are also an assumption, and the rigorous treatment as in the case of bridge designing can not be expected, and only approximate check calculations are possible.

By similar reasons, if the section has vertical sides and bottom surface curved or horizontal, their reactions to the bottom surface will be assumed to act as shown in **Fig. 8** and **9** respectively.

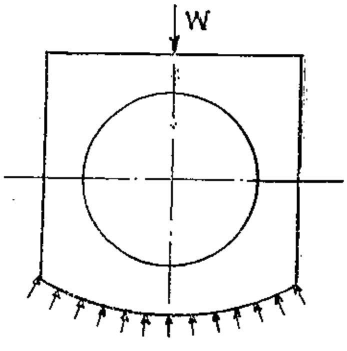

Fig. 8.

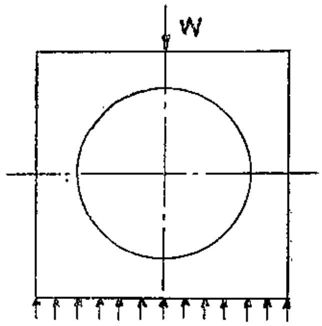

Fig. 9.

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