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GENERAL THEORY ON EARTH PRESSURE AND SEISMIC STABILITY OF RETAINING WALL AND DAM

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SYNOPSIS.

In this paper the following subjects are stated:— the solution of Coulomb's theory for earth pressure, where the cohesion of earth being considered: general theory on the seismic earth pressure; and the seismic stability of a retaining wall and a dam, and also-what kind of attention should be paid for the design of a retaining wall in future? —what was the real cause of the damage to the quay wall of Yokohama on September 1st. 1923?

Preface.

Remarkable progress on the theory of earth pressure and on the stability of a retaining wall and a dam, have been made under the statical condition up to date. In the country, where the destructive earthquake may occur, the stability of a retaining wall and a dam is required to be studied not only on statics but on dynamics. The writer, in this paper, will touch to this problem.

There are two theories for earth pressure, known as Coulomb's theory and Rankin's theory. In the former, the direction of earth pressure is assumed to make a certain angle (equivalent to the angle of repose between the wall and earth) to the wall; while in the latter, the direction of earth pressure is always parallel to its surface. If a wall tends to begin relative movement against earth, the line of pressure should be affected by the friction between them. Hence the Coulomb's theory is preferred to study the ultimate stability of a wall.

For the design of a smaller wall, the cohesion of earth must not be ignored, for which the writer has given the theoretical solution. In the seismic field, where the direction of the gravity being inclined due to the seismic force, the general solution for earth pressure has been deduced also by the writer.

The stability of a retaining wall or a gravity dam, on which a periodic earth pressure or a periodic water pressure and the seismic force on the wall itself being applied, is studied after the forced motion has been exactly known. In this paper, the retaining wall and the dam are assumed to be rigid, for elastic wall the writer will study in future.

The writer has been, for many years, in charge of the design and the construction of quay walls at Yokohama Harbour, and faced directly to the severe earthquake, which gave dreadful damage to the existing quay wall, hereby the writer has the honour to propose the gist for the ideal design of the quay wall recommendable in the country where severe earthquake may occur, and to manifest the cause of the damage to the quay wall of Yokohama Harbour due to the earthquake on September 1st. 1923.

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the design of the quay wall.

Art. 1. Suggestion for the ideal design of the retaining wall.

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Section I.

Solution of Coulomb's theory on earth pressurs, when the cohesion of earth being taken into account.

Notations used;

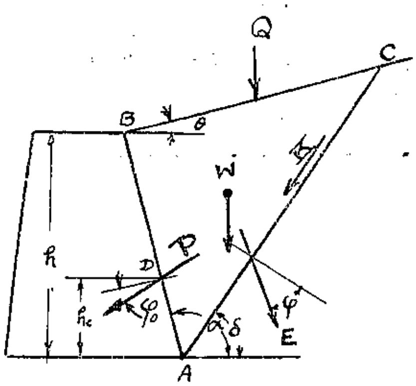


Fig. 1.

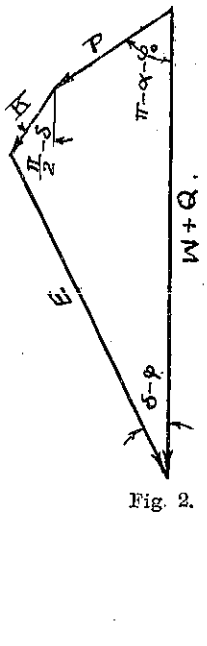


Fig. 2.

P=resultant earth pressure on the wall.

E=resultant earth preesure on the plane of rupture.

W=weight of earth in wedged section ΔABC.

Q=resultant of uniform surcharge on BC.

w=weight of earth per unit volume.

q=surcharge per unit area.

h=height of the wall.

α=angle between the backward plane of the wall and the horizon.

φ=angle of repose of earth.

φ_0=angle of repose between the wall and earth.

K=resultant force of cohesion along the plane of rupture.

k=intensity of cohesion per unit area.

θ=angle of inclination of the surface of earth to the horizon.

δ=angle of inclination of the plane of rupture to the horizon.

p=intensity of earth pressure at any depth η.

h_c=height of resultant earth pressure.

$$W=\frac{wh^2\sin(\alpha-\theta)\sin(\alpha-\delta)}{2\sin^2\alpha\sin(\delta-\theta)}$$

$$Q=qh\frac{\sin(\alpha-\delta)}{\sin\alpha\sin(\delta-\theta)} \qquad K=\frac{kh\sin(\alpha-\theta)}{\sin\alpha\sin(\delta-\theta)}$$

Hence, we have $$W+Q=\frac{h\sin(\alpha-\delta)}{\sin\alpha\sin(\delta-\theta)}\left[\frac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right]$$

By the force polygon shown in fig. 2, we can deduce

$$P=\frac{(W+Q)\sin(\delta-\varphi)-K\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)} \quad \ldots\ldots\ldots (1)$$

i. e.

$$P=\frac{h\left\{\frac{wh}{2}\cdot\frac{\sin(\alpha-\theta)}{\sin\alpha}+q\right\}\sin(\delta-\varphi)\sin(\alpha-\delta)-kh\sin(\alpha-\theta)\cos\varphi}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta).} \quad \ldots\ldots\ldots (2)$$

By Coulomb's theorem, the earth pressure must be equal to the maximum value of P concerning to δ. Therefore δ will be determined by the following equation.

$$\frac{dP}{d\delta}=0$$

while

$$\begin{aligned}\frac{dP}{d\delta}=\frac{h}{\sin\alpha}\Bigg[\Bigg[&\left\{\frac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right\}\Big\{\cos(\delta-\varphi)\sin(\alpha-\delta)\\ &-\sin(\delta-\varphi)\cos(\alpha-\delta)\Big\}\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)\\ &-\Bigg[\left\{\frac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right\}\sin(\delta-\varphi)\sin(\alpha-\delta)\\ &-k\sin(\alpha-\theta)\cos\varphi\Bigg]\Big\{\cos(\delta-\theta)\sin(\alpha-\delta+\varphi+\varphi_o)\\ &-\sin(\delta-\theta)\cos(\alpha-\delta+\varphi+\varphi_o)\Big\}\Bigg]\div\Big[\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)\Big]^2\end{aligned}$$

Then, we have

$$\left\{\frac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right\}\Big\{\cos(\delta-\varphi)\sin(\alpha-\delta)$$

$$- \sin(\delta-\varphi)\cos(\alpha-\delta)\Big\} \sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)$$

$$-\left[\left\{\frac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right\}\sin(\delta-\varphi)\sin(\alpha-\delta)\right.$$

$$\left.-k\sin(\alpha-\theta)\cos\varphi\right]\Big\{\cos(\delta-\theta)\sin(\alpha-\delta+\varphi+\varphi_o)$$

$$-\sin(\delta-\theta)\cos(\alpha-\delta+\varphi+\varphi_o)\Big\}=0$$

i. e.

$$\left\{\frac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right\}\sin(\alpha-2\delta+\varphi)\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)$$

$$-\left[\left\{\frac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right\}\sin(\delta-\varphi)\sin(\alpha-\delta)\right.$$

$$\left.-k\sin(\alpha-\theta)\cos\varphi\right]\sin(\alpha-2\delta+\varphi+\varphi_o+\theta)=0$$

While

$$\sin(\delta-\theta)\sin(\alpha-\delta+\varphi+\varphi_o)\equiv\frac{1}{2}\{\cos(\alpha-2\delta+\varphi+\varphi_o+\theta)$$

$$-\cos(\alpha+\varphi+\varphi_o-\theta)\}$$

$$\sin(\delta-\varphi)\sin(\alpha-\delta)\equiv\frac{1}{2}\{\cos(\alpha-2\delta+\varphi)-\cos(\alpha-\varphi)\}$$

Substituting these relations in the former equation, we have

$$-\sin(\varphi_o+\theta)-\sin(\alpha-2\delta+\varphi)\cos(\alpha+\varphi+\varphi_o-\theta)$$

$$+\cos(\alpha-\varphi)\sin(\alpha-2\delta+\varphi+\varphi_o+\theta)$$

$$-\frac{2k}{\left\{\dfrac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right\}}\sin(\alpha-\theta)\cos\varphi\sin(\alpha-2\delta+\varphi+\varphi_o+\theta)=0.$$

Let $\mu=\alpha-2\delta+\varphi$

Substituting μ in the former equation and by simplifying, we have

$$\sin(\varphi_o+\theta)=\cos\mu\left[\cos(\alpha-\varphi)\sin(\varphi_o+\theta)-\frac{2k\sin(\alpha-\theta)\cos\varphi}{\left\{\dfrac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right\}}\sin(\varphi_o+\theta)\right]$$

$$-\sin\mu\left[\cos(\alpha+\varphi+\varphi_o-\theta)-\cos(\alpha-\varphi)\cos(\varphi_o+\theta)\right.$$

$$\left.+\frac{2k\sin(\alpha-\theta)\cos\varphi}{\left\{\dfrac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right\}}\cos(\varphi_o+\theta)\right] \quad\cdots\cdots\cdots\cdots (3)$$

$$\left.\begin{aligned} \text{Let}\quad a&=\sin(\varphi_o+\theta)\\ b&=\cos(\alpha+\varphi+\varphi_o-\theta)-\cos(\alpha-\varphi)\cos(\varphi_o+\theta)\\ &\qquad+\frac{2k\sin(\alpha-\theta)\cos\varphi}{\left\{\frac{wh\sin(\alpha-\theta)}{\sin\alpha}+q\right\}}\cos(\varphi_o+\theta)\\ c&=\cos(\alpha-\varphi)\sin(\varphi_o+\theta)\\ &\qquad-\frac{2k\sin(\alpha-\theta)\cos\varphi}{\left\{\frac{wh}{2}\cdot\frac{\sin(\alpha-\theta)}{\sin\alpha}+q\right\}}\sin(\varphi_o+\theta) \end{aligned}\right\}\quad\cdots\cdots(4)$$

Substituting these values in eq. (3), we have

$$\tan^2\mu-\frac{2bc}{b^2-a^2}\tan\mu+\frac{c^2-a^2}{b^2-a^2}=0$$

$$\therefore\qquad \tan\mu=\frac{bc\pm a\sqrt{b^2-a^2+c^2}}{b^2-a^2}\quad\cdots\cdots\cdots(4)'$$

For maximum of P, $\tan\mu=\dfrac{bc+a\sqrt{b^2-a^2+c^2}}{b^2-a^2}$ should be taken.

Thus μ is determined, and consequently δ will be found as follows.

$$2\delta=\alpha+\varphi-\tan^{-1}\frac{bc+a\sqrt{b^2-a^2+c^2}}{b^2-a^2}\quad\cdots\cdots(5)$$

Then substituting the known value of δ in eq. (2), we have the maximum value of P, which indicates the theoretical earth pressure.

$$P=\frac{\left\{\frac{wh^2}{2}\cdot\frac{\sin(\alpha-\theta)}{\sin\alpha}+qh\right\}\sin(\delta-\varphi)\sin(\alpha-\delta)-k\,h\sin(\alpha-\theta)\cos\varphi}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}\quad\cdot(6)$$

Let P be the intensity of earth pressure at any depth η, the following relation will be held.

$$P=\int_o^h p\frac{d\eta}{\sin\alpha}\qquad\therefore\ p=\frac{dP}{d\eta}\sin\alpha.$$

Therefore, we have

$$p=\{w\eta\sin(\alpha-\theta)+q\sin\alpha\}\frac{\sin(\delta-\varphi)\sin(\alpha-\delta)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}-\frac{k\sin(\alpha-\theta)\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)} \quad \cdots\cdots (7)$$

The point of application of the resultant earth pressure will be easily found from the center of figure of the pressure intensity trapezoid, because the pressure is an uniformly varying traction.

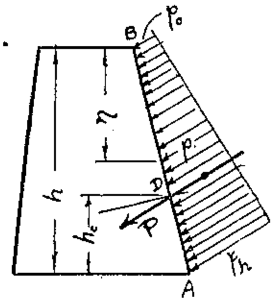


Fig. 3.

$$\frac{h_c}{\sin\alpha}=\frac{h}{\sin\alpha}\cdot\frac{(p_h+2p_o)}{3(p_h+p_o)}$$

$$p_o=\frac{q\sin(\alpha-\delta)\sin(\delta-\varphi)}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}-\frac{k\sin(\alpha-\theta)\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}$$

$$p_h=\{wh\sin(\alpha-\theta)+q\sin\alpha\}\times\frac{\sin(\alpha-\delta)\sin(\delta-\varphi)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}-\frac{k\sin(\alpha-\theta)\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}$$

Since, we have

$$h_c=h\frac{\{wh\sin(\alpha-\theta)+3q\sin\alpha\}\frac{\sin(\alpha-\delta)\sin(\delta-\varphi)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}-3\frac{k\sin(\alpha-\theta)\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}}{3\left[\{wh\sin(\alpha-\theta)+2q\sin\alpha\}\frac{\sin(\alpha-\delta)\sin(\delta-\varphi)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}-2\frac{k\sin(\alpha-\theta)\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}\right]}$$

$$=h\frac{\left\{\frac{1}{3}\cdot\frac{wh^2\sin(\alpha-\theta)\sin(\alpha-\delta)}{2\sin^2\alpha}+\frac{q}{2}\cdot\frac{\sin(\alpha-\delta)}{\sin\alpha\sin(\delta-\theta)}\right\}}{\left\{\frac{wh^2}{2}\cdot\frac{\sin(\alpha-\theta)\sin(\alpha-\delta)}{\sin^2\alpha}+q\frac{\sin(\alpha-\delta)}{\sin\alpha\sin(\delta-\theta)}\right\}}$$

$$\frac{\frac{\sin(\delta-\varphi)}{\sin(\alpha-\delta+\varphi+\varphi_o)}-\frac{1}{2}\frac{k\sin(\alpha-\theta)\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}}{\frac{\sin(\delta-\varphi)}{\sin(\alpha-\delta+\varphi+\varphi_o)}-\frac{k\sin(\alpha-\theta)\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}}$$

$$\therefore \quad h_c = h\frac{\left(\frac{1}{3}W+\frac{Q}{2}\right)\sin(\delta-\varphi)-\frac{1}{2}K\cos\varphi}{(W+Q)\sin(\delta-\varphi)-K\cos\varphi} \quad \cdots\cdots (8)$$

In the case, when $k=0$, we have

$$\left.\begin{aligned} a&=\sin(\varphi_o+\theta)\\ b&=\cos(\alpha+\varphi+\varphi_o-\theta)-\cos(\alpha-\varphi)\cos(\varphi_o+\theta)\\ c&=\cos(\alpha-\varphi)\sin(\varphi_o+\theta)\\ 2\delta&=\alpha+\varphi-\tan^{-}\frac{bc+a\sqrt{b^2-a^2+c^2}}{b^2-c^2}\end{aligned}\right\} \quad \cdots\cdots (9)$$

$$P=\left\{\frac{wh^2}{2}\cdot\frac{\sin(\alpha-\theta)}{\sin\alpha}+qh\right\}\frac{\sin(\delta-\varphi)\sin(\alpha-\delta)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)} \quad \cdots (10)$$

$$p=\{w\eta\sin(\alpha-\theta)+q\sin\alpha\}\frac{\sin(\delta-\varphi)\sin(\alpha-\delta)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)} \quad \cdots (11)$$

$$h_c=h\frac{\frac{W}{3}+\frac{Q}{2}}{W+Q} \quad \cdots\cdots (12)$$

In the case when $k=0$, eq. 264) in Prof. Shibata's Applied Mechanics Art. 157 is identical to eq. (3) in this section, hence by giving some reductions, eq. (9) and (10) are proved to be identical to the following formulae, which are written in Art. 157 as eq. 265) and 266).

$$\tan\delta=\frac{\sin\varphi\sqrt{\sin(\alpha-\theta)\sin(\varphi+\varphi_o)}+\sin\alpha\sqrt{\sin(\varphi-\theta)\sin(\alpha+\varphi_o)}}{\cos\varphi\{\sqrt{\sin(\alpha-\theta)\sin(\varphi+\varphi_o)}+\cos\alpha\sqrt{\sin(\varphi-\theta)\sin(\alpha+\varphi_o)}}$$

$$P=\left[\frac{wh^2}{2}\cdot\frac{\sin(\alpha-\theta)}{\sin\alpha}+qh\right]\frac{\sin^2(\alpha-\varphi)}{\sin\alpha\,\{\sqrt{\sin(\alpha-\theta)\sin(\alpha+\varphi_o)}+\sqrt{\sin(\varphi-\theta)\sin(\varphi+\varphi_o)}\}^2}$$

Section II.

General solution of earth pressure, where seismic force being considered.

Notations used;

All notations used in Section I will be used.

Fig. 4.

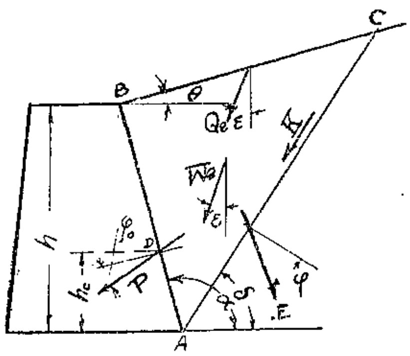
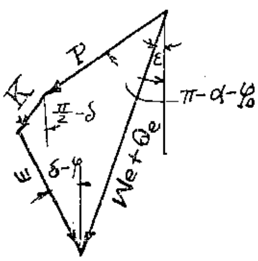


Fig. 5.



ε=deflection of gravity line due to the seismic force.

Q_e=resultant of Q and seismic force on it.

W_e= ,, ,, W ,, ,, ,,

w_e= ,, ,, w ,, ,, ,,

q_e= ,, ,, q ,, ,, ,,

By the force polygon shown in fig. 5, we have

$$P=\frac{(W_e+Q_e)\sin(\varepsilon+\delta-\varphi)-K\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)} \cdot \quad (13)$$

W_e, Q_e and K can be substituted by w_e, q_e, and k, then we have

$$P=\frac{h}{\sin\alpha}\cdot\frac{\left\{\frac{w_e h\sin(\alpha-\theta)}{2\sin\alpha}+q_e\right\}\sin(\varepsilon+\delta-\varphi)\sin(\alpha-\delta)-k\sin(\alpha-\theta)\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)} \cdots (14)$$

and

$$\frac{dP}{ds}=\frac{h}{\sin\alpha}\Bigg[\left\{\frac{w_e h\sin(\alpha-\theta)}{2\sin\alpha}+q_e\right\}\{\cos(\varepsilon+\delta-\varphi)\sin(\alpha-\delta)$$
$$-\sin(\varepsilon+\delta-\varphi)\cos(\alpha-\delta)\}\sin(\delta-\theta)\sin(\alpha-\delta+\varphi+\varphi_o)$$
$$-\Bigg[\left\{\frac{w_e\sin(\alpha-\theta)}{2\sin\alpha}+q_e\right\}\sin(\varepsilon+\delta-\varphi)\sin(\alpha-\delta)$$
$$-k\sin(\alpha-\theta)\cos\varphi\Bigg]\{\cos(\delta-\theta)\sin(\alpha-\delta+\varphi+\varphi_o)$$
$$-\sin(\delta-\theta)\cos(\alpha-\delta+\varphi+\varphi_o)\}\Bigg]\div[\sin(\delta-\theta)\sin(\alpha-\delta+\varphi+\varphi_o)]^2$$

By putting $\frac{dP}{ds}=0$, we have

$$\left\{\frac{w_e h\sin(\alpha-\theta)}{2\sin\alpha}+q_e\right\}\sin(\alpha-2\delta-\varepsilon+\varphi)\sin(\delta-\theta)\sin(\alpha-\delta+\varphi+\varphi_o)$$
$$-\Bigg[\left\{\frac{w_e h\sin(\alpha-\theta)}{2\sin\alpha}+q_e\right\}\sin(\varepsilon+\delta-\varphi)\sin(\alpha-\delta)$$

$$-k\sin(\alpha-\theta)\cos\varphi\Big]\sin(\alpha-2\delta+\varphi+\varphi_o+\theta)=0$$

That is

$$-\sin(\varphi_o+\theta+\varepsilon)-\sin(\alpha-2\delta+\varphi-\varepsilon)\cos(\alpha+\varphi+\varphi_o-\theta)$$

$$+\cos(\alpha-\varphi+\varepsilon)\sin(\alpha-2\delta+\varphi+\varphi_o+\theta)$$

$$-\frac{2k\sin(\alpha-\theta)\cos\varphi}{\left\{\dfrac{w_e h\sin(\alpha-\theta)}{2\sin\alpha}+q_e\right\}}\sin(\alpha-2\delta+\varphi+\varphi_o+\theta)=0.$$

Put $\alpha-2\delta+\varphi=\mu$, then we have

$$\sin(\varphi_o+\theta+\varepsilon)=\cos\mu\Big[\sin\varepsilon\cos(\alpha+\varphi+\varphi_o-\theta)+\cos(\alpha-\varphi+\varepsilon)\sin(\varphi_o+\theta)$$

$$-\frac{2k\sin(\alpha-\theta)\cos\varphi}{\left\{\dfrac{w_e h\sin(\alpha-\theta)}{2\sin\alpha}+q_e\right\}}\sin(\varphi_o+\theta)\Big]$$

$$-\sin\mu\Big[\cos\varepsilon\cos(\alpha+\varphi+\varphi_o-\theta)-\cos(\alpha-\varphi+\varepsilon)\cos(\varphi_o+\theta)$$

$$+\frac{2k\sin(\alpha-\theta)\cos\varphi}{\left\{\dfrac{w_e h\sin(\alpha-\theta)}{2\sin\alpha}+q_e\right\}}\cos(\varphi_o+\theta)\Big] \quad \cdots\cdots (15)$$

Let

$$\left.\begin{aligned}
a&=\sin(\varphi_o+\theta+\varepsilon)\\
b&=\cos\varepsilon\cos(\alpha+\varphi+\varphi_o-\theta)-\cos(\alpha-\varphi+\varepsilon)\cos(\varphi_o+\theta)\\
&\quad+\frac{2k\sin(\alpha-\theta)\cos\varphi}{\left\{\dfrac{w_e h\sin(\alpha-\theta)}{2\sin\alpha}+q_e\right\}}\cos(\varphi_0+\theta)\\
c&=\sin\varepsilon\cos(\alpha+\varphi+\varphi_o-\theta)+\cos(\alpha-\varphi+\varepsilon)\sin(\varphi_o+\theta)\\
&\quad-\frac{2k\sin(\alpha-\theta)\cos\varphi}{\left\{\dfrac{w_e h\sin(\alpha-\theta)}{2\sin\alpha}+q_e\right\}}\sin(\varphi_o+\theta)
\end{aligned}\right\} \cdot (16)$$

Substituting these values in eq. (15) and by the same reductions as shown in Section I, we have

$$2\delta=\alpha+\varphi-\tan^{-1}\frac{bc+a\sqrt{b^2-a^2+c^2}}{b^2-a^2} \quad \cdots\cdots (16)'$$

Substituting the known δ in eq. (14), we have the exact formula for earth pressure, where seismic force being considered.

$$P=\frac{\left\{\frac{w_e h^2 \sin(\alpha-\theta)}{2\sin\alpha}+q_e h\right\}\sin(\varepsilon+\delta-\varphi)\sin(\alpha-\delta)-kh\sin(\alpha-\theta)\cos\varphi}{\sin\alpha\sin(\delta-\theta)\sin(\alpha-\delta+\varphi+\varphi_o)} \quad \cdot \; (17)$$

Intensity of earth pressure at any depth η will be.

$$p=\frac{dP}{d\eta}\sin\alpha$$

i. e.

$$p=\{w_e\,\eta\sin(\alpha-\theta)+q_e\sin\alpha\}\frac{\sin(\delta-\varphi+\varepsilon)\sin(\alpha-\delta)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}$$

$$-\frac{k\sin(\alpha-\theta)\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)} \quad \cdots\cdots\cdots (18)$$

By the same reduction as shown in Section I, we have the height of the point of application of the earth pressure.

$h_c=h\frac{(p_h+2p_o)}{3(p_h+p_o)}$, where p_o and p_h will be as follows.

$$p_o=\frac{q_e\sin(\delta-\varphi+\varepsilon)\sin(\alpha-\delta)}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}-\frac{k\sin(\alpha-\theta)\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}$$

$$p_h=\frac{\{w_e h\sin(\alpha-\theta)+q_e\sin\alpha\}\sin(\delta-\varphi+\varepsilon)\sin(\alpha-\delta)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}$$

$$-\frac{k\sin(\alpha-\theta)\cos\varphi}{\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}$$

Therefore, we have

$$h_c=h\frac{\left(\frac{1}{3}W_e+\frac{1}{2}Q_e\right)\sin(\delta-\varphi+\varepsilon)-\frac{1}{2}K\cos\varphi}{(W_e+Q_e)\sin(\delta-\varphi+\varepsilon)-K\cos\varphi} \quad \cdots\cdots\cdots (19)$$

In the case when $\varepsilon=0$, formulae (16), (17), (18) and (19) will be identical to (4), (5), (6), (7) and (8).

In the case when $k=0$, we have

$$\left.\begin{aligned}a&=\sin(\varphi_o+\theta+\varepsilon)\\ b&=\cos\varepsilon\cos(\alpha+\varphi+\varphi_o-\theta)-\cos(\alpha-\varphi+\varepsilon)\cos(\varphi_o+\theta)\\ c&=\sin\varepsilon\cos(\alpha+\varphi+\varphi_o-\theta)+\cos(\alpha-\varphi+\varepsilon)\sin(\varphi_o+\theta)\end{aligned}\right\} \quad \cdots\cdots (20)$$

$$2\delta=\alpha+\varphi-\tan^{-1}\frac{bc+a\sqrt{b^2-a^2+c^2}}{b^2-a^2} \quad \cdots\cdots (21)$$

$$P=\left\{\frac{w_e h^2 \sin(\alpha-\theta)}{2\sin\alpha}+q_e h\right\}\frac{\sin(\delta-\varphi+\varepsilon)\sin(\alpha-\delta)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)} \quad \cdots (22)$$

$$p=\{w_e\eta\sin(\alpha-\theta)+q_e\sin\alpha\}\frac{\sin(\delta-\varphi+\varepsilon)\sin(\alpha-\delta)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)} \quad \cdot (23)$$

$$h_e=h\frac{\left(\frac{1}{3}W_e+\frac{1}{2}Q_e\right)}{(W_e+Q_e)} \quad \cdots\cdots (24)$$

In the case when $k=0$, $\alpha=\frac{\pi}{2}$ and $\theta=0$, we have

$$\left.\begin{aligned} a&=\sin(\varepsilon+\varphi_o)\\ b&=-\{\cos\varepsilon\sin(\varphi+\varphi_o)+\sin(\varphi-\varepsilon)\cos\varphi_o\}\\ c&=-\sin\varepsilon\sin(\varphi+\varphi_o)+\sin(\varphi-\varepsilon)\sin\varphi_o \end{aligned}\right\} \quad \cdots\cdots (25)$$

$$2\delta=\frac{\pi}{2}+\varphi-\tan^{-1}\frac{bc+a\sqrt{b^2-a^2+c^2}}{b^2-a^2} \quad \cdots\cdots (26)$$

$$P=\left\{\frac{w_e h^2}{2}+q_e h\right\}\frac{\sin(\delta-\varphi+\varepsilon)\cos\delta}{\cos(\delta-\varphi-\varphi_o)\sin\delta} \quad \cdots\cdots (27)$$

Section III.
Stability of retaining wall and gravity dam, where seismic force being considered.

Retaining wall and gravity dam in this section are assumed to be rigid bodies.

Art. 1. The direction and the magnitude of the résultant force of gravity and earthquake.

Notations used;

g=acceleration due to gravity.

α_e=maximum acceleration due to earthquake.

g_e=resultant of g and α_e.

α_v=vertical component of α_e.

α_h = horizontal component of α_e

B = amplitude of seismic vibration.

B_v = vertical component of B.

B_h = horizontal component of B.

S = relative movement of a body against the seismic vibration.

S_v = vertical component of S.

S_h = horizontal component of S.

m = mass of a body. $W_e = mg_e$, $W = mg$.

T = period of seismic vibration.

Let the equation of a seismic vibration be

$$y_e = \frac{B}{2} \sin \frac{2\pi}{T} t \quad \cdots\cdots\cdots\cdots (28)$$

If a body displaces S against the earth, which is forced to vibrate in the motion indicated by $y_e = \frac{B}{2} \sin \frac{2\pi}{T} t$, the motion of the body will be expressed by

$$y = \frac{(B-S)}{2} \sin \frac{2\pi}{T} t \quad \cdots\cdots\cdots\cdots (29)$$

Since, we have

$$\frac{dy}{dt} = \frac{\pi(B-S)}{T} \cos \frac{2\pi}{T} t$$

$$\frac{d^2y}{dt^2} = -\frac{2\pi^2(B-S)}{T^2} \sin \frac{2\pi}{T} t \quad \cdots\cdots (30)$$

Therefore the maximum acceleration will be

$$\alpha_e = \frac{2\pi^2(B-S)}{T^2} \quad \cdots\cdots\cdots\cdots (31)$$

The horizontal and vertical components are

$$\left.\begin{aligned} \alpha_h &= \frac{2\pi^2(B_h - S_h)}{T^2} \\ \alpha_v &= \frac{2\pi^2(B_v - S_v)}{T^2} \end{aligned}\right\} \cdots (31)'$$

In the case when there is no relative displacement, we have

$$\left.\begin{aligned} \alpha_e &= \frac{2\pi^2 B}{T^2} \\ \alpha_h &= \frac{2\pi^2 B_h}{T^2} \qquad \alpha_v = \frac{2\pi^2 B_v}{T^2} \end{aligned}\right\} \cdots (32)$$

By applying the vector for these accelerations, we have the resultant acceleration of gravity and earthquake as folllows,

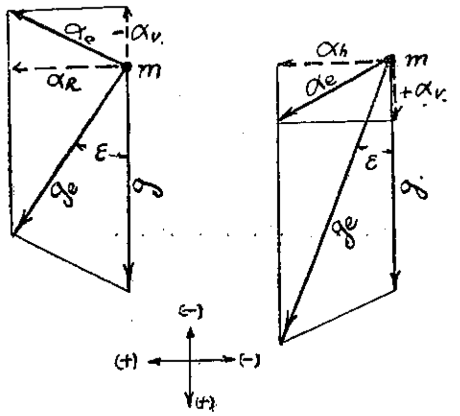


Fig. 6.

$$g_e = \sqrt{(g \pm \alpha_v)^2 + \alpha_h^2}$$

$$\varepsilon = \tan^{-1} \frac{\alpha_h}{g \pm \alpha_v} \cdots (33)$$

Let $\lambda_o = \frac{g_e}{g}$, we have

$$\lambda_o = \frac{\sqrt{(g \pm \alpha_v)^2 + \alpha_h^2}}{g} \cdots (34)$$

Then we have the resultant force of gravity and earthquake,

$$W_e = \lambda_o W \quad \text{and} \quad Q_e = \lambda_o Q \quad \cdot (35)$$

Art. 2. Application of general solution of earth pressure to a retaining wall, where seismis force being considered.

If a wall be rigid and fixed to the foundation so firmly that there is no relative displacement, the earth contained in ABC will be forced to vibrate similar to the wall and the foundation.

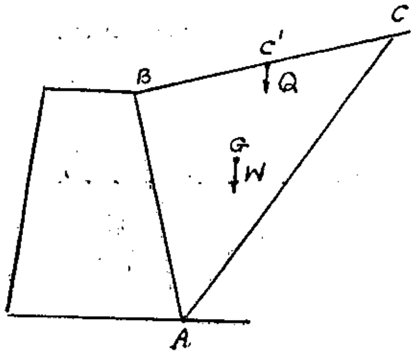


Fig. 7.

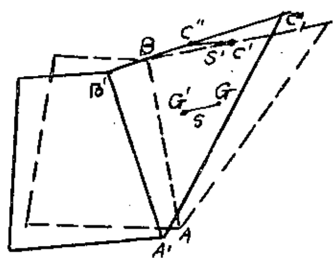


Fig. 8.

Let G be the center of gravity of ΔABC and C' be the center of surcharge Q; the vibration of G and Q is the same to that of the wall and the foundation.

Therefore following formulae denote the maximum acceleratioon to be applied for W and Q.

$$\alpha_h=\frac{2\pi^2 B_h}{T^2} \qquad \alpha_v=\frac{2\pi^2 B_v}{T^2} \quad \cdots\cdots (36)$$

If a wall be displaced S relatively during one half period of seismic vibration, the following acceleration should be applied for W and Q.

$$s \doteqdot s' \qquad \left.\begin{aligned} \alpha_h&=\frac{2\pi^2(B_h-S_h)}{T^2} \\ \alpha_v&=\frac{2\pi^2(B_v-S_v)}{T^2} \end{aligned}\right\} \cdots\cdots (36)'$$

If the amplitude, horizontal and vertical, of an earthquake and the displacement of the wall during every half period of the former be known, we can easily find out the amount of horizontal and vertical accelerations which will be acting to W and Q.

The period of the earthquake should be measured.

Let α_v and α_h be the vertical and horizontal accelerations acting on W and Q, we have

$$W_e=\lambda_o W \qquad w_e=\lambda_o w$$

$$Q_e=\lambda_o Q \qquad q_e=\lambda_o q$$

in which,

$$\lambda_o=\frac{\sqrt{(g\pm\alpha_v)^2+\alpha_h^2}}{g} \qquad \varepsilon=\tan^{-1}\frac{\alpha_h}{g\pm\alpha_v}$$

Substituting these relations in eqs. (16), (17) $\cdots$ and (24), we have the maximum earth pressure and its height from the base.

$$P_e=\frac{\lambda_o\left\{\dfrac{wh^2\sin(\alpha-\theta)}{2\sin\alpha}+qh\right\}\sin(\varepsilon+\delta-\varphi)\sin(\alpha-\delta)-kh\sin(\alpha-\theta)\cos\varphi}{\sin\alpha\sin(\delta-\theta)\sin(\alpha-\delta+\varphi+\varphi_o)} \cdot (38)$$

$$h_e=h\frac{\lambda_o\left(\dfrac{1}{3}W+\dfrac{1}{2}Q\right)\sin(\delta-\varphi+\varepsilon)-\dfrac{1}{2}K\cos\varphi}{\lambda_o(W+Q)\sin(\delta-\varphi+\varepsilon)-K\cos\varphi} \cdots (39)$$

in which,

$$2\delta=\alpha+\varphi-\tan^{-1}\frac{bc+a\sqrt{b^2-a^2+c^2}}{b^2-a^2} \cdots\cdots (40)$$

$$\left.\begin{aligned}
a&=\sin(\varphi_o+\theta+\varepsilon)\\
b&=\cos\varepsilon\cos(\alpha+\varphi+\varphi_o-\theta)-\cos(\varphi_o+\theta)\cos(\alpha-\varphi+\varepsilon)\\
&\quad+\frac{2k\sin(\alpha-\theta)\cos\varphi}{\lambda_o\left\{\dfrac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right\}}\cos(\varphi_o+\theta)\\
c&=\sin\varepsilon\cos(\alpha+\varphi+\varphi_o-\theta)+\sin(\varphi_o+\theta)\cos(\alpha-\varphi+\varepsilon)\\
&\quad-\frac{2k\sin(\alpha-\theta)\cos\varphi}{\lambda_o\left\{\dfrac{wh\sin(\alpha-\theta)}{2\sin\alpha}+q\right\}}\sin(\varphi_o+\theta)
\end{aligned}\right\}\quad\cdots(40)'$$

The cohesion of earth will be disturved owing to earthquake, and k tends to zero according to the increase of α_e.

Since in the case of severe earthquake, it will be safe side if the earth pressure be calculated by putting $k=0$. Thus we have the maximum earth pressure and its point of action for a destructive earthquake.

$$P_e=\lambda_o\left\{\frac{wh^2\sin(\alpha-\theta)}{2\sin\alpha}+qh\right\}\frac{\sin(\varepsilon+\delta-\varphi)\sin(\alpha-\delta)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}\quad\cdots(41)$$

$$h_c=h\frac{\left(\dfrac{1}{3}W+\dfrac{1}{2}Q\right)}{(W+Q)}\quad\cdots\cdots\cdots(42)$$

in which,

$$2\delta=\alpha+\varphi-\tan^{-1}\frac{bc+a\sqrt{b^2-a^2+c^2}}{b^2-a^2}\quad\cdots\cdots\cdots(43)$$

$$\left.\begin{aligned}
a&=\sin(\varphi_o+\theta+\varepsilon)\\
b&=\cos\varepsilon\cos(\alpha+\varphi+\varphi_o-\theta)-\cos(\varphi_o+\theta)\cos(\alpha-\varphi+\varepsilon)\\
c&=\sin\varepsilon\cos(\alpha+\varphi+\varphi_o-\theta)+\sin(\varphi_o+\theta)\cos(\alpha-\varphi+\varepsilon)
\end{aligned}\right\}\quad\cdots(43)'$$

Let P_o be the earth pressure in ordinary condition, where $\varepsilon=0$ and $k=0$, by eq. (10) we have

$$P_o=\left\{\frac{wh^2\sin(\alpha-\theta)}{2\sin\alpha}+qh\right\}\frac{\sin(\delta-\varphi)\sin(\alpha-\delta)}{\sin\alpha\sin(\alpha-\delta+\varphi+\varphi_o)\sin(\delta-\theta)}\quad\cdot(44)$$

It will be assumed that the variation of earth pressure during an earthquake be expressed by a sine function whose period is the same as that of the seismic vibration.

Since, we have P, which expresses the amount of earth pressure at any instant t

$$P=P_o+(P_e-P_o)\sin\frac{2\pi}{T}t \quad \cdots\cdots\cdots\cdots (45)$$

This is the fundamental formula to determine the dynamical stability of a retaining wall, when the seismic force is concerned.

Art. 3. Water pressure on a quay wall or on a dam, when seismic action being considered.

a). Horizontal vibration is concerned.

The motiom of water will be indifferent to the earth, so far as the horizontal vibration of the latter is concerned, because the friction (viscosity) between water and the earth is too small to transmit the vibration to water. In consequence, the horizontal seismic force for water will be nearly equal to zero.

But the horizontal vibration of a wall in water will give some shock to the latter, and the water level will consequently rise corresponding to the change of velocity of the vibration.

Let the equation for the vibration of a wall be

$$y=\frac{B_h}{2}\sin\frac{2\pi}{T}t$$

Then we have

$$\frac{dy}{dt}=\frac{\pi}{T}B_h\cos\frac{2\pi}{T}t, \quad \frac{d^2y}{dt^2}=-\frac{2\pi^2B_h}{T^2}\sin\frac{2\pi}{T}t=-\alpha_h\sin\frac{2\pi}{T}t$$

Since we have the maximum velocity change,

$$v=2\frac{\pi}{T}B_h$$

By the theory of hydraulics, we have

$$H'=\frac{v^2}{2g}=\frac{2\times2\pi^2B_h^2}{2g\,T^2}=\frac{2\pi^2B_h}{T^2}.\frac{B_h}{g}=\frac{\alpha_h}{g}B_h \quad \cdots\cdots (46)$$

Thus, the water pressure will increase according to the elevation of water level by this additional head H'.

In practical case, H' will be so small that the horizontal vibration of an earthquake can be neglected to determine the water pressure.

b). Vertical vibration is concerned.

Let w' be the weight of water per unit volume, and w_e' be the resultant weight of water per unit volume, when the vertical vibration is considered.

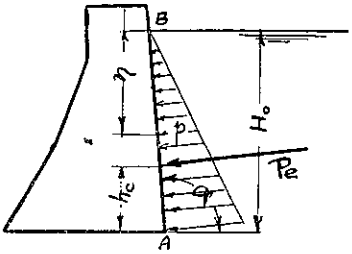


Fig. 9

By eq. (37), we have

$$w_e' = \lambda_o w' \text{ while } \lambda_o = \frac{\sqrt{(g \pm \alpha_v)^2 + \alpha_h^2}}{g} = \frac{g \mp \alpha_v}{g}$$

Since, $$w'_e = w'\left(1 \mp \frac{\alpha_v}{g}\right)$$

The pressure intensity at any depth η will be

$$p = w_e' \eta$$

Therefore, the total water pressure,

$$P_e = \int_0^{H_o} \frac{p}{\sin\alpha} d\eta = \frac{H_o^2 w_e'}{2\sin\alpha} = \frac{H_o^2 w'}{2\sin\alpha}\left(1 \mp \frac{\alpha_v}{g}\right) \quad \cdots (47)$$

Let P_o be the water pressure in ordinary condition, when $\alpha_v = 0$, we have

$$P_o = \frac{H_o^2 w'}{2\sin\alpha} \quad \cdots\cdots (47)'$$

Let P be the water pressure at any instant t during an earthquake, we have

$$P = P_o + P_o \frac{\alpha_v}{g} \sin\frac{2\pi}{T} t = \frac{H_o^2 w'}{2\sin\alpha}\left(1 + \frac{\alpha_v}{g} \sin\frac{2\pi}{T} t\right) \quad \cdots (48)$$

$$h_c = \frac{H_o}{3}$$

c). Effect of seismic tidal wave.

If the water level be changed suddenly by a seismic tidal wave, the equilibrium of water surface will be disturved, and in consequence, some difference of water level will take place everywhere; such as, the level difference between the back and the front of a quay wall between inner and outer harbour bounded by break-water.

In ordinary case, the seismic tidal wave will arrive when the main vibration of an earthquake is over, since it will be scarcely be happened that both effects coincide at the same time.

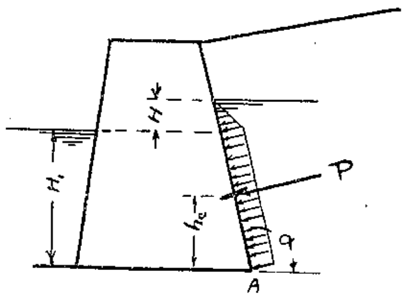


Fig. 10.

Let H be the head difference due to a seismic tidal wave, we have

$$P=\frac{w'H}{\sin\alpha}\left(\frac{H}{2}+H_1\right)$$

$$h_c=\frac{\frac{H^2}{3}+H_1H+H_1^2}{(H+2H_1)}$$

Art. 4. Resultant force acting on a retaining wall.

a). Rigid foundation, where resultant force acting on the base.

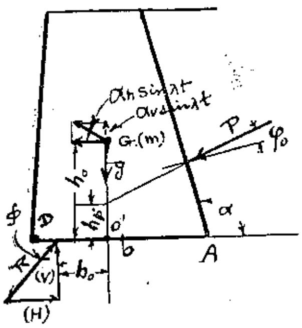


Fig. 11

If the foundation is rigid and the relation $b_o < O'D$ is held, the motion of the wall will be the same as the vibration of the earth, consequently the seismic force will be uniformly distributed on every parts of the wall. Since the seismic force acting on a wall is the product of the seismic acceleration and its mass.

Taking moment about O' we have

$$M=Ph_p\sin(\alpha+\varphi_o)+mh_o\alpha_h\sin\frac{2\pi}{T}t$$

In which,

$$P=P_o+(P_e-P_o)\sin\frac{2\pi}{T}t$$

Therefore, we have

$$M=P_oh_p\sin(\alpha+\varphi_o)+\{(P_e-P_o)h_p\sin(\alpha+\varphi_o)+mh_o\alpha_h\}\sin\frac{2\pi}{T}t \quad (50)$$

Let the vertical component of resultant force be (V),

$$(V)=P\cos(\alpha-\varphi_o)+m\left(g\pm\alpha_v\sin\frac{2\pi}{T}t\right)$$

$$=P_o\cos(\alpha-\varphi_o)+\{(P_e-P_o)\cos(\alpha-\varphi_o)\pm m\alpha\}\sin\frac{2\pi}{T}t+mg \cdot\cdot (50)'$$

Let the horizontal component of resultant force be (H),

$$(H)=P_o\sin(\alpha-\varphi_o)+\{(P_e-P_o)\sin(\alpha-\varphi_o)+m\alpha_h\}\sin\frac{2\pi}{T}t \cdot\cdot (50)''$$

Then we have

$$b_o=\frac{M}{(V)}=\frac{P_oh_p\sin(\alpha+\varphi_o)+\{(P_e-P_o)h_p\sin(\alpha+\varphi_o)+mh_o\alpha_h\}\sin\frac{2\pi}{T}t}{P_o\cos(\alpha-\varphi_o)+\{(P_e-P_o)\cos(\alpha-\varphi_o)\pm m\alpha_v\}\sin\frac{2\pi}{T}t+mg} \cdot\cdot (51)$$

$$\tan\phi=\frac{(H)}{(V)}=\frac{P_o\sin(\alpha-\varphi_o)+\{(P_e-P_o)\sin(\alpha-\varphi_o)+m\alpha_h\}\sin\frac{2\pi}{T}t}{P_o\cos(\alpha-\varphi_o)+\{(P_e-P_o)\sin(\alpha-\varphi_o)\pm m\alpha_v\}\sin\frac{2\pi}{T}t+mg}\quad\cdots\cdots(51)'$$

Above eqs. become maximum when $t=\frac{T}{4}$, since we have

$$\left.\begin{aligned}
M&=P_eh_p\sin(\alpha+\varphi_o)+mh_o\alpha_h\\
(V)&=P_e\cos(\alpha-\varphi_o)+m(g\pm\alpha_v)\\
(H)&=P_e\sin(\alpha-\varphi_o)+m\alpha_h\\
b_o&=\frac{M}{(V)}=\frac{P_eh_p\sin(\alpha+\varphi_o)+mh_o\alpha_h}{P_e\cos(\alpha-\varphi_o)+m(g\pm\alpha_v)}\\
\tan\phi&=\frac{(H)}{(V)}=\frac{P_e\sin(\alpha-\varphi_o)+m\alpha_h}{P_e\cos(\alpha-\varphi_o)+m(g\pm\alpha_v)}
\end{aligned}\right\}\quad\cdots\cdots(52)$$

In the case when the wall begins displacement-sliding or sinking, which is not elastic but permanent; eq. (36)′ will be substituted in eq. (52).

b). Rigid foundation, where resultant force passes out of the base.

In this case, the wall will begin a rocking motion, and at first we will study the motion of its free rocking.

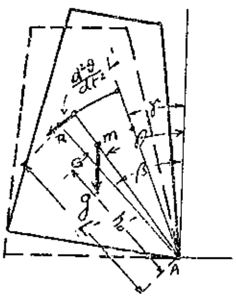


Fig. 12

Notations used,

G=center of gravity

A=center of percussion

R=center of spontaneous rotation

$h'=AG$

$L'=AR$

If a body rock from γ to β, we have the differential equation for the motion as follows.

$$-L'\frac{d^2\theta}{dt^2}+g\sin\theta=0\cdots\cdots\cdots\cdots\cdots\cdots(53)$$

$$\frac{d^2\theta}{dt^2}=\frac{g}{L'}\sin\theta$$

$$\frac{1}{2}\left(\frac{d\theta}{dt}\right)^2=-\frac{g}{L'}\cos\theta+C_1$$

When $$\frac{d\theta}{dt}=0, \qquad \theta=\gamma$$

Since $$C_1=\frac{g}{L'}\cos\gamma$$

Therefore, we have

$$t=\sqrt{\frac{L'}{2g}}\int_\gamma^\theta \frac{d\theta}{\sqrt{\cos\gamma-\cos\theta}}=\sqrt{\frac{L'}{4g}}\int_\gamma^\theta \frac{d\theta}{\sqrt{\sin^2\frac{\theta}{2}-\sin^2\frac{\gamma}{2}}}\cdots\cdots(54)$$

If $\theta=$small, we can simplify this integral.

$$\sqrt{\frac{L'}{4g}}\int_\gamma^\theta \frac{d\theta}{\sqrt{\left(\frac{\theta}{2}\right)^2-\left(\frac{\gamma}{2}\right)^2}}=\sqrt{\frac{L'}{g}}\int_\gamma^\theta \frac{d\theta}{\sqrt{\theta^2-\gamma^2}}=\sqrt{\frac{L'}{g}}\left\{\cosh^{-1}\frac{\theta}{\gamma}-\cosh^{-1}\frac{\gamma}{\gamma}\right\}$$

$$=\sqrt{\frac{L'}{g}}\cosh\frac{\theta}{\gamma}$$

$$\therefore \qquad \theta=\gamma\cosh\sqrt{\frac{g}{L'}}\,t=\frac{\gamma}{2}\left(e^{\sqrt{\frac{g}{L'}}t}+e^{-\sqrt{\frac{g}{L'}}t}\right)\cdots\cdots\cdots\cdots(54)'$$

Period $$T=4\sqrt{\frac{L'}{g}}\cosh^{-1}\frac{\beta}{\gamma} \quad\cdots\cdots\cdots\cdots\cdots\cdots(54)''$$

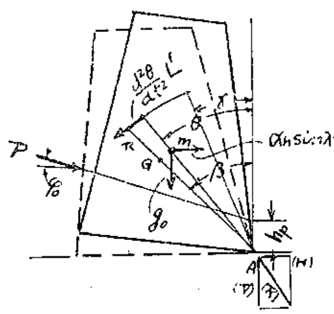


Fig. 13

The forced rocking of a wall due to earth pressure and the seismic force will be expressed by the following differetial equation.

Taking moment about A, we have

$$-mL'^2\frac{d^2\theta}{dt^2}+mg_oh'\sin\theta=Ph_p\sin(\alpha+\varphi_o)$$
$$+mh_o\alpha_h\sin\lambda t \quad\cdots\cdots\cdots\cdots\cdots\cdots(55)$$

In which,

$$P=P_o+(P_e-P_o)\sin\lambda t$$

$$g_o=g+\alpha_v\sin\lambda t$$

$$\lambda=\frac{2\pi}{T}$$

Then we have

$$-\frac{d^2\theta}{dt^2}+\frac{\alpha_vh'}{L'^2}\sin\theta\sin\lambda t+\frac{gh'}{L'^2}\sin\theta=\frac{P_oh_p\sin(\alpha+\varphi_0)}{mL'^2}$$

$$+\left\{\frac{(P_e-P_o)h_p\sin(\alpha+\varphi_o)+mh_o\alpha_h}{mL'^2}\right\}\sin\lambda t \quad \cdots\cdots\cdots\cdots(56)$$

Put

$$A=\frac{\alpha_v h'}{L'^2}, \qquad B=\frac{gh'}{L'^2}$$

$$C=\frac{P_o h_p\sin(\alpha+\varphi_o)}{mL'^2}, \quad D=\frac{(P_e-P_o)h_p\sin(\alpha+\varphi_o)+mh_o\alpha_h}{mL'^2}$$

We have

$$\frac{d^2\theta}{dt^2}-A\sin\theta\sin\lambda t-B\sin\theta=-(C+D\sin\lambda t)\cdots\cdots\cdots(56)'$$

This equation will be solved when $\theta\doteqdot\sin\theta$, and $A=0$, in other word, when the wall is comparatively high and vertical acceleration is small. The former equation will be written,

$$\frac{d^2\theta}{dt^2}-B.\theta=-(C+D\sin\lambda t)\cdots\cdots\cdots\cdots\cdots\cdots(57)$$

The general solution of eq. (57) is

$$\theta=C'_1e^{\sqrt{B}t}+C_2'e^{-\sqrt{B}t}$$

Where,

$$C_1'=C_1+\int\frac{-(C+D\sin\lambda t)e^{-\sqrt{B}t}}{e^{-\sqrt{B}t}\dfrac{d(e^{\sqrt{B}t})}{dt}-e^{\sqrt{B}t}\dfrac{d(e^{-\sqrt{B}t})}{dt.}}dt=C_1+\frac{C}{2B}e^{-\sqrt{B}t}$$

$$+\frac{De^{-\sqrt{B}t}(\sqrt{B}\sin\lambda t+\lambda\cos\lambda t)}{2\sqrt{B}(\lambda^2+B)}$$

$$C_2'=C_2+\int\frac{(C+D\sin\lambda t)e^{\sqrt{B}t}}{e^{-\sqrt{B}t}\dfrac{d(e^{\sqrt{B}t})}{dt}-e^{\sqrt{B}t}\dfrac{d(e^{\sqrt{B}t})}{dt}}dt=C_2+\frac{C}{2B}e^{\sqrt{B}t}$$

$$+\frac{De^{\sqrt{B}t}(\sqrt{B}\sin\lambda t-\lambda\cos\lambda t)}{2\sqrt{B}(\lambda^2+B)}$$

Since, we have

$$\theta=C_1e^{\sqrt{B}t}+C_2e^{-\sqrt{B}t}+\frac{C}{B}+\frac{D}{\lambda^2+B}\sin\lambda t \quad\cdots\cdots\cdots\cdots(58)$$

$$\frac{d\theta}{dt}=\left\{C_1e^{\sqrt{B}t}-C_2e^{-\sqrt{B}t}\right\}\sqrt{B}+\frac{D}{\lambda^2+B}\sin\lambda t\cdots\cdots\cdots(58)'$$

Let T_o be the period, and t_o be the time when the wall begins the rocking, we have the following boundary conditions,

$$1)\left\{\begin{array}{l}\theta=\beta\\ t=t_o\\ \dfrac{d\theta}{dt}=0\end{array}\right. \qquad 2)\left\{\begin{array}{l}\theta=\gamma\\ t=\dfrac{T_o}{4}\\ \dfrac{d\theta}{dt}=0\end{array}\right.$$

By substituting these conditions in eqs. (58) and (58)′, we have

$$\left.\begin{array}{l}\beta=C_1e^{\sqrt{B}t_o}+C_2e^{-\sqrt{B}t_o}+\dfrac{C}{B}+\dfrac{D}{\lambda^2+B}\sin\lambda t_o\\ O=C_1e^{\sqrt{B}t_o}-C_2e^{-\sqrt{B}t_o}+\dfrac{D\lambda}{\sqrt{B}(\lambda^2+B)}\cos\lambda t_o\\ \gamma=C_1e^{\sqrt{B}\frac{T_o}{4}}+C_2e^{-\sqrt{B}\frac{T_o}{4}}+\dfrac{C}{B}+\dfrac{D}{\lambda^2+B}\sin\lambda t_o\\ O=C_1e^{\sqrt{B}\frac{T_o}{4}}-C_2e^{-\sqrt{B}\frac{T_o}{4}}\dfrac{D\lambda}{\sqrt{B}(\lambda^2+B)}\cos\lambda t_o\end{array}\right\}\cdots\cdots(58)''$$

By solving these symultaneous equations, we can find the values of C_1, C_2, t_o and T_o.

The resultant force acting at the heel A will be maximum at the instant, when the wall begins rocking, since by substituting t_o for t we have the maximum horizontal and vertical components of the resultant force.

$$(V)=P_o\cos(\alpha-\varphi_o)+(P_e-P_o)\cos(\alpha-\varphi_o)\sin\lambda t_o+m(g+\alpha_v\sin\lambda t_o)\cdots(59)$$

$$(H)=P_o\sin(\alpha-\varphi_o)+(P_e-P_o)\sin(\alpha-\varphi_o)\sin\lambda t_o+m\alpha_h\sin\lambda t_o\cdots\cdots(59)'$$

No wall is safe, when it is designed to begin rocking motions, since the base at least must be so designed that the resultant force does not pass out of the base.

c). Elastic foundation, where resultant force acting on the base.

Notations used,

m=mass of a wall

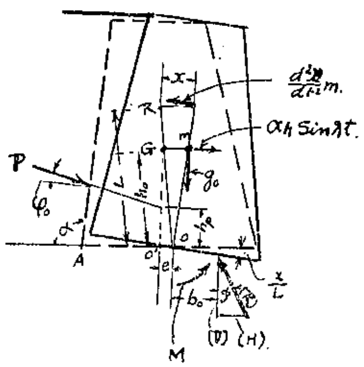


Fig. 14.

E=force per unit area required to depress an elastic base for unit depth.

I=horizontal sectional moment of inertia of a wall at its base.

O=center of the base, and also center of percussion.

r=radius of gyration around G in the cross section of a wall.

$$L=h_o+\frac{r^2}{h_o}$$

M_o=resisting moment about O in statical condition.

The following differential equation will express the self oscillation of a wall.

$$-mL\frac{d^2x}{dt^2}+mgh_o\frac{x}{L}=EI\frac{x}{L}$$

Put $B=\dfrac{EI-mgh_o}{mL^2}$, then we have a differential equation of a simple harmonic motion,

$$\frac{dx^2}{dt^2}=-Bx \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(60)$$

Solving, we have

$$x=\frac{a}{2}\sin\sqrt{B}\,t, \text{ in which } a=\text{amplitude} \quad \cdots\cdots\cdots(61)$$

For the period of the self oscillation, we have

$$T=\frac{2\pi}{\sqrt{B}}=\frac{2\pi L\sqrt{m}}{\sqrt{EI-mgh_o}} \quad \cdots\cdots\cdots\cdots\cdots\cdots(61)'$$

The forced oscillation of a wall due to the periodic earth pressure and the earthquake will be expressed by the following differential equation.

By taking moment about O, we have

$$-mL\frac{d^2x}{dt^2}+mg_o\left(h_o\frac{x}{L}-e\right)+Ph_p\sin(\alpha+\varphi_o)+h_om\alpha_h\sin\lambda t=EI\frac{x}{L}+M_o$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(62)$$

In which,

$$P=P_o+(P_e-P_o)\sin\lambda t$$

$$g_o=g+\alpha_v\sin\lambda t$$

$$M_o = P_o h_p \sin(\alpha+\varphi_o) - mge.$$

By substituting in (62), we have

$$\frac{d^2x}{dt^2} - \left\{ \frac{(P_e - P_o)h_p \sin(\alpha+\varphi_o) + mh_o\alpha_h - me\alpha_v}{mL} \right\} \sin\lambda t$$

$$- \frac{h_o\alpha_v}{L^2} x \sin\lambda t + \frac{x}{mL^2}(EI - mgh_o) = 0 \cdots\cdots\cdots\cdots (62)'$$

Put

$$\left.\begin{aligned} D &= \frac{(P_e - P_o)h_p \sin(\alpha+\varphi_o) + mh_o\alpha_h - me\alpha_v}{mL} \\ B &= \frac{EI - mgh_o}{mL^2}, \qquad A = \frac{h_o\alpha_v}{L^2} \end{aligned}\right\} \cdots\cdots (63)$$

Then we have

$$\frac{d^2x}{dt^2} + (B - A\sin\lambda t)x = D\sin\lambda t \quad \cdots\cdots\cdots\cdots (63)'$$

This is non-homogeneous linear equation, and its corresponding homogeneous linear equation is

$$\frac{d^2x}{dt^2} + (B - A\sin\lambda t)x = 0$$

Let $\omega = \sqrt{B - A\sin\lambda t}$, we have the particular solution.

$$x = C'\cos\omega t + C''\sin\omega t.$$

Where C' and C'' will be determined as follows.

$$C' = C_1 + D\int \frac{\sin\lambda t \sin\omega t}{\sin\omega t \dfrac{d(\cos\omega t)}{dt} - \cos\omega t \dfrac{d(\sin\omega t)}{dt}} dt$$

$$= C_1 - D\int \frac{2\omega \sin\lambda t \sin\omega t}{2\omega - A\lambda t \cos\lambda t} dt$$

$$C'' = C_2 - D\int \frac{\sin\lambda t \cos\omega t}{\sin\omega t \dfrac{d(\cos\omega t)}{dt} - \cos\omega t \dfrac{d(\sin\omega t)}{dt}} dt$$

$$= C_2 + D\int \frac{2\omega \sin\lambda t \cos\omega t}{2\omega - A\lambda t \cos\lambda t} dt$$

Since we have the general solution.

$$x=\left\{C_1-D\int\frac{2\omega\sin\lambda t\sin\omega t}{2\omega-A\lambda t\cos\lambda t}dt\right\}\cos\omega t$$

$$+\left\{C_2+D\int\frac{2\omega\sin\lambda t\cos\omega t}{2\omega-A\lambda t\cos\lambda t}dt\right\}\sin\omega t\quad\cdots\cdots\cdots\cdots\cdots\cdots(64)$$

Also,

$$\frac{dx}{dt}=\left\{C_2+D\int\frac{2\omega\sin\lambda t\cos\omega t}{2\omega-A\lambda t\cos\lambda t}dt\right\}\left\{\frac{2\omega-A\lambda t\cos\lambda t}{2\omega}\right\}\cos\omega t$$

$$-\left\{C_1-D\int\frac{2\omega\sin\lambda t\sin\omega t}{2\omega-A\lambda t\cos\lambda t}dt\right\}\left\{\frac{2\omega-A\lambda t\cos\lambda t}{2\omega}\right\}\sin\omega t\cdot\cdot(64)'$$

If $\int\frac{2\omega\sin\lambda t\cos\omega t}{2\omega-A\lambda t\cos\lambda t}dt$ and $\int\frac{2\omega\sin\lambda t\sin\omega t}{2\omega-A\lambda t\cos\lambda t}dt$ be calculated, C_1 and C_2 will be found by the boundary conditions.

$$(1)\quad\begin{cases}x=0\\t=0\end{cases}\qquad\qquad(2)\quad\begin{cases}\frac{dx}{dt}=0\\t=0\end{cases}$$

Thus the general solution will be determined, but the calculation is too difficult; since the following method, which will be delivered from the special case, is preferred to find the approximate solution.

In the special case when $A=0$ (i. e. $\alpha_v=0$), the general solution will be found at once.

$$\omega=\sqrt{B},$$

$$x=\left[C_1-\frac{D}{2\sqrt{B}}\left\{\frac{\sin(\lambda-\sqrt{B})t}{\lambda-\sqrt{B}}-\frac{\sin(\lambda+\sqrt{B})t}{\lambda+\sqrt{B}}\right\}\right]\cos\sqrt{B}\,t$$

$$+\left[C_2-\frac{D}{2\sqrt{B}}\left\{\frac{\cos(\lambda+\sqrt{B})t}{\lambda+\sqrt{B}}+\frac{\cos(\lambda-\sqrt{B})t}{\lambda-\sqrt{B}}\right\}\right]\sin\sqrt{B}\,t$$

$$\frac{dx}{dt}=\left[C_2-\frac{C}{2\sqrt{B}}\left\{\frac{\cos(\lambda+\sqrt{B})t}{\lambda+\sqrt{B}}+\frac{\cos(\lambda-\sqrt{B})t}{\lambda-\sqrt{B}}\right\}\right]\sqrt{B}\cos\sqrt{B}\,t$$

$$-\left[C_1-\frac{D}{2\sqrt{B}}\left\{\frac{\sin(\lambda-\sqrt{B})t}{\lambda-\sqrt{B}}-\frac{\sin(\lambda+\sqrt{B})t}{\lambda+\sqrt{B}}\right\}\right]\sqrt{B}\sin\sqrt{B}\,t$$

By the boundary conditions $\begin{cases} x=0 \\ t=0 \end{cases}$ $\begin{cases} \frac{dx}{dt}=0 \\ t=0 \end{cases}$ we have

$$C_1=0 \quad \text{and} \quad C_2=\frac{D\lambda}{\sqrt{B}(\lambda^2-B)}$$

Since we have

$$x=\frac{D}{\sqrt{B}(\lambda^2-B)}\{\lambda \sin\sqrt{B}t-\sqrt{B}\sin\lambda t\} \quad \cdots\cdots\cdots\cdots(65)$$

$$\frac{dx}{dt}=\frac{D\lambda}{(\lambda^2-B)}\{\cos\sqrt{B}t-\cos\lambda t\}\cdots\cdots\cdots\cdots(65)'$$

$$\frac{d^2x}{dt^2}=-\frac{D\lambda}{(\lambda^2-B)}\{\sqrt{B}\sin\sqrt{B}t-\lambda\sin\lambda t\}\cdots\cdots\cdots\cdots(65)''$$

By putting $\frac{dx}{dt}=0$, we will find t, which gives maximum or minimum for x, and x is minimum when $t=0$, and also x is maximum when

$$t=t_o=\frac{2\pi}{\sqrt{B}+\lambda} \quad \cdots\cdots\cdots\cdots(66)$$

The period of the forced oscillation will be,

$$T_o=T.\frac{2\pi L\sqrt{m}}{\sqrt{EI-mgh_o}}, \text{ in which } T \text{ is the seismic period.} \cdots\cdots(67)$$

The resisting moment about O will be maximum when x is maximum, i. e. when $t=t_0$.

$$M=M_o+EI\frac{x}{L}=\{P_o+(P_e-P_o)\sin\lambda t\}h_p\sin(\alpha+\varphi_o)+\alpha_h mh_o\sin\lambda t$$

$$+mg_o\left(h_o\frac{x}{L}-e\right)-mL\frac{d^2x}{dt^2}$$

Where,

$$M_o=P_oh_p\sin(\alpha+\varphi_o)-mge$$

$$x_o=\frac{D}{\sqrt{B}(\lambda^2-B)}\{\lambda\sin\sqrt{B}\,t_o-\sqrt{B}\sin\lambda\,t_o\}$$

$$\frac{d^2x}{dt^2}=-\frac{D\lambda}{(\lambda^2-B)}\{\sqrt{B}\sin\sqrt{B}\,t_o-\lambda\sin\lambda t_o\}$$

$$t_o=\frac{2\pi}{\sqrt{B}+\lambda} \qquad B=\frac{EI-mgh_o}{mL^2}$$

$$g_0 = g \qquad D = \frac{(P_e - P_o)h_p \sin(\alpha + \varphi_o) + mh_o\alpha_h}{mL}$$

Since, we have

$$M = P_o h_p \sin(\alpha + \varphi_o) - mge + \frac{EID}{L\sqrt{B}(\lambda^2 - B)}[\lambda \sin\sqrt{B}\,t_o - \sqrt{B}\sin\lambda t_o] \cdots (68$$

or,

$$M = \{P_o + (P_e - P_o)\sin\lambda t_o\}\,h_p \sin(\alpha + \varphi_o) + \alpha_h m h_o \sin\lambda t_o$$

$$+ mg\Big[\frac{D}{L\sqrt{B}(\lambda^2 - B)}\{\lambda \sin\sqrt{B}\,t_o - \sqrt{B}\sin\lambda t_o\} - e\Big]$$

$$- \frac{LmD\lambda}{(\lambda^2 - B)}[\sqrt{B}\sin\sqrt{B}\,t_o - \lambda\sin\lambda t_o] \cdots\cdots\cdots (68)'$$

The horizontal component and the vertical component of the maximum resultant force will be

$$(H) = P_o \sin(\alpha - \varphi_o) + \{(P_e - P_o)\sin(\alpha - \varphi_o) + m\alpha_h\}\sin\lambda t_o \cdots\cdots (69)$$

$$(V) = P_o \cos(\alpha - \varphi_o) + (P_e - P_o)\cos(\alpha - \varphi_o)\sin\lambda t_o + mg \cdots\cdots (69)'$$

And also the leverage from 0 will be,

$$b_o = \frac{M}{(V)} = \frac{P_o h_p \sin(\alpha + \varphi_o) - mge + \frac{EID}{L\sqrt{B}(\lambda^2 - B)}[\lambda\sin\sqrt{B}t_o - \sqrt{B}\sin\lambda t_o]}{P_o \cos(\alpha - \varphi_o) + (P_e - P_o)\cos(\alpha - \varphi_o)\sin\lambda t_o + mg} \cdots\cdots (69)''$$

If α_v is small comparing to g, A will be negligibly small against B, since $\sqrt{B - A\sin\lambda t}$ will be nearly equal to $\sqrt{B}$, in consequence, eq. (65) will express the general solution of a forced oscillation where vertical vibration being considered.

Therefore, we have

$$M = P_o h_p \sin(\alpha + \varphi_o) - m\,ge + \frac{EID}{L\sqrt{B}(\lambda^2 - B)}[\lambda\sin\sqrt{B}t_o - \sqrt{B}\sin\lambda t_o]$$

or, $$= \{P_o + P_o - P_e)\sin\lambda t_o\}h_p \sin(\alpha + \varphi_o) + \alpha_h m h_o \sin\lambda t_o$$

$$+ m(g + \alpha_v \sin\lambda t_o)\Big[\frac{D}{L\sqrt{B}(\lambda^2 - B)}\{\lambda\sin\sqrt{B}\,t_o - \sqrt{B}\sin\lambda\,t_o\} - e\Big]$$

$$+\frac{mLD\lambda}{\lambda^2-B}[\sqrt{B}\sin\sqrt{B}\,t_0-\lambda\sin\lambda t_0] \cdots\cdots (70)$$

$$(H)=P_o\sin(\alpha-\varphi_o)+\{(P_e-P_o)\sin(\alpha-\varphi_o)+m\alpha_h\}\sin\lambda t_0 \cdots\cdots (71)$$

$$(V)=P_o\cos(\alpha-\varphi_o)+\{(P_e-P_o)\cos(\alpha-\varphi_o)+m\alpha_v\}\ \sin\lambda t_0+mg\cdots (71)'$$

$$b_0=\frac{M}{(V)}=\frac{P_oh_p\sin(\alpha+\varphi_o)-mge+\dfrac{EID}{L\sqrt{B}(\lambda^2-B)}[\lambda\sin\sqrt{B}\,t_0-\sqrt{B}\sin\lambda t_0]}{P_o\cos(\alpha-\varphi_o)+\{(P_e-P_o)\cos(\alpha-\varphi_o)+m\alpha_v\}\sin\lambda t_0+mg} \cdots\cdots (71)''$$

where

$$t_0=\frac{2\pi}{\sqrt{B}+\lambda}$$

$$D=\frac{(P_e-P_o)h_p\sin(\alpha+\varphi_o)+mh_o\alpha_h-me\alpha_v}{mL} \qquad B=\frac{EI-mgh_o}{mL^2}$$

Art. 5. Resultant force acting on a dam.

a). Rigid foundation where resultant force acting on the base.

All relations held in Art. 4 will be applied in the case of a dam, because a dam will be forced by the periodical water pressure instead of the earth pressure.

By the similar reductions, we have

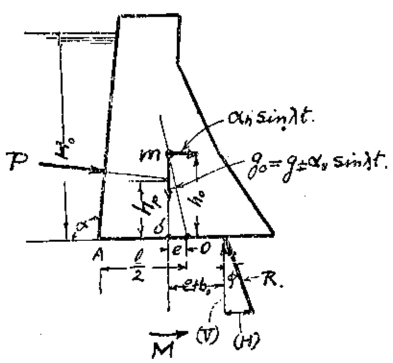


Fig. 15.

$$P=\frac{H_0^2w'}{2\sin\alpha}\left\{1+\frac{\alpha_v}{g}\sin\frac{2\pi}{T}t\right\}$$

$$=P_o\left\{1+\frac{\alpha_v}{g}\sin\frac{2\pi}{T}t\right\}$$

$$M=P_oh_p\sin\alpha\left\{1+\frac{\alpha_v}{g}\sin\frac{2\pi}{T}t\right\}$$

$$+mh_o\alpha_h\sin\frac{2\pi}{T}t$$

$$\left.\begin{aligned}(H)&=P_o\left\{1+\frac{\alpha_v}{g}\sin\frac{2\pi}{T}t\right\}\sin\alpha\\&\quad+m\alpha_h\sin\frac{2\pi}{T}t\\(V)&=(P_o\cos\alpha+mg)\left\{1+\frac{\alpha_v}{g}\sin\frac{2\pi}{T}t\right\}\end{aligned}\right\} \quad (72)$$

$$e+b_o=\frac{M}{(V)}=\frac{P_o h_p \sin\alpha\left\{1+\frac{\alpha_v}{g}\sin\frac{2\pi}{T}t\right\}+mh_o\alpha_h\sin\frac{2\pi}{T}t}{(P_o\cos\alpha+mg)\left\{1+\frac{\alpha_v}{g}\sin\frac{2\pi}{T}t\right\}}$$

We have maximum values when $t=\frac{T}{4}$

$$\left.\begin{aligned} M&=P_o h_p \sin\alpha\left(1\pm\frac{\alpha_v}{g}\right)+mh_o\alpha_h \\ (H)&=P_o\left(1\pm\frac{\alpha_v}{g}\right)\sin\alpha+m\alpha_h \\ (V)&=\left(1\pm\frac{\alpha_v}{g}\right)(P_o\cos\alpha+mg) \end{aligned}\right\}\cdots\cdots\cdots\cdots(73)$$

$$\left.\begin{aligned} e+b_o&=\frac{P_o h_p\sin\alpha\left(1\pm\frac{\alpha_v}{g}\right)+mh_o\alpha_h}{\left(1\pm\frac{\alpha_v}{g}\right)(P_o\cos\alpha+mg)} \\ \tan\phi&=\frac{(\cos\alpha+P_o\,mg)\left(1\pm\frac{\alpha_v}{g}\right)}{P_o\left(1\pm\frac{\alpha_v}{g}\right)\sin\alpha+m\alpha_h} \end{aligned}\right\}\cdots\cdots\cdots\cdots(73)'$$

b). Rigid foundation where resultant force passes out of the base.

If a dam be forced by such a periodic pressure as to begin a rocking motion the water penetrates under its base to cause the sudden increase of upward pressure, by which a fatal damage will sometimes be given to the dam. Therefore, for the design of a dam the following condition is absolutely necessary.

$$l>2(b_o-e)\quad\text{i. e.}\quad l>2\left\{\frac{P_o h_p\sin\alpha\left(1\pm\frac{\alpha_v}{g}\right)+mh_o\alpha_h}{(P_o\cos\alpha+mg)\left(1\pm\frac{\alpha_v}{g}\right)}-e\right\}\cdots\cdots(74)$$

c). Elastic foundation, where resultant force acting on the base.

All relations held in Art. 4 will be also applied for a dam.

$$P=P_o\left(1+\frac{\alpha_v}{g}\sin\lambda t\right)$$

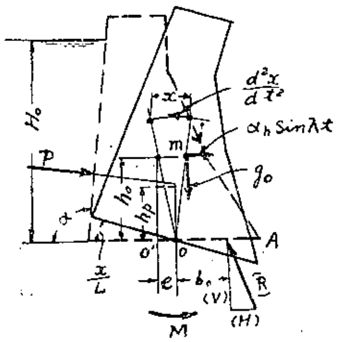


Fig. 16.

$$P_o = \frac{w' H_o}{2 \sin\alpha}$$

$$g_o = g + \alpha_v \sin \lambda t.$$

$$M_o = P_o h_p \sin\alpha - mge$$

Taking moment about 0, we have

$$M = M_o + EI\frac{x}{L} = -mL\frac{d^2x}{dt^2} + mg_o\left(h_o\frac{x}{L} - e\right)$$

$$+ Ph_p \sin\alpha + mh_o\alpha_h \sin\lambda t$$

i. e.

$$\frac{d^2x}{dt^2} - \left\{\frac{h_p\frac{\alpha_v}{g}P_o \sin\alpha + mh_o\alpha_h - me\alpha_v}{mL}\right\}\sin\lambda t - \frac{h_o\alpha_v}{L^2}x\sin\lambda t + \frac{EI - mgh_o}{mL^2}x = 0$$

Put

$$\left.\begin{aligned} D &= \frac{P_o h_p\frac{\alpha_v}{g}\sin\alpha + mh_o\alpha_h - me\alpha_v}{mL} \\ A &= \frac{h_o\alpha_v}{L^2} \qquad B = \frac{EI - mgh_o}{mL^2} \end{aligned}\right\}\cdots\cdots\cdots\cdots(75)$$

Then we have

$$\frac{d^2x}{dt^2} + (B - A\sin\lambda t)x = D\sin\lambda t \cdots\cdots\cdots\cdots\cdots(75)'$$

General solution is,

$$x = \left\{C_1 - D\int\frac{2\omega\sin\lambda t\sin\omega t}{2\omega - A\lambda t\cos\lambda t}dt\right\}\cos\omega t + \left\{C_2 + D\int\frac{2\omega\sin\lambda t\cos\omega t\,dt}{2\omega - A\lambda t\cos\lambda t}\right\}\sin\omega t$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(75)''$$

in which, $\omega = \sqrt{B - A\sin\lambda t}$

In special case, when $A=0$ i. e. $\alpha_v = 0$ and $\omega = \sqrt{B}$, we have at once,

$$x = \frac{D}{\sqrt{B}(\lambda^2 - B)}\{\lambda\sin\sqrt{B}\,t - \sqrt{B}\sin\lambda t\} \cdots\cdots\cdots (76)$$

x will be maximum when $$t=t_o=\frac{2\pi}{\sqrt{B}+\lambda}$$

Since we have the maximum moment and resultant force as the following.

$$M=P_oh_p\sin\alpha-mge+\frac{EID}{L\sqrt{B}(\lambda^2-B)}\{\lambda\sin\sqrt{B}\,t_o-\sqrt{B}\sin\lambda t_o\}$$

$$\text{or }=P_oh_p\sin\alpha+\alpha_h mh_o\sin\lambda t_o+mg\Big[\frac{D}{L\sqrt{B}(\lambda^2-B)}\{\lambda\sin\sqrt{B}\,t_o-\sqrt{B}\sin\lambda t_o\}-e\Big]$$

$$+\frac{mLD\lambda}{(\lambda^2-B)}\{\sqrt{B}\sin\sqrt{B}\,t_o-\lambda\sin\lambda t_o\}\cdots\cdots\cdots\cdots(77)$$

$$\left.\begin{aligned}(H)&=P_o\sin\alpha+m\alpha_h\sin\lambda t_o\\(V)&=P_o\cos\alpha+mg\\b_o&=\frac{M}{(V)}=\frac{P_oh_p\sin\alpha-mge+\dfrac{EID}{L\sqrt{B}(\lambda^2-B)}\{\lambda\sin\sqrt{B}\,t_o-\sqrt{B}\sin\lambda t_o\}}{P_o\cos\alpha+mg}\end{aligned}\right\}(77)'$$

$$\text{where, }\quad B=\frac{EI-mgh_o}{mL^2}\quad\text{and}\quad D=\frac{h_o\alpha_h}{L}$$

If α_v be relatively small, eq. (76) will nearly express the forced oscillation of a dam for which the vertical vibration is acting. Then we have the maximum resisting moment about 0 and the resultant force as follows.

$$M=P_oh_p\sin\alpha-mge+\frac{EID}{L\sqrt{B}(\lambda^2-B)}\{\lambda\sin\sqrt{B}\,t_o-\sqrt{B}\sin\lambda t_o\}\quad(78)$$

$$\text{or, }=P_o\Big(1+\frac{\alpha_v}{g}\sin\lambda t_o\Big)h_p\sin\alpha+mh_o\alpha_h\sin\lambda t_o$$

$$+m(g+\alpha_v\sin\lambda t_o)\Big[\frac{D}{L\sqrt{B}(\lambda^2-B)}\{\lambda\sin\sqrt{B}\,t_o-\sqrt{B}\sin\lambda t_o\}-e\Big]$$

$$+\frac{mLD\lambda}{(\lambda^2-B)}[\sqrt{B}\sin\sqrt{B}\,t_o-\lambda\sin\lambda t_o]$$

$$(H)=\Big(P_o+\frac{\alpha_v}{g}\sin\lambda t_o\Big)\sin\alpha+m\alpha_h\sin\lambda t_o$$

$$(V)=P_o\Big(1+\frac{\alpha_v}{g}\sin\lambda t_o\Big)\cos\alpha+mg\Big(1+\frac{\alpha_v}{g}\sin\lambda t_o\Big)-$$

$$b_o=\frac{P_oh_p\sin\alpha-mge+\dfrac{EID}{L\sqrt{B}(\lambda^2-B)}\{\lambda\sin\sqrt{B}\,t_o-\sqrt{B}\sin\lambda t_o\}}{(P_o\cos\alpha+mg)\left(1+\dfrac{\alpha_v}{g}\sin\lambda t_o\right)}$$

In which $t_o=\dfrac{2\pi}{\sqrt{B}+\lambda}$, and also D and B will be determined by eq. (75).

d). Numerical example of a dam, where the upward water pressure on the base being neglected.

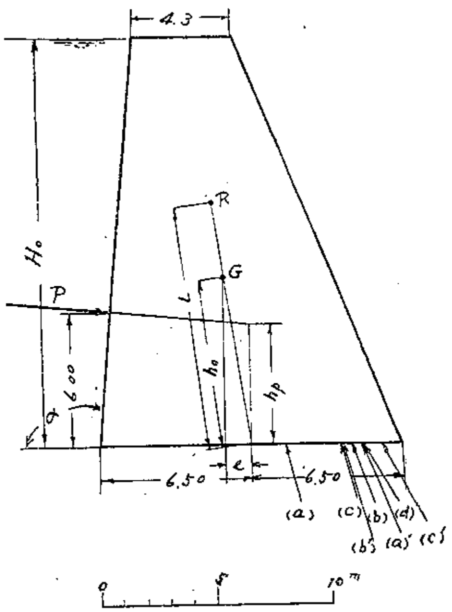


Fig. 17.

Height of dam = 18 meters.
Base " " = 13 meters
Top " " = 4.3 meters
α = 95 degrees.
Cross sectional area = 155.6 sq. m.
Moment of inertia = 3,850 m^4
h_o = 7.5 m. h_p = 5.45 m. e = 1.1 m.

$$r=\sqrt{\frac{3,850}{155.6}}=4.97 \text{ m.}$$

$$L=h_o+\frac{r^2}{h_o}=10.8 \text{ m.}$$

$$P_o=\frac{1\times 18^2}{2\times\sin 95^\circ}=162.6 \text{ Kg. tons.}$$

$$W=358 \text{ tons.}$$

$$m=\frac{W}{g}=36.5$$

Case 1. Rigid foundation.

(*a*) $\alpha_h=\alpha_v=0$

$M=P_o\,h_p\sin 95^\circ-eW$ $=489$ m. t.

$$b_o=\frac{489}{372.2}=1.313 \text{ m.}$$

(*b*) $\alpha_h=4$ m/sec². $\alpha_v=0$

$M=P_oh_p\sin 95^\circ+mh_o\alpha_h-We.$ $=1,858$ m. t.

$$b_o=\frac{1,584}{372.2}=4.255 \text{ m.}$$

(*c*) $\alpha_h=4$ m/sec². $\alpha_v=1.8$ m/sec².

$M=P_oh_p\sin 95^\circ\left(1+\dfrac{\alpha_n}{g}\right)+mh_o\alpha_h-m(g+\alpha_v)e$ $=1,678$ m. t.

$$b_o=\frac{1,678}{372.2\times 1.1837}=3.82 \text{ m.}$$

(d). $\alpha_h=4.$ m/sec². $\alpha_v=-1.8$ m./sec².

$$M=P_oh_p\sin 95°\left(1-\frac{\alpha_v}{g}\right)+mh_o\alpha_h-m(g-\alpha_v)e \qquad =1,494.7 \text{ m. t.}$$

$$b_o=\frac{1,494.7}{372.2\times 0.8163}=4.915 \text{ m}$$

Case 2. Elastic foundation.

Let

$E=50$ t./sq. m. for 1 cm. depression $=5,000$ t./sq. m./m. dep.

$$I=\frac{1\times 13^3}{12}=183 \text{ m}^4.$$

$EI=915,000$ t. m².

(a)'. $\alpha_h=4$ m/sec². $\alpha_v=0$

$$D=\frac{h_o\alpha_h}{L}=2.778$$

$$B=\frac{EI-mgh_o}{mL^2}=214.6 \qquad \sqrt{B}=14.65$$

$$T=1.8 \text{ sec.} \qquad \lambda=\frac{2\pi}{1.8}=3.49 \qquad t_o=\frac{2\tau}{3.49+14.65}=0.3463 \text{ sec.}$$

$$x=\frac{D}{\sqrt{B}(\lambda^2-B)}\{\lambda\sin\sqrt{B}\,t_o-\sqrt{B}\sin\lambda t_o\} \qquad =0.0159 \text{ m.}$$

$$M=P_oh_p\sin 95°-We+EI\frac{x}{L}=883-394+915,000\times\frac{0.0159}{10.8}=1,834 \text{ t. m.}$$

$$b_o=\frac{1,834}{372.2}=4.92 \text{ m.}$$

(b)' $\alpha_h=4$ m/sec². $\alpha_v=1.8$ m/sec².

$$A=\frac{h_o\alpha_v}{L^2}=0.1157$$

$$\omega=\sqrt{B-A\sin\lambda t}=\sqrt{214.6-0.1557\sin\lambda t}\doteqdot\sqrt{B}=14.65$$

$$D=\frac{P_oh_p\sin 95°\frac{\alpha_v}{g}+mh_o\alpha_h-me\alpha_v}{mL}=3.025$$

$x=0.01733$ m.

$$M=883-394+915,000\times\frac{0.01733}{10.8}=1,957 \text{ m. t.}$$

$$b_o=\frac{1,957}{372.2\left(1+\frac{1.8}{9.8}\right)}=4.43 \text{ m.}$$

(c)' $\alpha_h=4$ m./sec². $\alpha_v=-1.8$ m./sec².

$$D=\frac{-162+1,095+65.8}{394}=2.535 \qquad x=0.01452 \text{ m.}$$

$M=1,719$ m. t. $b_o=5.65$ m.

Art. 6. Stability of a retaining wall and a dam.

a). Stability against overturning.

No wall having a certain larger dimension will be overturned, but it will begin a rocking motion, when the wall is standing on a solid foundation and is acted by severe earthquake so that the resultant force passes out of the base.

The relation between the seismic force and the absolute dimensions of a wall, which is merely standing on a solid foundation, will easily be found as follows.

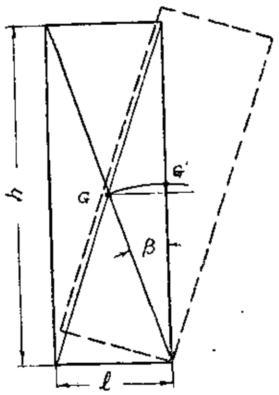


Fig. 18.

Let W_e=work done by an earthquake during a quarter cycle.

W =work done required to raise the center of gravity as much as the wall is overturned.

$$dW_e=-m\frac{d^2y}{dt^2}dy=-m\frac{d^2y}{dt^2}\cdot\frac{dy}{dt}dt$$

$$=m\left(\alpha_h\sin\frac{2\pi}{T}t\right)\left(\frac{\beta\pi}{T}\cos\frac{2\pi}{T}t\right)dt$$

$$=m\frac{\pi^3\beta^2}{T^3}\sin\frac{4\pi}{T}t$$

$$W_e=m\frac{\pi^3\beta^2}{T^3}\int_0^{\frac{T}{4}}\sin\frac{4\pi}{T}t.\,dt=\frac{m}{8}\left(\frac{T\alpha_h}{\pi}\right)^2 \cdots\cdots\cdots\cdots(80)$$

Also we have

$$W=\frac{1}{2}\sqrt{h^2+l^2}(1-\cos\beta)mg=mg\frac{h}{2}\left[\sqrt{1+\left(\frac{l}{h}\right)^2}-1\right] \cdots\cdots(80)'$$

Since if $W_e<W$, the wall will not overturn.

i. e. $$\frac{1}{8}\left(\frac{\alpha_h T}{\pi}\right)^2<g\frac{h}{2}\left[\sqrt{1+\left(\frac{l}{h}\right)^2}-1\right]$$

For example, T=1.8 sec. =4,000 mm/sec². $\frac{l}{h}=\frac{1}{3}$

When $h>2.4$ m., the wall will not overturn.

In ordinary case, therefore, an instantaneous force as earthquake need not be considered for the overturning of a retaining wall, but constant forces as water pressure and earth pressure should be taken into account to study the stability against overturning.

A dam, however, may be overturned by the same instantaneous force, because the dam in consequence of its rocking motion is forced by an upward pressure of water rushed in cracks.

If the resultant force, though instantaneous, passes near the corner of the base, an enormous pressure caused on the heel will become a motive power of its overturning.

A retaining wall, by these reasons, should be so designed as $\frac{l}{2} \leqq b_0$; and a dam should have its base so large as $\frac{l}{2} = nb_0$, in which n=from 3 to 1.5.

b). Stability against sliding.

Taking any horizontal section of a wall or a dam, the horizontal component of all forces on the upper portion must be smaller than the friction or the resisting shear of the said section. If a wall be constructed in one mass, weakest section will be at the base, in this case the frictional resistance at the base should not be exceeded by the horizontal component, which is known as (H) in Section III.

In the case of severe earthquake, the horizontal force will sometimes exceeds the frictional resistance at the base, especially when the execution of the wall is carried in water as ordinary quay wall.

Consequently the designer must pay attention to the following articles.

A wall or a dam should be built in one mass for a certain length.

A wall or a dam must provide suitable front apron or a foothold excavated in the foundation in order to increase the stability against sliding.

c). Bearing power of the foundation.

The width of the base of a wall or a dam must be so designed that the maximum pressure on its toe does not exceed the ultimate bearing power of the foundation.

If a wall be fixed in the foundation as shown in fig. 19, the maximum pressure on the toe will be determined as the following.

ϕ_1 = angle of repose between wall and apron.

ϕ = inclination of the resultant to vertical.

ϕ_2 = inc. of the resultant on the base to vertical.

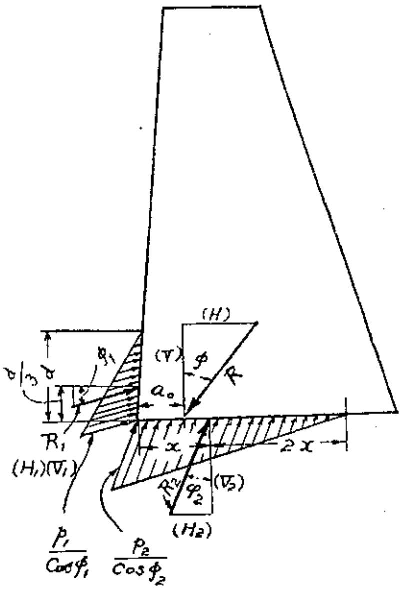


Fig. 19.

R =resultant force.

R_2=resultant force on the base.

R_1=resultant force on the apron.

p_1=maximum pressure on the apron

P_2=maximum pressure on the foundation

(H) and (V) are horizontal and vertical componets respectively.

(1), When p_1 and ϕ_2 are given, ϕ_1, p_2 and x will be found.

$$\left.\begin{aligned} &(H_1)=\frac{p_1d}{2} \quad (H_2)=\frac{3p_2x}{2}\tan\phi_2 \\ &(V_1)=\frac{p_1d}{2}\tan\phi_2 \quad (V_2)=\frac{3p_2x}{2} \end{aligned}\right\} \cdots (82)$$

Then by the equiliblium of forces, we have

$$\left.\begin{aligned} &(H)=(H_1)+(H_2) \qquad (V)=(V_1)+(V_2) \\ &(V_2)x=a_oV+\frac{d}{3}(H_1), \end{aligned}\right\} \cdots\cdots\cdots (82)'$$

From eq. (82) and (82)′ we can reduce the following equations.

$$\left.\begin{aligned} &2(H)=p_1d+3p_2x\tan\phi_2 \\ &2(V)=p_1d\tan\phi_1+3p_2x, \\ &a_o(V)+\frac{p_1d}{6}=\frac{3}{2}p_2x^2 \end{aligned}\right\} \cdots\cdots\cdots\cdots (83)$$

By solving eq. (83), we have

$$\left.\begin{aligned} &x=\frac{\{6a_o(V)+p_1d^2\}\tan\phi_2}{3\{2(H)-p_1d\}} \\ &p_2=\frac{\{2(H)-p_1d\}^2}{\{6a_o(V)+p_1d^2\}\tan^2\phi_2} \\ &\tan\phi_1=\frac{2(V)\tan\phi_2-\{2(H)-p_1d\}}{p_1d\tan\phi_2} \end{aligned}\right\} \cdots\cdots\cdots\cdots (84)$$

(2) When $\phi_1=0$, and ϕ_2 is given, we can find p_1, p_2 and x.

By solving eq. (83), we have

$$\left.\begin{aligned} p_1 &= \frac{2\{(H)-(V)\tan\phi_2\}}{d} \\ x &= a_0 + \frac{\{(H)-(V)\tan\phi_2\}d}{3(V)} \\ p_2 &= \frac{(V)^2}{3a_0(V)+\{(H)-(V)\tan\phi_2\}d} \end{aligned}\right\} \cdots\cdots\cdots\cdots (85)$$

d) Strength of a retaining wall and a dam.

A wall or a dam must be so designed that it has no special weak points at any sections, and if possible, the internal stress should be equally distributed on all sections.

Section IV.

Suggestion for the ideal design of the retaining wall and examples on the new design of the quay wall.

Art. 1. Suggestion for the ideal design of the retaining wall.

a). The upward pressure on the heel of the base, caused by its back filling material, is dangerous against overturning; since we must pay attention to eliminate the cause.

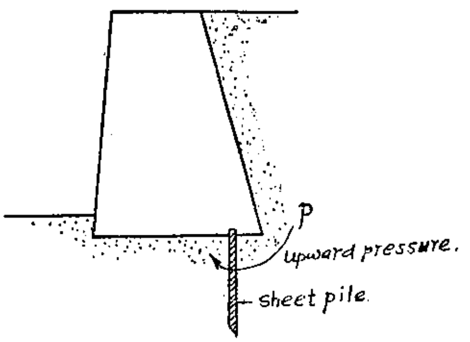


Fig. 20.

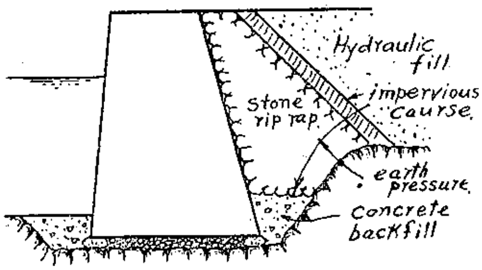


Fig. 21.

As shown in fig. 20, the upward earth pressure due to the wet earthen back fill will be checked by sheet piles driven under the heel (instead of toe).

As shown in fig. 21, the upward earth pressure on the base of a quay wall which is built in water, will be eliminated by concrete deposited near the heel or by an impervious course laid between the stone riprap and reclaimed quick sand.

Generally speaking, a retaining wall is but a kind of a dam, and we must pay the same attention to their design.

b). The wall should be perfectly fixed to the foundation. The execution is very simple for dry works, but pretty difficult for wet works.

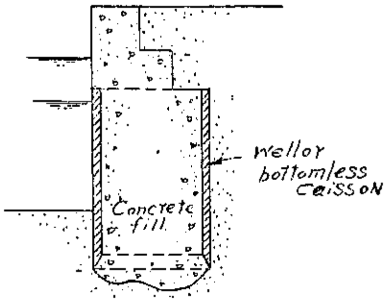


Fig 22.

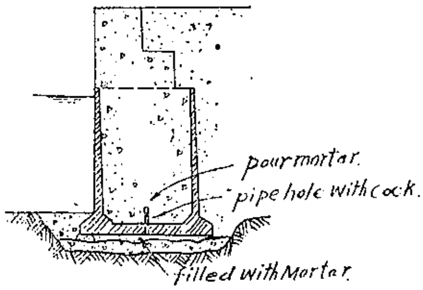


Fig. 23.

As shown in fig. 22, a well foundation or a bottomless caisson foundation will be preferable for this object, because the earth under the wall is replaced by concrete filling, if the piles are driven as shown in Pl. 6, the consequence will be good.

As shown in fig. 23, the space between the base of a caisson and the foudation can be filled with mortar by a grout mixer in order to fix the concrete caisson on the ground.

c). The wall must provide suitable apron or a foothold under the heel in order to resist the sliding motion at the base. As shown in fig. 22., a well foundation or a bottomless caisson will be sufficiently safe against the sliding motion due to the earthquake.

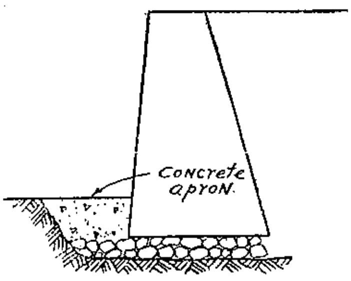


Fig. 24.

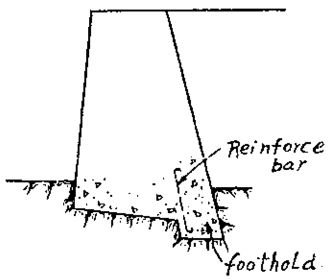


Fig. 25.

Fig. 24 shows a concrete apron laid in front of the wall.

Fig. 25 shows a foothold built in the foundation under the heel.

No retaining walls built on the hard foundation will be damaged by ordinary destructive earthquake, if they are well designed providing suitable apron or a foothold.

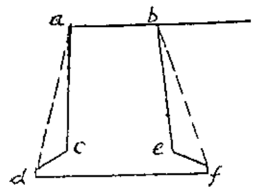


Fig. 26.

d). It is well known that the footings of a wall added to both sides increase the stability.

The wall **acdfeb** shown in fig. 26, has the equal stability to **adfb**, if their own weights are not considered.

Also the footing on the rear side will increase the stability of a wall, because in the case shown in fig. 27, the angle of repose between the wall and the earth becomes nearly equal to that of earth itself.

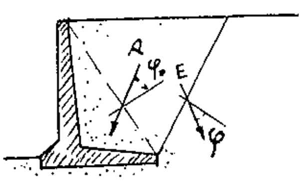


Fig. 27.

In this case $\varphi_o \doteqdot \varphi$.

e). Back filling materials.

Back filling materials should have smaller specific gravity, larger angle of repose and larger cohesion in wet and dry. In the case of severe earthquake, the cohesion of back filling materials, except the compact clay, will be disturved by the shock.

Engineers must pay attention in the design of a retaining wall or a quay wall that the total sum of the cost of the wall and the back filling materials should be minimum.

Art. 2. Examples on the new design of the quay wall and barge wharf.

a). Cellular block type-quay wall. (Refer Pl. 1.)

This design was preferred to the reconstruction of Yokohama Harbour in 1924, though some dimentions are not the same.

Hollow bottomless reinforced concrete boxes, say, ceilular blocks, will be moulded on shore, where a crane barge being approached. Cellular blocks, weighing up to 50 tons each, will be placed by the same crane barge in site and then their interior will be filled with concrete. Their vertical joints are stiffened by concrete as shown in the sectional plan. Horizontal joints are stiffened by steel bars or old rails, in consequence we have the quay wall of one mass.

The wall is fixed to the ground by concrete placed in the front and on the back in order to guaranty the stability.

b). Multiple arch caisson type-quay wall. (Refer Pl. 2.)

This design was reported to the navigation congress in London in 1923.

The quay wall shown in Pl. 2 will be adapted to the site where the bed is not so hard.

The reinforced concrete caisson has the cylindrical wall, whose horizontal section being multiple arch. The water pressure on the outer surface will act normally on the wall, and the resultant pressure line will pass through the neutral axis of the arched wall. In consequence of this reason, the water pressure on the wall, in its floating stage, will give uniform compressive stress to the wall, causing no bending moment on it, (this fact is impossible for rectangular caissons) thus great many reinforcing steel bars will be saved, and the wall itself can be very thin. Only a little amount of steel is required to resist against the internal stress of concrete wall owing to its setting shrinkage and temperature change, and also against the expansive force of the filling materials in the caisson. The depth of draught will be greatly saved comparing to the rectangular caisson, since the execution is more simple, and the cost of construction of quay wall will be cheaper.

On the base of the caisson many longitudinal grooves are moulded in order to increase the friction on the base when the space between the base and the foundation is filled with cement mortar by a grout mixer. The holes (providing sluice valves) in the base of the caisson, will be used not only to fill the mortar but also to pour water in the chamber when the caisson is required to be sunken in the position.

c). Bottomless caisson type—Quay wall. (Refer Pl. 3.)

The reinforced concrete caisson having temporary bottom in its floating stage will be transported to the site. When the caisson has been placed in the site, the temporary bottom is removed and the wall of the front chamber will be heighten by reinfoced concrete in site. The foundation under the caisson must be dredged, and by giving some load on the caisson, it will be sunk to a certain position. Then the interior will be filled with concrete.

d). Multiple arch type—quay wall. (Refer Pl. 4.)

This design is adapted to the site, where the execution is carried in dry condition. This type, if the coffer dam be cheap, will be most economical and will be sufficiently strong against an earthquake.

e). Well sinking type-barge wharf. (Refer Pl. 5.)

The pre-cast reinfored concrete well (or cylinder) will be placed by a crane barge in site, and will be lowered by the ordinary method of well sinking. The pre-cast reinforced concrete wall will be placed between the wells in order to hold the back filling materials in their proper position. Each pier (well) provides an ancher stay near the top, thus the safety factor will be increased.

f). Pre-cast buttress wall. (Refer Pl. 6.)

This type will be adopted for rapid works, and is very cheap if a crane barge, having larger capacity, be provided.

This buttress wall weighing 50 tons each will be moulded on shore by reinforced concrete, then it will be placed by a crane barge in site. Each wall will be fixed by cement mortar, which is filled in a cambas bag placed in the vertical groove.

This type, if properly designed, can be guarantied against ordinary destructive earthquake.

g). Sheet pile type-wall. (Refer Pl. 7.)

This design is a kind of a retaining wall or a wharf, which is built using reinforced concrete sheet piles.

Appendix.

Study on the cause of the damage to the quay wall of Yokahame Harbour due to the earthquake on September 1st. 1923.

a). General descriptions.

The quay wall of the Yokohama Custom House had the length of 1,110 ken (6,620 ft.) allowing 13 vessels to moor at the same time. The height of the wall was ranging from 32.5 ft. to 45 ft., and the width of the base from

15 ft. to 18 ft.

The greater part was built on the hard clay strata, and the other was constructed on the rubble mound. The hard clay strata was excavated and massive concrete was deposited by means of a pneumatic caisson to build the strong foundation, on which many concrete blocks (4.5 ft. × 5 ft. × 7 ft. and 4 ft. × 6 ft. × 9 ft.) were laid in several layers. The wall was constructed in several sections each having 36 ft. in length. The vertical bond of each block was reinforced by the cement mortar filled in the vertical grooves, and nothing was attempted to stiffen the horizontal bond, since the bonding power between horizontal layers was so poor that it was easily overcome by the seismic force.

No wall overturned from its root, but all quay walls except the foundation of stationary wharf cranes, was forced to slide out to the front step by step according to the vibration of the earthquake. Lower two or three courses, covering 800 ken in length, were remained because they were fixed to the hard clay strata by concrete, while upper layers above them were forced to silde out so far as they were thrown down in the water. The other part, covering 240 ken in length, was forced to slide out from the root, which was merely lying on the rubble mound or on the hard clay strata; and moreover relative displacement between horizontal layers had taken place, and the upper part was remaining on the lower layers by overhanging about 5 feet.

Pl. 8 and Pl. 9 show the typical section of the failure of the quay, which was built on hard clay strata.

Pl. 10 and Pl. 11 show the damage to the quay walls which were built on the rubble mound.

Pl. 13 shows the section of the foundation of 20 ton stationary wharf crane, which was perfectly safe during the earthquake.

b). Calculations on the stability of the quay wall.

Pl. 12 denotes the force diagram for the typical section of the quay, same as shown in Pl. 8.

Pl. 13 indicates the force diagram for the foundation of the stationary wharf crane, which was perfectly safe.

Several experiments were carried to determine the bonding force (friction) between horizontal courses of blocks, and the angle of repose of the back filling material. And no great error will occur if the following results are used.

$\varphi=40°$ $\varphi_o=22°30'$ (frictional angle between horizontal courses) $=35°$

i. e. $\tan\phi_o=0.7$

Pl. 12.

$h=29.25$ ft. $w_1=115$ lbs./cu. ft. Weight of back fill above water.

$\alpha=96°24'$ $w_2=75$ „ „ „ „ „ submerged.

$l=14.4'$ $w=140$ „ „ „ wall.

$l'=9.5'$ $w'=62.5$ „ „ „ water.

Weight of wall = 48,250 lbs.

Buoyancy of wall = 15,820 lbs.

Case II. $\alpha_h=3{,}600$ mm/sec². $\alpha_v=\pm2{,}100$ mm/sec².

$\lambda_o\begin{cases}=1.181\\=0.952\end{cases}$ $\varepsilon=\begin{cases}=18°8'\\=22°42'\end{cases}$

For $\lambda_o=1.181$, $\varepsilon=18°8'$, we have

$a=\sin 40°38'$ $=0.6512$

$b=\cos 18°8' \cos 158°54'-\cos 22°30' \cos 74°32'=-1.1336$

$c=\sin 18°8' \cos 158°54'+\sin 22°30' \cos 74°32'=-0.1886$

$\delta=46°2'$

$P_1=1.181\times4{,}554\times0.475=2{,}554$ lbs.

$P_2=1.181\times36{,}360\times0.475=20{,}370$ lbs.

$h_c=8.725$ ft. for P_2

Weight of wall $=1.181\times48{,}250$

Buoyancy $=1.181\times15{,}820$

Back water pressure by eq. (46) $=1.181\times930$

Resultant- $P=22{,}924$ lbs.
Wall $=38{,}320$ lbs.
Water pressure $=1{,}100$ lbs. } $R_2=58{,}200$ lbs.

R_2 makes inclination of 34°30′ to the vertical and makes 31°30′ to the normal line of the base.

For $\lambda_o=0.952$, $\varepsilon=22°42'$

$a=\sin 45°12'$ $=0.7096$

$b=\cos 22°42' \cos 158°54'-\cos 22°30' \cos 79°6'=-1.026$

$c=\sin 22°42' \cos 158°54'+\sin 22°30' \cos 79°6'=-0.2876$

$\delta=37°52'$

$P_1=0.952\times4{,}554\times0.573=2{,}480$ lbs.

$P_2=$ „ $\times36{,}360\times$ „ $=19{,}800$ lbs.

Wall $=45{,}850$ lbs.

Back water $=885$ lbs.

Resultant $P=22{,}280$ lbs.
Wall $=30{,}850$ lbs.
Water pressure $=885$ lbs } $R_3=51{,}100$ lbs.

R_3 makes the inclination of 39° to the vertical, and 36° to the normal line of the base.

Case. III. $\alpha_h = 4{,}000$ mm/sec.^{2} $\alpha_v = \pm 1{,}800$ mm/sec.^{2}

$$\lambda_0 \begin{cases} = 1.252 \\ = 0.913 \end{cases} \qquad \varepsilon \begin{cases} = 19°2' \\ = 26°34' \end{cases}$$

For $\lambda_0 = 0.913$, $\varepsilon = 26°34'$, we have

$a = 0.7558$ $b = -0.9482$ $c = -0.3708$

$\delta = 33°36'$

$P_1 = 0.913 \times 4{,}554 \times 0.678$

$P_2 = 0.913 \times 36{,}360 \times 0.678$

Wall $= 0.913 \times 48{,}250$

Buoyancy $= 0.913 \times 15{,}820$

Back water $= 0.913 \times 1{,}140$

$R_4 = 52{,}000$ lbs.

R_4 makes the inclination of 43° to the vertical and makes 40° to the normal line of the base.

Case IV. $\alpha_h = \alpha_v = 0$ **Ordinary condition with no earthquake.**

For $k = 0$, we have

$a = 0.3827$ $b = -1.445$ $c = 0.2117$

$\delta = 64°46'$

$P_1 = 4{,}554 \times 0.245 = 1{,}115$ lbs.

$P_2 = 36{,}360 \times 0.245 = 8{,}900$ lbs.

Wall $= 48{,}250$. lbs.

Buoyancy $= 15{,}820$ lbs.

Resultant $P = 10{,}015$ lbs., Wall $= 32{,}430$ lbs. } $R_5 = 38{,}400$ lbs.

For $k = 0$, we have

$k = \dfrac{wh \sin(\delta - \varphi) \cos \delta}{2 \cos \varphi} = 0.1165\ wh$ $h = 6$ ft. when the excavated plane is vertical. Hence,

$k = 0.7\ w$

$k = 80$ lbs/sq. ft. above water line.

$k = 50$ lbs/sq. ft. below water line.

For upper section above water level.

$a = 0.3827$ $b = -1.225$ $c = 0.1205$

$\delta = 61°57'$

$P_1 = 0.242 \times 4{,}554 - 8.9 \times 80 \times 0.878 = 478$ lbs.

$h_c \doteqdot 0$

For lower section below water line

$a = 0.3827$ $b = -1.4365$ $c = 0.2083$

$\delta = 64°41'$

$P_2 = 0.245 \times 36{,}360 - 20.3 \times 50 \times 0.85 = 8{,}035$ lbs.

$h_c = 8.65$ ft.

Resultant $P = 8{,}513$ lbs, Wall $= 32{,}430$ lbs. } $R_6 = 37{,}600$ lbs.

Pl. 13.

$\varphi = 40°$

$\varphi_0 = 22°30'$

$h = 19.5$ ft

$\alpha = 90°$

$l = 18.75$ ft.

$l' = 18$ ft.

$\alpha_h = 4{,}000$ mm sec² $\qquad \alpha_v = \pm 1{,}800$ mm/sec².

For $\lambda_0 = 0.913 \qquad \varepsilon = 26°34'$, we have

$a = 0.7558$

$b = -1.0075$

$c = -0.3125$

$\delta = 33°29'$

$P_1 = 0.913 \times 4{,}554 \times 0.593$

$P_2 = 0.913 \times 15{,}060 \times 0.593$

Weight of wall $= 0.913 \times 50{,}150$

Buoyancy $= 0.913 \times 12{,}290$

Weight of crane and its bed $= 0.913 \times 8{,}700$

Resultant $\qquad R_7 = 51{,}000$ lbs.

Resultant makes the inclination of 34° to the normal of the base.

c). Conclusion.

It is sure that all quay walls at some weakest horizontal joint were forced to slide out more or less by the earthquake, whose acceleration is $\alpha_h = 3{,}600$ mm/sec² and $\alpha_v = 1{,}200$ mm/sec², when α_v acted downwards, because the resultant force makes larger inclination than 35 degrees to the normal line of the base. (tan 35° is assumed to be the frictional coefficient between concrete blocks).

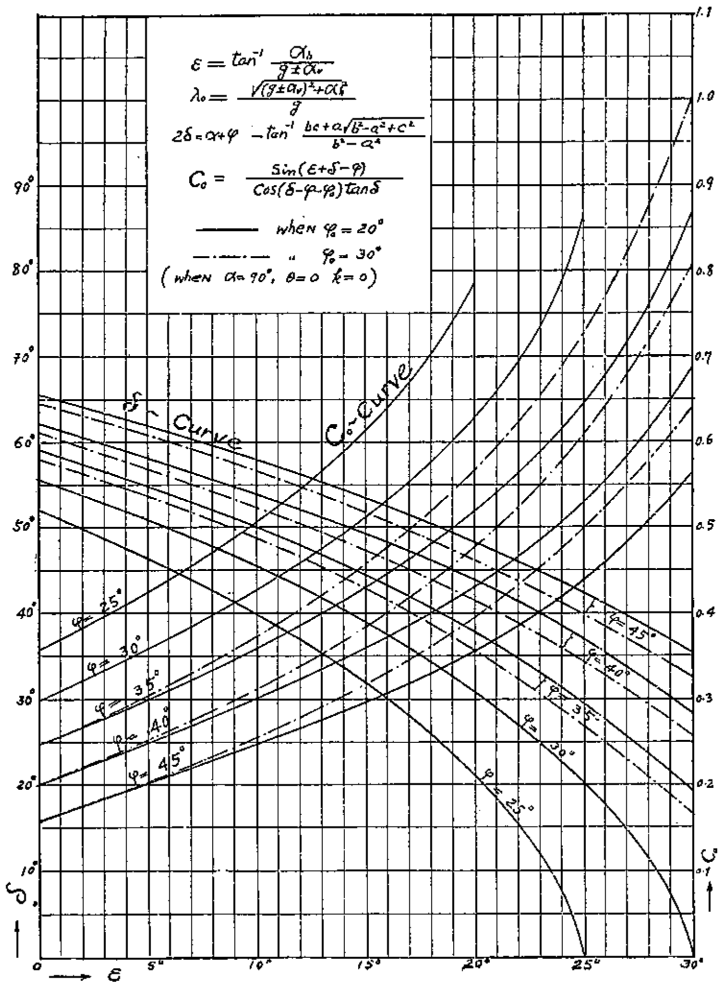
The foundation of the wharf crane proved to be perfectly safe against the earthquake whose acceleration is $\alpha_h = 4{,}000$ mm/sec² and $\alpha_v = -1{,}800$ mm/sec², because the resultant in this case makes 34 degrees to the normal of the base. Since it is supposed that the maximum acceleration of the earthquake on Sep. 1st., at the site was nearly $\alpha_h = 4{,}000$ mm/sec² and $\alpha_v = 1{,}800$ mm/sec².

Generally speaking, the foundation of the quay wall was sufficiently hard, and the execution was carried in good condition, but no attention was paid for the bondings between horizontal layers. If the horizontal bondings had been stiffened by some means (concrete blocks providing grooves and tenons etc.) the quay wall would have been not so radically damaged.

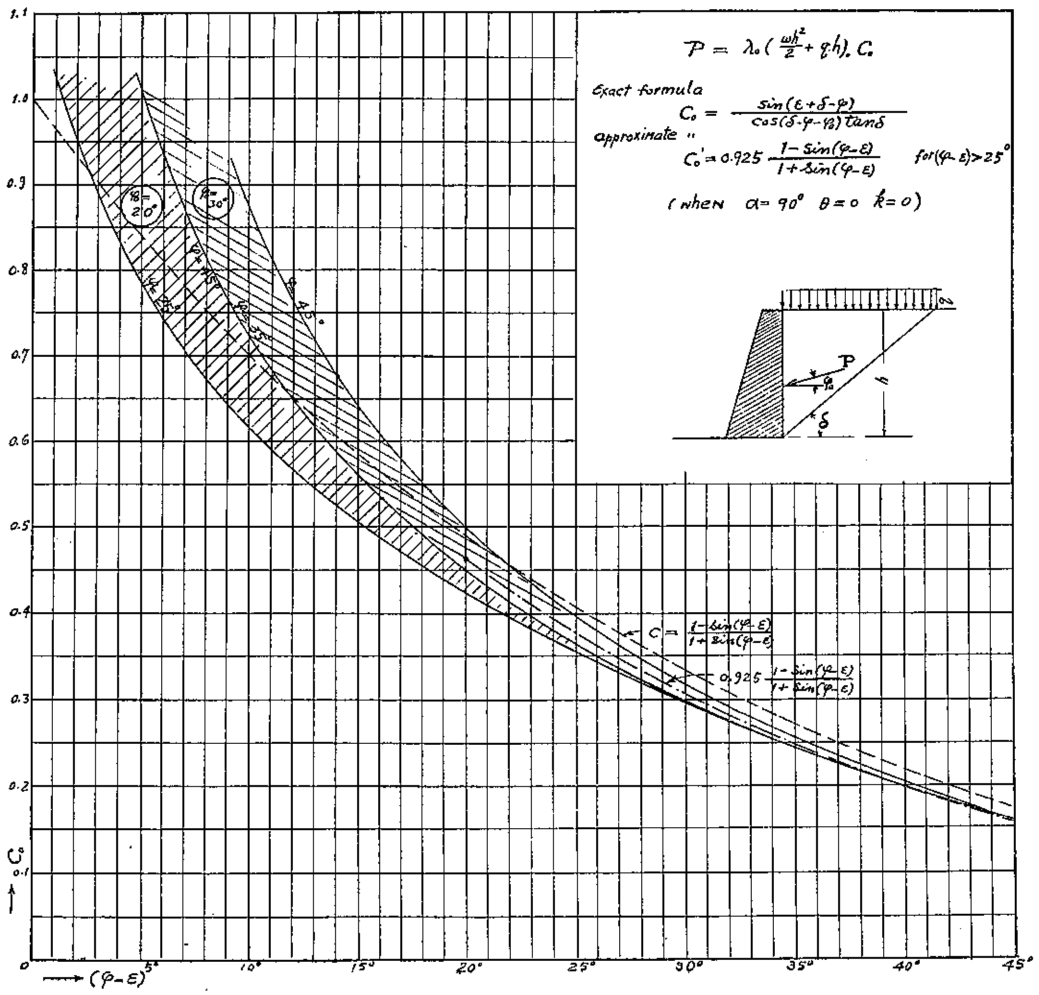
By some superior design for which kinematic consideration is paid, a solid

Pl. O. Diagram of Seismic Earth Pressure.

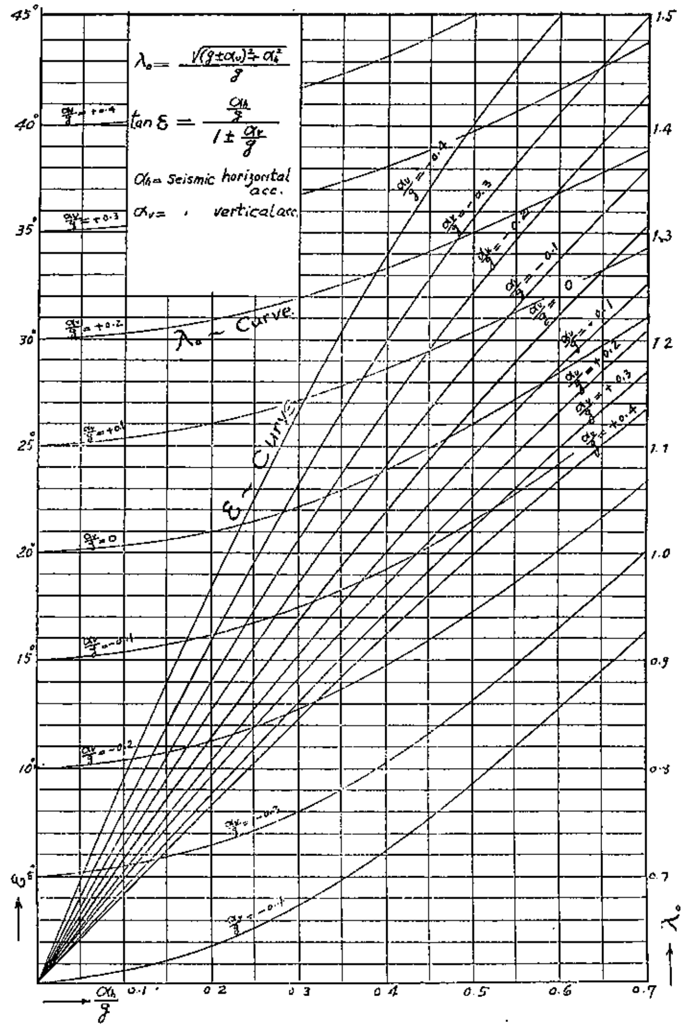
Slope of Plane of Rupture and Coeff. of Seismic Earth Pressure.



Coeff. of Seismic Earth Pressure. (φ-ε) and C_0 Curve.

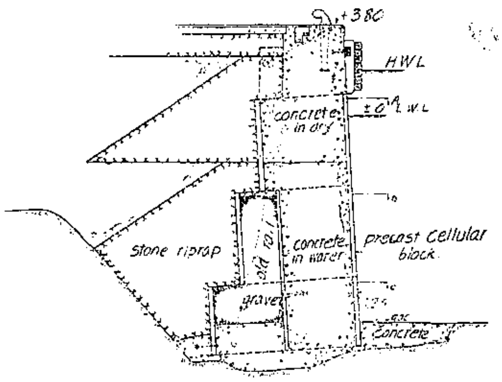


Coeff. of Resultant Force and Seismic Intensity and Inclination of Gravity Line.

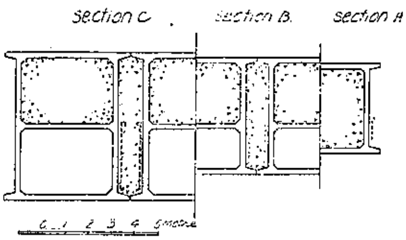


Pl. 1. Cellular Block Type – Quay Wall.

Cross section.

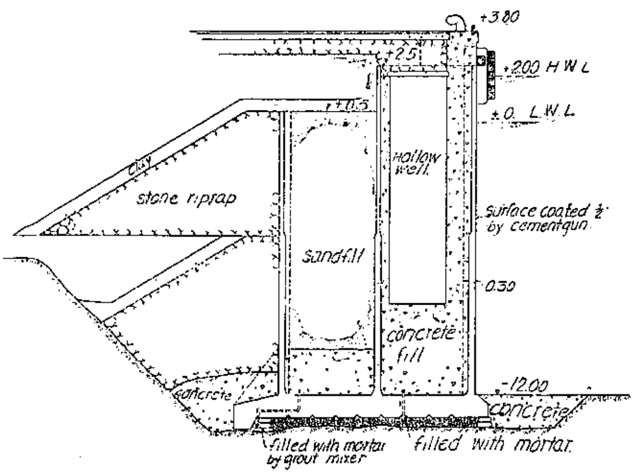


Sectional Plan.

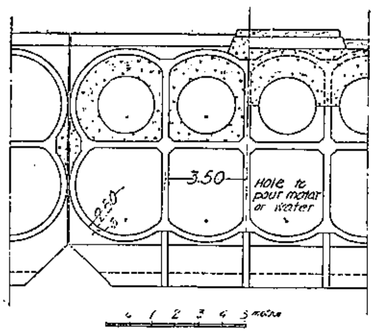


Pl. 2. Multiple Arched Caisson Type – Quay Wall.

Cross Section.

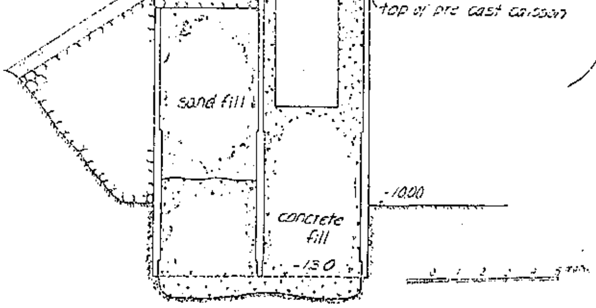


Sectional Plan.

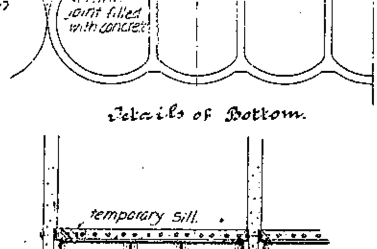


Pl. 3 Bottomless Caisson Type – Quay Wall.

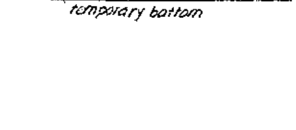
Cross Section.



Sectional Plan.



Details of Bottom.



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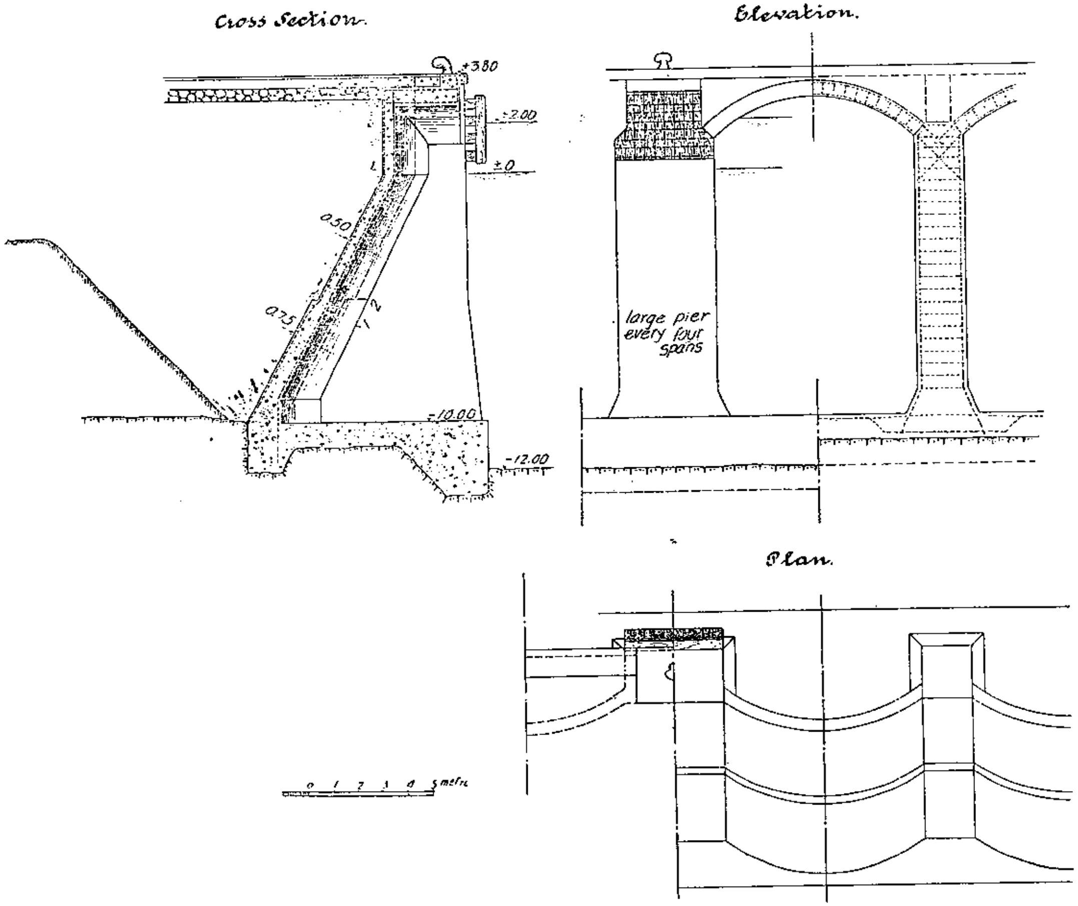
Pl. 4. **Multiple Arch Type – Quay Wall.**



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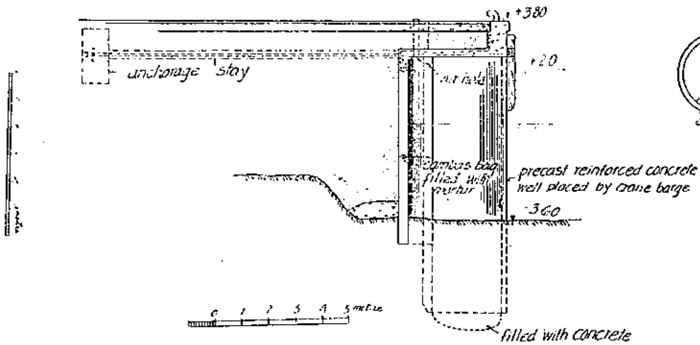
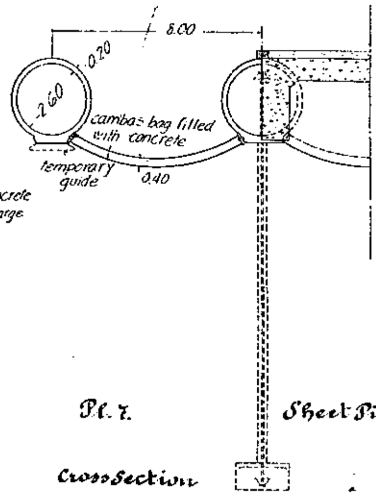
Pl. 5. Well Sinking Type - Barge Wharf.

Cross Section.

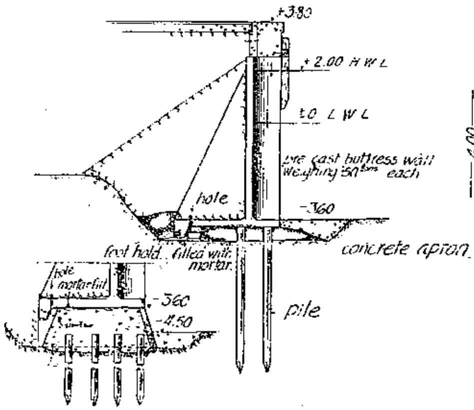


Sectional Plan

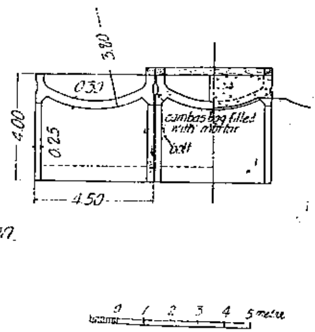


Pl. 6. Pre-Cast Buttress Wall.

Cross Section.

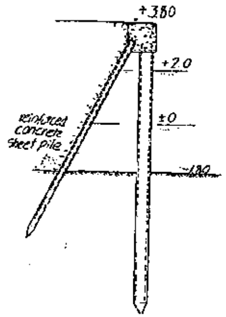


Sectional Plan.

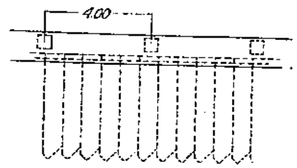


Pl. 7. Sheet Pile Type - Wall.

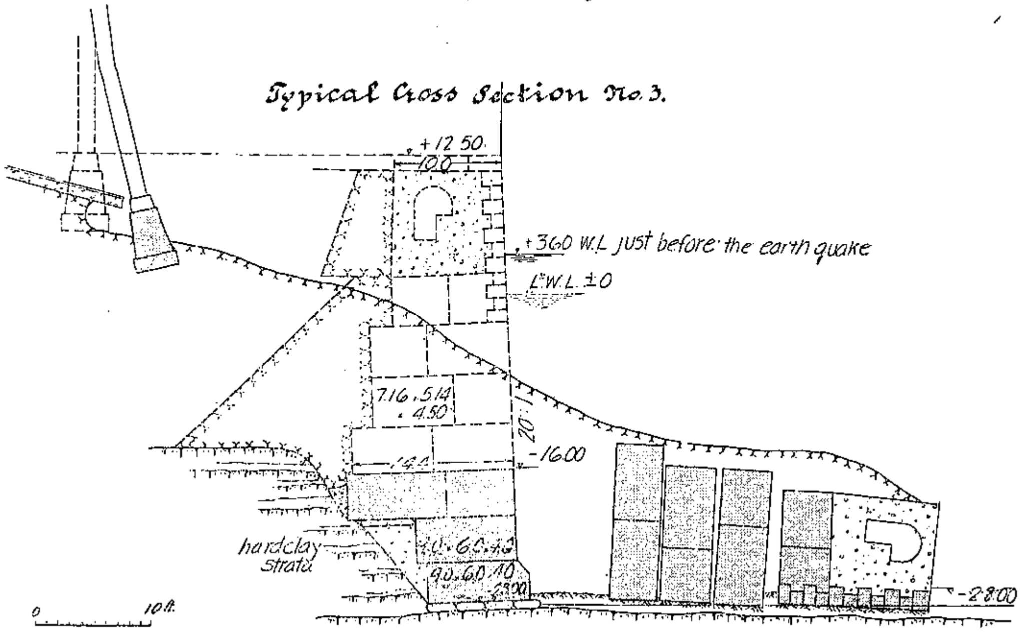
Cross Section



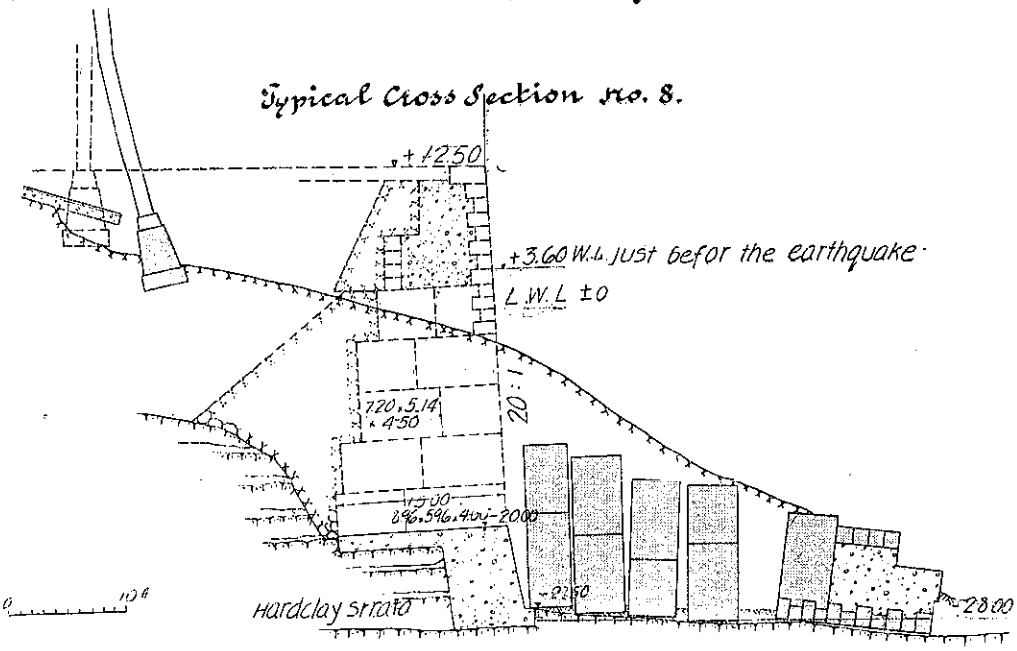
Plan.



Pl. 8. Failure of Quay Wall.

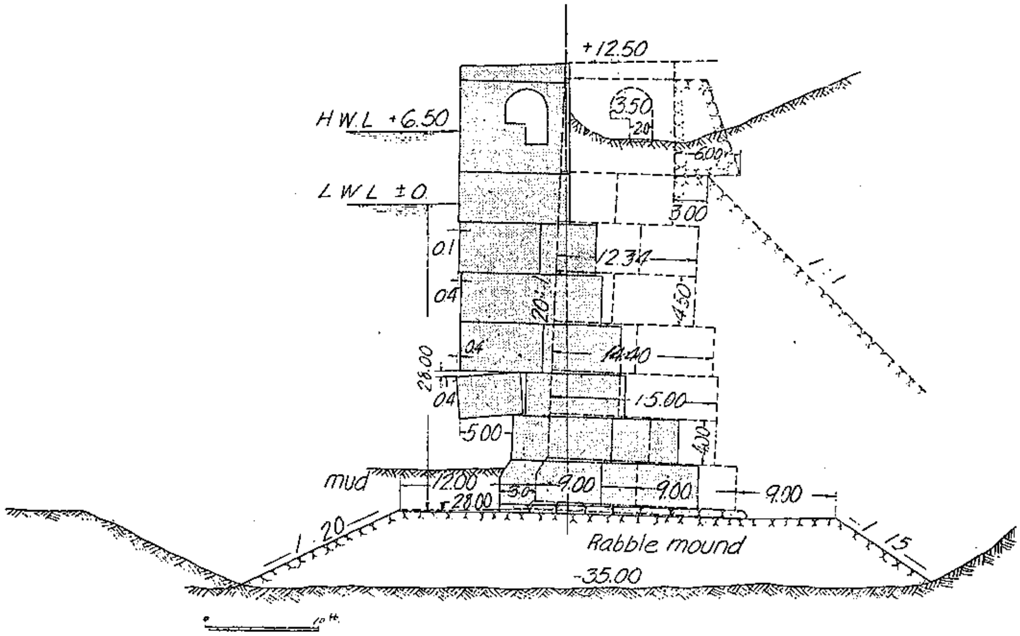


Pl. 9. Failure of Quay Wall.

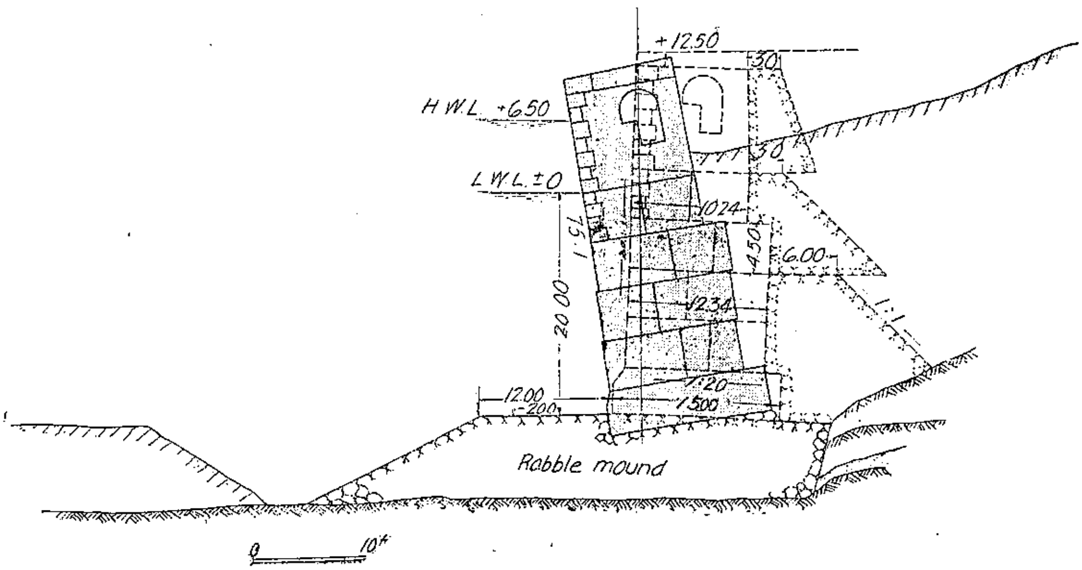


(土木學會誌第十卷第六號附圖)

Pl. 10. Damage to Quay Wall. No. 6.

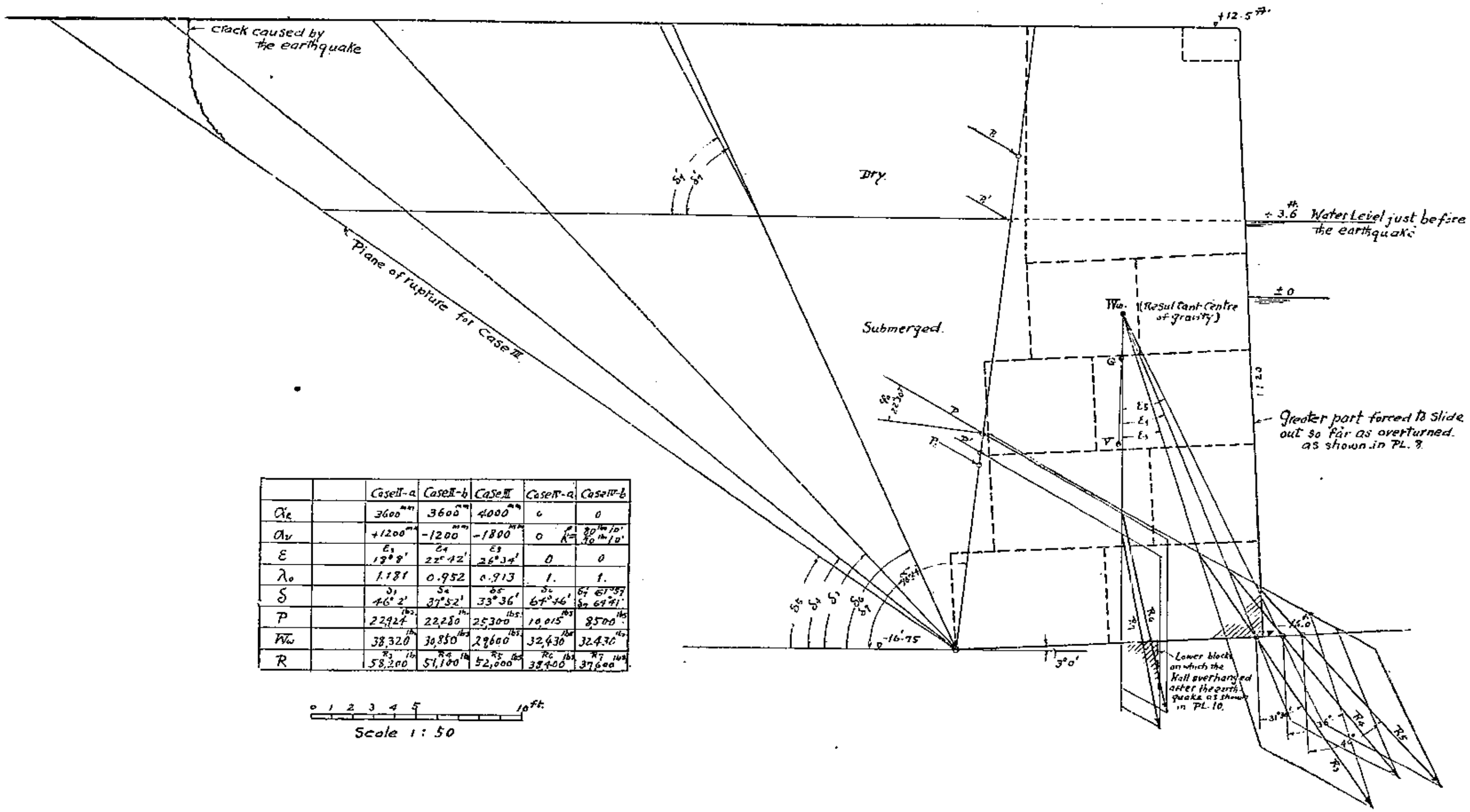


Pl. 11. Damage to Quay Wall. No. 1.



(土木學會誌第十卷第六號附圖)

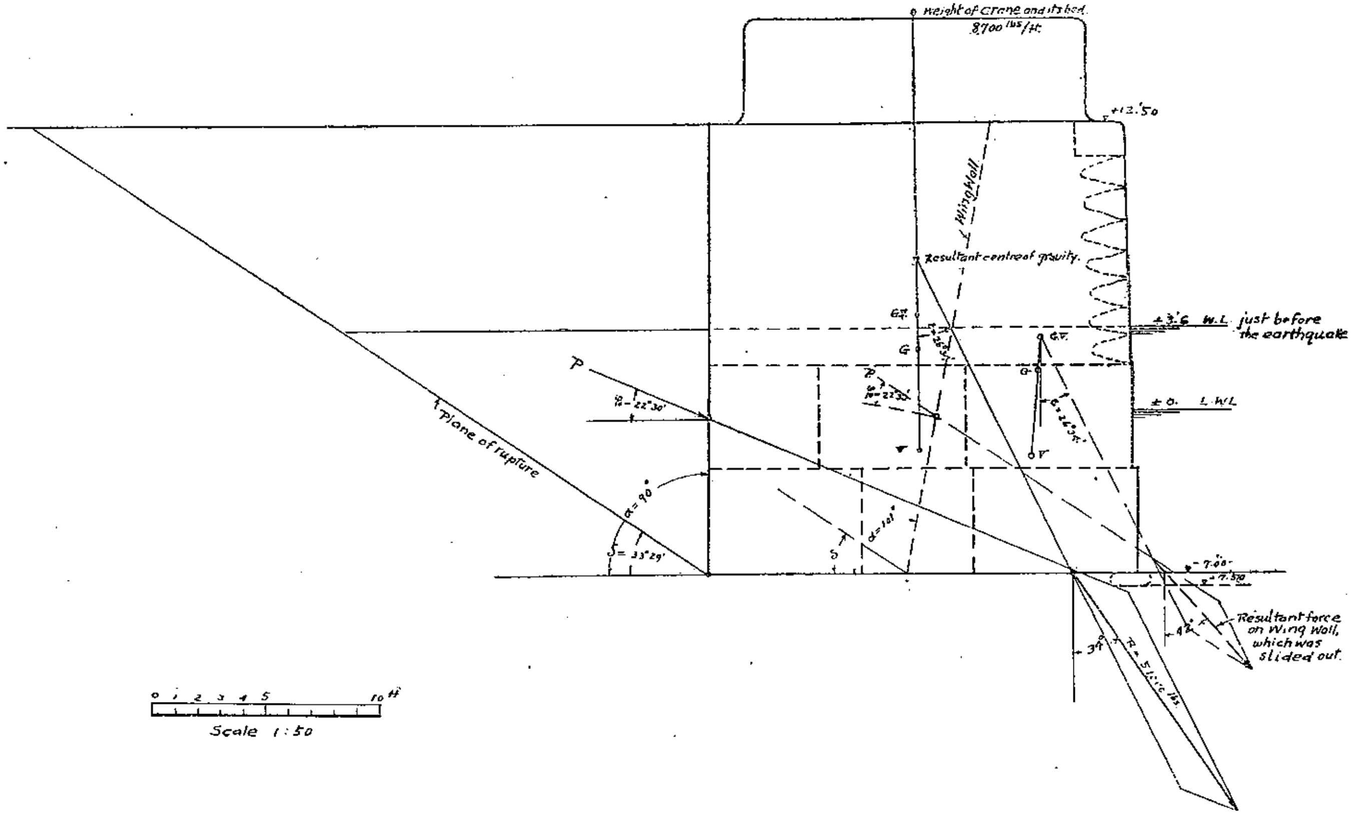
Pl. 12. Force Diagram for the Stability of the Quay Wall.



		Case II-a	Case II-b	Case III	Case IV-a	Case IV-b
α_h		3600 mm	3600 mm	4000 mm	0	0
α_v		+1200 mm	-1200 mm	-1800 mm	0	$k = \frac{80 \text{ lbs}/10'}{30 \text{ lbs}/10'}$
ε		ε_3 18°8'	ε_4 22°42'	ε_5 26°34'	0	0
λ_0		1.181	0.952	0.913	1.	1.
δ		δ_3 46°2'	δ_4 37°52'	δ_5 33°36'	δ_6 64°46'	δ_7' 61°57' δ_7 64°41'
P		22,924 lbs.	22,280 lbs.	25,300 lbs.	10,015 lbs	8,500 lbs
W_w		38,320 lbs	30,850 lbs	29,600 lbs.	32,430 lbs	32,430 lbs
R		R_3 58,200 lbs	R_4 51,100 lbs	R_5 52,000 lbs	R_6 38,400 lbs	R_7 37,600 lbs

(土木學會誌第十卷第六號附圖)

Pl. 18. Force Diagram for the Stability of Crane Base.



（土木學會誌第十卷第六號附圖）

quay wall as well as a gravity dam can be constructed by moderate cost and also can be guarantied against such an earthquake as that on September 1st. 1923.

—(The End)—