土木學會誌　第七卷第三號　大正十年六月

羽越北線折渡隧道用しーるど設計概要

會員　工學士　田中　豐

大正八年七月二日鐵道院官房研究所ニ於テ設計ヲ了シタル羽越北線折渡隧道用しーるどノ設計概要次ノ如シ

一　設計要領

目下建設工事中ナル羽越北線第五工區內折渡隧道ハ其掘鑿施工中南坑ニ於テ導坑ヲ掘鑿シ引續キ丸形ヲ切擴ケ完全ナル支保工ヲ施シタルニ數日ニシテ路盤隆起約三呎ニ達シ柱下土臺木中央部ヨリ上部ニ向ヒ挫折シ丸形先ツ壓潰シ續テ導坑亦全ク閉塞スルニ至レリ右ノ狀況ニ徵スレハ該地盤ハ普通ノ支保工ヲ以テシテハ到底掘鑿遂行ノ見込覺束ナク結局現在ノ施工方法ヲ變更シ簡易ナル鐵製構盾ヲ使用シ掘鑿並ニ疊築ヲ進工スルノ利益ナルヲ認メ左記ノ條項ニヨリテ其結構部ノ設計ヲナセリ

設計資料

一　隧道ノ斷面ハ現在設計ニ依リ仰拱ヲ附シタルモノ又ハ之ニ相當スル圓形トス

一　覆工ノ厚サハ一呎十吋二分ノ一トシ混凝土塊ヲ用ヒ一個ノ長サハ一呎六吋トス

一　構盾ノ長サハ十二呎乃至十五呎トス

一　構盾ハ水壓扛重器ニ依リ進行セシメ一日ノ進程ハ一呎六吋以上トス

一　土壓ハ構盾ノ外周每平方呎三噸ヲ下ラサル見込

一 水壓扛重器其他ノ裝置ハ相當設計ノコト

○附記 上記覆工ノ厚サ一呎十吋二分ノ一ハ其後二呎ト變更スルコト、ナリタリ

而シテしーるど推進用ノ水壓機ハ壓力二十五噸ノモノ三十二個迄使用スルモノトシしーるどノ形狀及寸法ヲ別紙附圖第一ノ如ク決定セリ

今試ニ本しーるどト外國ニテ使用セラレタル外徑二十呎以上ノしーるどトヲ對照比較スレハ次ノ如シ

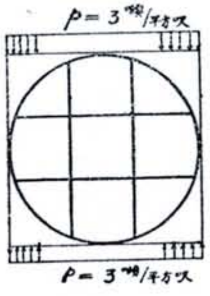
外徑二十呎以上ノしーるど一覽

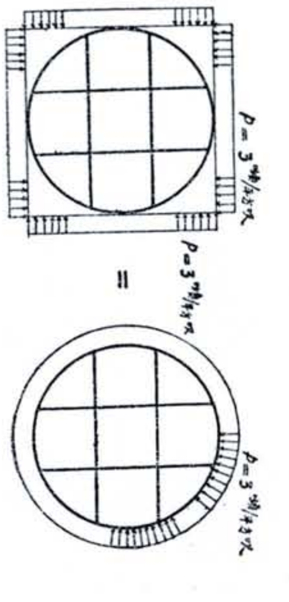
隧道名	外徑	外鈑ノ厚	長	重量	年代
St. Clair, Canada	21呎—6吋	1吋	15呎—3吋	92(米噸)	1889
Waterloo & City, London	24—10	1	10—0	112(〃)	1893
Central London	22—10	1	6—10	—	1896
East River, P. R. R.	23—6¼	2¼	18—0	240(〃)	1903—1909
North River, P. R. R.	23—6¼	2¼	17—3⅜	193(〃)	1903—1909
Rotherhithe, London	30—8	2¼	18—0	—	1904
折渡	24—2	1½	12—0	86(英噸)	1919

（備考 上記外國ノ諸例ハ Am. C. E. Pocket Book ヨリ摘記シタルモノナリ）

二 假定外壓

しーるどノ受ク可キ外壓ノ强度ハ建設局工事課長太田技師及秋田建設事務所長八田技師ト協議ノ上最大平均外壓力强度ヲ毎平方呎三噸ト假定シ之ニ依リテ設計上次ノ如キ外壓力ヲ考フルコトヽ爲シタリ


(I)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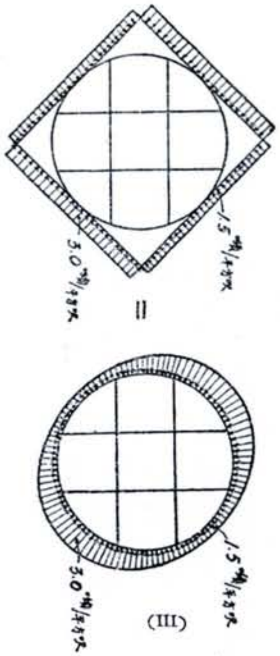

(III)

（I） 上下ヨリ毎平方呎三噸ノ等布壓力ヲ受クル場合

（II） 周圍ヨリ毎平方呎三噸ノ等布壓力ヲ受クル場合

（III） 左四十五度ノ方向ヨリ毎平方呎三噸ノ壓力ヲ受ケ右四十五度方向ヨリ毎平方呎、一噸半ノ壓力ヲ受クル場合

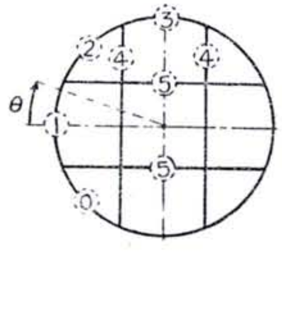
上記ノ假定外壓中（III）ハ本しーるどカ山岳隧道用タルノ故ヲ以テ特ニ傾斜偏壓ヲモ考慮シタル次第ナリ

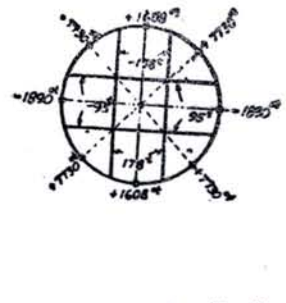
三　外壓ノ爲メニ生スヘキ主應力計算ノ概要

前記(I)(II)(III)ノ假定外壓ニヨリテしーるど結構內ニ生スヘキ主應力ハ先ツ(一)上下ヨリ單位等布荷重ヲ受クル場合及(二)四十五度ノ單位等布荷重ヲ受クル場合ノ二ツノ場合ノ主應力ヲ計算シ其結果ヨリ所要應力ヲ算出シタリ即チ單位等布荷重ニアリテ前記(一)及(二)ノ場合ノ彎曲率及直力ハ附圖第二及附圖第三ノ如シ四十五度ノ單位等布荷重ヲ受クル場合ノ彎曲率及直力ハ井狀ノ結構ナキ場合ト大差ナキヲ以テ其彎曲率ハ簡單ニ附圖第四ノ如ク假定スルヲ得ヘク又此ノ場合ニ於ケル直力ハ附圖第五ノ如シ但シ傾斜荷重ニ對シ井狀結構ノ影響ヲ考慮シタル場合ノ計算ハ參考トシテ之ヲ本文ノ附錄中ニ記載スヘク又第二圖ニ示ス應力ノ計算法ハ稍々複雜ナルヲ以テ之カ解法ハ附錄トシテ添付スルコトヽナセリ

上記ノ結果ヨリ前記(I)(II)(III)ノ場合ニ於ケル最大彎曲率直力竝ニ之カ爲メニ生スヘキ最大應力强度ヲ求ムレハ次ノ如シ

荷重狀態/部分	彎曲率（吋噸）				直壓力（噸）					
	(0)	(1)	(2)	(3)	(0)	(1)	(2)	(3)	(4)	(5)
I	+588	−1,890	+588	+1,608	−141	−236	−141	−83	−178	+83
II	+1,176	−282	+1,176	−282	−282	−319	−282	−319	−95	−95
III	+588	−141	+588	−141	−345	−263	−141	−263	−48	−48
	−7,142	0	+7,142	0	—	—	—	—	0	0
	−6,554	−141	+7,730	−141	—	—	—	—	−48	−48

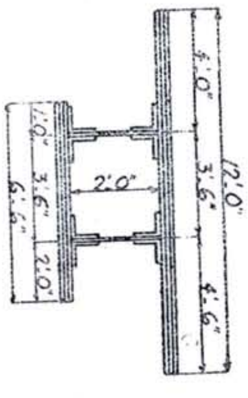




故ニ一般ニ最大應力ヲ示セハ上圖ノ如シ

但シ上表及上圖ニ於テ彎曲率ハ外側ニ應壓力ヲ生スルモノヲ(＋)トシ直力ハ張力ヲ(＋)トス

外殼使用斷面



3－蓋　鈑　144"×½"
4－山　形　6"×4"×$\frac{9}{16}$"
2－腹　鈑　23½"×½"
4－山　形　6"×4"×$\frac{9}{16}$"
3－蓋　鈑　78"×½"
總斷面積　380□"

物量力率　55,620$^{(")4}$

但シ純斷面積ハ上記ノ($\frac{4}{9}$)即チ　$380\times\frac{4}{9}=169$□"　トス

又斷面ノ中立軸ヨリ纖維ニ到ル距離ハ $\begin{cases} y_i = 16''.82 & \text{（內側迄）} \\ y_o = 10''.18 & \text{（外側迄）} \end{cases}$

前記(I)(II)(III)ノ場合ノ最大直力强度ヲ求ムレハ

θ＝0　ノ點ニテハ　$f=\dfrac{-319\times2{,}240}{169}=-4{,}230$ #/□"

θ＝45°　ノ點ニテハ　$f=\dfrac{-345\times2{,}240}{169}=-4{,}570$ #/□"

θ＝90°　ノ點ニテハ　$f=\dfrac{-319\times2{,}240}{169}=-4{,}230$ #/□"

外殼ノ最大彎曲應力

θ＝0　ノ點ニテハ

$$f=\frac{-1,890\times 2,240}{55,620}\times\begin{cases}16.82\times(\frac{9}{4})=-2,880\ \#/\square'' & \text{内側}\\ 10.18\times(\frac{9}{4})=+1,740\ \#/\square'' & \text{外側}\end{cases}$$

$\theta=45^\circ$ ノ點ニテハ

$$f=\frac{+7,730\times 2,240}{55,620}\times\begin{cases}16.82\times(\frac{9}{4})=+11,800\ \#/\square'' & \text{内側}\\ 10.18\times(\frac{9}{4})=-7,130\ \#/\square'' & \text{外側}\end{cases}$$

$\theta=90^\circ$ ノ點ニテハ

$$f=\frac{+1,608\times 2,240}{55,620}\times\begin{cases}16.82\times(\frac{9}{4})=+2,450\ \#/\square'' & \text{内側}\\ 10.18\times(\frac{9}{4})=-1,480\ \#/\square'' & \text{内側}\end{cases}$$

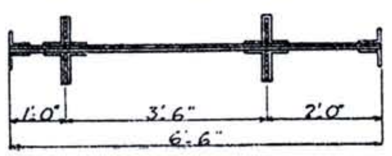
之ニ依リテ最大合成應力ヲ求ムレハ次ノ如シ

$$\theta=0^\circ\begin{cases}-\ 2,880-4,230=-\ 7,110\ \#/\square''\\ +\ 1,740-4,230=-\ 2,490\ 〃\end{cases}$$

$$\theta=5^\circ\begin{cases}+11,800-4,570=+\ 7,230\ 〃\\ -\ 7,130-4,570=-11,700\ 〃\end{cases}$$

$$\theta=90^\circ\begin{cases}+\ 2,450-4,230=-\ 1,780\ 〃\\ -\ 1,480-4,230=-\ 5,710\ 〃\end{cases}$$

支柱ノ使用斷面並ニ最大應力强度



8－山形　$6''\times 4''\times\frac{1}{2}''$

4－山形　$3\frac{1}{2}''\times 3\frac{1}{2}''\times\frac{3}{8}''$

2－鈑　$78''\times\frac{1}{2}''$

4－填材　$6''\times\frac{1}{4}''$

總斷面積$=138\square''$,　純斷面積$\doteqdot 138\times\frac{2}{3}=92\square''$,　物量力率$=528''^4$,　最小環動半徑$=1.''96$

今支柱ノ支持セラレサル長ヲ84吋トスレハ

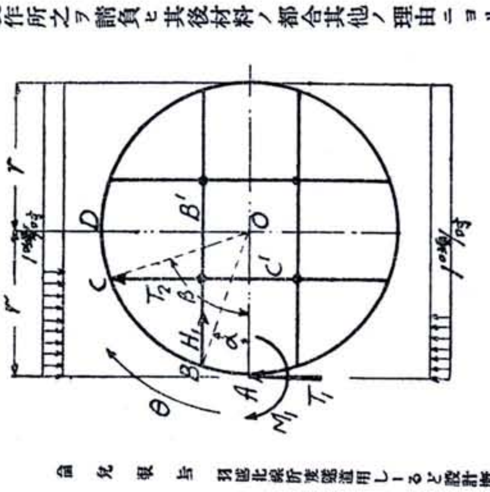
$$\text{許容應壓力强度}=16,000-70\times\frac{84}{1.96}=13,000\ \#/\square''$$

$$\text{計算應壓力强度}=\frac{178\times 2,240}{138}=2,900\ \#/\square''<13,000\ \#/\square''$$

$$\text{計算應張力强度}=\frac{178\times 2,240}{92}=4,300\ \#/\square''<16,000\ \#/\square''$$

四　製　作

製作ハ大正八年九月十六日附契約ニヨリ材料請負人持ニテ製作費金六萬五千二百餘圓製作期限六十日以内ニテ横河橋梁製作所之ヲ請負ヒ其後材料ノ都合其他ノ理由ニヨリ多少ノ設計變更ヲナシタル爲メ製作期限ヲ二十五日間延期シ工費ニ於テモ亦約一千六百圓ノ追加ヲナシ十月八日其製作ヲ了シタリ



附　錄

(I)　上下ヨリ單位等布荷重ヲ受クル場合ノ主應力ノ解法

此ノ場合ニ於ケル假定荷重ハ上下相等シク且ツ與ヘラレタル結構モ水平軸ニ就キテ對稱ナルヲ以テ上圖 AOD ヲ以テ區劃シタル四分ノ一結構ニ就テ論スルヲ便トス

故ニ便宜上 D ヲ固定點ト假定シ A 點ニ作用スル彎曲率ヲ M_1 同點ニ於ケル反力ヲ T_1 BB' 内ノ張力ヲ H_1 CC' 内ノ壓力ヲ T_2 ト假定シ此等ノ諸力ハ圓形外殼ニ對スル外力トシ上圖ニ示ス如キ方向ニ作用

スルモノト假定シ之ニ正ノ符號ヲ附ス尚 $ABCD$ 上ニ於ケル外力ノ內彎曲率ハ時針廻轉ノ方向ト等シキ方向ヲ有スルモノヲ正トシ直力 (Direct force) ハ上方ニ向フモノヲ正トス

上記ノ假定ニヨリ先ツ $ABCD$ 間ニ於ケル彎曲率ヲ求ムレハ次ノ如シ

$$\theta=(0\ 乃至\ \alpha),\quad M'=M_1+rmT_1-\tfrac{1}{2}r^2m^2 \qquad (1)$$

$$\theta=(\alpha\ 乃至\ \beta),\quad M''=M_1+rmT_1-\tfrac{1}{2}r^2m^2-H_1rs \qquad (2)$$

$$\theta=(\beta\ 乃至\ \tfrac{\pi}{2}),\quad M'''=M_1+rmT_1-\tfrac{1}{2}r^2m^2-H_1rs+T_2rc$$

但シ　$m=(1-\cos\theta)$,　$s=(\sin\theta-\sin\alpha)$,　$c=(\cos\beta-\cos\theta)$

上式ニ於テ T_2 ハ當然 $(r-T_1)$ ナル可キヲ以テ

$$\theta=(\beta\ 乃至\ \tfrac{\pi}{2}),\quad M'''=M_1+rmT_1-\tfrac{1}{2}r^2m^2-H_1rs-T_1rc+r^2c \qquad (3)$$

ヲ得

次ニ $ABCD$ 內ニ於ケル直力ヲ求ムレハ次ノ如シ

$$\theta=(0\ 乃至\ \alpha),\quad T'=(T_1-rm)\cos\theta \qquad (4)$$

$$\theta=(\alpha\ 乃至\ \beta),\quad T''=(T_1-rm)\cos\theta+H_1\sin\theta \qquad (5)$$

$$\theta=(\beta\ 乃至\ \tfrac{\pi}{2}),\quad T'''=(T_1-rm)\cos\theta+H_1\sin\theta+T_2\cos\theta=r(1-m)\cos\theta+H_1\sin\theta \qquad (6)$$

又 BB' 及 CC' 內ニ於ケル直力ハ BB' ニ在リテハ張力ヲ $(+)$, CC' ニ在リテハ壓力ヲ $(+)$ トシ假定ニヨリ

$$\overline{BB'}=H_1,\qquad \overline{CC'}=T_2=(r-T_1)$$

上記ノ諸外力ニヨル四分ノ一部分ノ各部材ノ內働ノ和ヲ ω トスレハ

$$\omega=\int_0^{\frac{\pi}{2}}\frac{M^2}{2EI}ds+\int_0^{\frac{\pi}{2}}\frac{T^2}{2EA}ds+\frac{H^2}{2EA}l_1+\frac{(r-T_1)^2}{2EA_2}l_2+\varepsilon$$

$$=\frac{1}{2E}\left[\int_0^{\alpha}\frac{M'^2}{I}ds+\int_{\alpha}^{\beta}\frac{M''^2}{I}ds+\int_{\beta}^{\frac{\pi}{2}}\frac{M'''^2}{I}ds\right]+\frac{1}{2E}\left[\int_0^{\alpha}\frac{T'^2}{A}ds+\int_{\alpha}^{\beta}\frac{T''^2}{A}ds+\int_{\beta}^{\frac{\pi}{2}}\frac{T'''^2}{A}ds\right]$$

$$+\frac{H_1^2}{2EA_1}l_1+\frac{(r-T_1)^2}{2EA_2}l_2+\varepsilon \qquad (7)$$

但 ε ハ剪力及不平均溫度等ニヨル內働ニシテ此ノ場合溫度ノ變化ナキモノトスルトキハ其値ノ影響ハ他ノ諸項ニ比シテ甚小ナルモノト假定シ得ヘキヲ以テ

$$\varepsilon=0$$

トシ又 I 及 A_0 ハ圓形外殼ノ斷面ノ物量力率及斷面積ニシテ全周ニ通シ一定ト假定スルトキハ

$$\omega=\frac{1}{2E}\left[\frac{1}{I}\left\{\int_0^{\alpha}M'^2ds+\int_{\alpha}^{\beta}M''^2ds+\int_{\beta}^{\frac{\pi}{2}}M'''^2ds\right\}+\frac{1}{A_0}\left\{\int_0^{\alpha}T'^2ds+\int_{\alpha}^{\beta}T''^2ds+\int_{\beta}^{\frac{\pi}{2}}T'''^2d\right\}+\frac{H_1^2l_1}{A_1}+\frac{(r-T_1)^2l_2}{A_2}\right]$$

即チ最小働ノ原理ニヨリテ M_1 T_1 H_1 ヲ求メ得ルコト次ノ如シ

$$\frac{\partial\omega}{\partial M_1}=0 \quad (8),\qquad \frac{\partial\omega}{\partial T_1}=0 \quad (9),\qquad \frac{\partial\omega}{\partial H_1}=0 \quad (10)$$

但シ $ds=rd\theta$ ナルヲ以テ (8) ヨリ

$$\int_0^{\alpha}M'\frac{\partial M'}{\partial M_1}d\theta+\int_{\alpha}^{\beta}M''\frac{\partial M''}{\partial M_1}d\theta+\int_{\beta}^{\frac{\pi}{2}}M'''\frac{\partial M'''}{\partial M_1}d\theta=0 \qquad (11)$$

又 (9) ヨリ

$$\frac{1}{I}\left\{\int_0^{\alpha} M'\frac{\partial M'}{\partial T_1}d\theta+\int_{\alpha}^{\beta} M''\frac{\partial M''}{\partial T_1}d\theta+\int_{\beta}^{\frac{\pi}{2}} M'''\frac{\partial M'''}{\partial T_1}d\theta\right\}+\frac{1}{A_0}\left\{\int_0^{\alpha} T\frac{\partial T}{\partial T_1}d\theta+\int_{\alpha}^{\beta} T''\frac{\partial T''}{\partial T_1}d\theta\right.$$
$$\left.+\int_{\beta}^{\frac{\pi}{2}} T'''\frac{\partial T'''}{\partial T_1}d\theta\right\}-\frac{(r-T_1)l_2}{rA_2}\quad\cdots\cdots\cdots\quad(12)$$

(10) ヨリ

$$\frac{1}{I}\left\{\int_0^{\alpha} M'\frac{\partial M'}{\partial H_1}d\theta+\int_{\alpha}^{\beta} M''\frac{\partial M''}{\partial H_1}d\theta+\int_{\beta}^{\frac{\pi}{2}} M'''\frac{\partial M'''}{\partial H_1}d\theta\right\}+\frac{1}{A_0}\left\{\int_0^{\alpha} T'\frac{\partial T'}{\partial H_1}d\theta+\int_{\alpha}^{\beta} T'\frac{\partial T'}{\partial H_1}d\theta\right.$$
$$\left.+\int_{\beta}^{\frac{\pi}{2}} T'''\frac{\partial T'''}{\partial H_1}d\theta\right\}+\frac{H_1l_1}{rA_1}=0\quad\cdots\cdots\cdots\cdots\cdots\quad(13)$$

然ルニ(1)(2)(3)(4)(5)(6)ニヨリ

$$\frac{\partial M'}{\partial M_1}=\frac{\partial M''}{\partial M_1}=\frac{\partial M'''}{\partial M_1}=1$$

$$\frac{\partial M'}{\partial T_1}=\frac{\partial M''}{\partial T_1}=rm,\quad\frac{\partial M'''}{\partial T_1}=r(m-c)$$

$$\frac{\partial M'}{\partial H_1}=0,\quad\frac{\partial M''}{\partial H_1}=\frac{\partial M'''}{\partial H_1}=-rs$$

又 $\frac{\partial T'}{\partial T_1}=\frac{\partial T'}{\partial T_1}=\cos\theta,\quad\frac{\partial T'''}{\partial T_1}=0$

$$\frac{\partial T'}{\partial H_1}=0,\quad\frac{\partial T''}{\partial H_1}=\frac{\partial T'''}{\partial H_1}=\sin\theta$$

故ニ(11)(12)(13)式ハ次ノ如ク書キ代フルコトヲ得

$$\int_0^{\frac{\pi}{2}}\left(M_1+rmT_1-\frac{r^2}{2}m^2\right)d\theta-\int_0^{\frac{\pi}{2}}rsH_1d\theta+\int_{\beta}^{\frac{\pi}{2}}r(r-T_1)cd\theta=0\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\quad(14)$$

$$\int_0^{\frac{\pi}{2}}\left(M_1+rmT_1-\frac{r^2}{2}m^2\right)rmd\theta+\int_{\alpha}^{\frac{\pi}{2}}(-r^2smH_1)d\theta+\int_{\beta}^{\frac{\pi}{2}}(r^2r-T_1)(cm-c^2)d\theta$$
$$+\int_{\beta}^{\frac{\pi}{2}}\left(M_1+rmT_1-\frac{r^2}{2}m^2-H_1rs\right)(-rs)d\theta+\frac{I}{A_3}\left\{\int_0^{\beta}(T_1-rm)\cos^2\theta d\theta\right.$$
$$\left.+\int_{\alpha}^{\beta}\sin\theta\cos\theta H_1\right\}-\frac{I(r-T_1)l_2}{rA_2}=0\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\quad(15)$$

$$\int_{\alpha}^{\frac{\pi}{2}}\left(M_1+rmT_1-\frac{r^2}{2}m^2-H_1rs\right)(-rs)d\theta+\int_{\beta}^{\frac{\pi}{2}}\left\{-r^2(r-T_1)\right\}csd\theta+\frac{I}{A_3}\left[\int_{\alpha}^{\beta}\left\{(T_1-rm)\cos\theta+H\sin\theta\right\}\sin\theta d\theta\right.$$
$$\left.+\int_{\beta}^{\frac{\pi}{2}}\left\{r\cos^2\theta+H\sin\theta\right\}\sin\theta d\theta\right]+\frac{IH_1l_1}{rA_1}=0\quad\cdots\cdots\cdots\cdots\quad(16)$$

(14)(15)(16)式ヲ積分シテ

$$a_1M_1+a_2T_1+a_3H_1=k_1\cdots\quad\cdots\cdots\cdots\quad(17),\qquad a_4M_1+a_5T_1+a_6H_1=k_2\quad\cdots\cdots\cdots\cdots\quad(18)$$

$$a_7M_1+a_8T_1+a_9H_1=k_3\cdots\quad\cdots\cdots\cdots\quad(19)$$

ヲ得ルモノトスルトキハ $a_1\ a_2\cdots\cdots a_9$ 及 $k_1\ k_2\ k_3$ ノ値ハ次ノ如キモノタルコトヲ要ス

$$a_1=\int_0^{\frac{\pi}{2}}d\theta,\qquad a_2=r\left\{\int_0^{\frac{\pi}{2}}md\theta-\int_{\beta}^{\frac{\pi}{2}}cd\theta\right\}\qquad a_3=-r\int_{\alpha}^{\frac{\pi}{2}}sd\theta$$

$$-k_1=r^2\left\{-\frac{1}{2}\int_0^{\frac{\pi}{2}}m^2d\theta+\int_{\beta}^{\frac{\pi}{2}}cd\theta\right\}$$

$$a_4=r\left\{\int_0^{\frac{\pi}{2}}md\theta-\int_{\beta}^{\frac{\pi}{2}}cd\theta\right\}=a_2$$

$$a_5=r^2\left\{\int_0^{\frac{\pi}{2}}m^2d\theta-\int_{\beta}^{\frac{\pi}{2}}(2cm-c^2)d\theta\right\}+\frac{I}{A_3}\int_0^{\beta}\cos^2\theta d\theta+\frac{Il_2}{rA_2}$$

$$a_6=r^2\left\{-\int_0^{\frac{\pi}{2}}sm\,d\theta+\int_\beta^{\frac{\pi}{2}}sc\,d\theta\right\}+\frac{I}{A_0}\int_\alpha^\beta\sin\theta\cos\theta\,d\theta$$

$$-k_2=r^2\left\{-\frac{1}{2}\int_0^{\frac{\pi}{2}}m^2d\theta+\int_\beta^{\frac{\pi}{2}}\left(\frac{1}{2}m^2c+cm-c^2\right)d\theta\right\}+\frac{I}{A_0}r\int_0^\beta m\cos^2\theta\,d\theta-\frac{Il_2}{A_2}$$

$$a_7=-r^2\int_\alpha^{\frac{\pi}{2}}s\,d\theta=a_2$$

$$a_8=r^2\left\{-\int_\alpha^{\frac{\pi}{2}}sm\,d\theta+\int_\beta^{\frac{\pi}{2}}cs\,d\theta\right\}+\frac{I}{A_0}\int_\alpha^\beta\cos\theta\sin\theta\,d\theta=a_6.$$

$$a_9=r^2\int_\alpha^{\frac{\pi}{2}}s^2d\theta+\frac{I}{A_0}\int_\alpha^{\frac{\pi}{2}}\sin^2\theta\,d\theta+\frac{Il_1}{rA_1}$$

$$k_3=r^3\left\{\frac{1}{2}\int_\alpha^{\frac{\pi}{2}}s\,m^2d\theta-\int_\beta^{\frac{\pi}{2}}c\,s\,d\theta\right\}-\frac{I}{A_0}r\int_\alpha^{\frac{\pi}{2}}\cos^2\theta\sin\theta\,d\theta+\frac{I}{A_0}r\int_\alpha^\beta\sin\theta\cos\theta\,d\theta$$

但　$m=(1-\cos\theta)$,　　$s=(\sin\theta-\sin\alpha)$,　　$c=(\cos\beta-\cos\theta)$

ナルコト前述ノ如シ

之ニヨリテ所要ノ値ヲ求ムルコトヲ得

今上式中ニ所要ノ積分ヲ摘記スレハ次ノ如シ但シ上記ノ諸項中其上方ニ横棒ヲ引キタルモノハ圓形外殼ニ對スル直力ノ影響ナリ

$$\int d\theta=\theta,\qquad \int m\,d\theta=\theta-\sin\theta,\qquad \int c\,d\theta=\theta\cos\beta-\sin,\qquad \int s\,d\theta=-\cos\theta-\theta\sin\alpha$$

$$\int m^2d\theta=\frac{3}{2}\theta-2\sin\theta+\frac{1}{4}\sin 2\theta,\qquad \int c^2d\theta=\theta\cos^2\beta-2\cos\beta\sin\theta+\frac{1}{4}\sin 2\theta+\frac{\theta}{2}$$

$$\int s^2d\theta=\frac{\theta}{2}-\frac{1}{4}\sin 2\theta+2\sin\alpha\cos\theta+\theta\sin^2\alpha,\qquad \int cm\,d\theta=\theta\cos\beta-\sin\theta(1+\cos\beta)+\frac{\sin 2\theta}{4}+\frac{\theta}{2}$$

$$\int sc\,d\theta=-\frac{1}{2}\sin^2\theta-\cos\beta\cos\theta-\theta\sin\alpha\cos\beta+\sin\alpha\sin\theta,\qquad \int sm\,d\theta=-\cos\theta-\frac{\sin^2\theta}{2}-\theta\sin\alpha+\sin\alpha\sin\theta$$

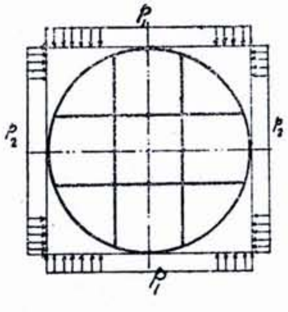
$$\int sm^2d\theta=-\frac{4}{3}\cos\theta-\sin^2\theta+\frac{\cos\theta\sin^2\theta}{3}$$

$$\int cm^2d\theta=\cos\beta\left(\frac{3}{2}\theta-2\sin\theta+\frac{1}{4}\sin 2\theta\right)-\left(\frac{5}{3}\sin\theta-\frac{1}{2}\sin 2\theta-\theta+\frac{\sin\theta\cos^2\theta}{3}\right)$$

$$\int m^3d\theta=\frac{5}{2}\theta-4\sin\theta+\frac{3}{4}\sin 2\theta+\frac{1}{3}\sin^3\theta,\qquad \int m\cos^2\theta\,d\theta=\int\cos^2\theta\,d\theta-\int\cos^3\theta\,d\theta$$

……………………………………………………………etc.

但シ上記ノ積分ニハ總テ任意ノ積分定數ヲ附加スヘキモノナルモ此ノ場合積分ハ總テ定積分トシテ計算セラルヘキモノナルヲ以テ特ニ之ヲ省略シタリ



即チ上記ノ積分ニ所定ノ限界ヲ挿入シテ　a_1　a_2　……　k_1　k_2　k_3　等ヲ求ムルコトヲ得之ニヨリテ(17)(18)(19)式ヲ得

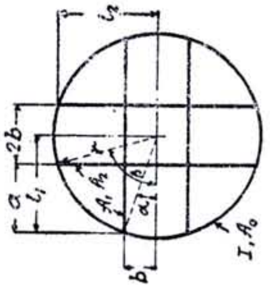
卽チ之ニヨリテ單位等布荷重ニ對スル　M_1, H_1, T_1　及　$T_2=(r-T_1)$　ヲ求メ得ヘク從テ結構ノ各部ニ於ケル彎曲率及直力ヲ求ムルコトヲ得又之ニヨリテ彎曲應力竝ニ直應力强度ヲ求ムルコトヲ得

荷重　$P^{t/m}$ ナルトキハ之等ノ値ヲP倍スレハ可ナルコトハ明ナリ卽之ヲ以テ此ノ如キ荷重狀態ニ於ケル應力ノ一般解法トナスコトヲ得ヘシ

尙其他ノ荷重狀態ニ於ケル應力モ本解法ノ結果ニヨリ誘導セラル丶場合少ナカラス

（系）　**上下及兩側荷重ヲ受クル場合**

今上下ヨリ p_1 ナル壓力ヲ受ケ水平ニ p_2 ナル壓力ヲ受ケタリトスルトキハ之ヲ單ニ p_1 ノミヲ受ケタル場合及 p_2 ノミヲ受ケタル場合トシテ應力ヲ求メ之ヲ加算スレハ可ナリ $p_1=p_2$ ナルトキハ卽チ均等水壓力ヲ受クル場合ニ等シ



上記ノ諸式ヲ使用シ羽越北線折渡隧道用盾構ノ應力ヲ計算シタル所次ノ如シ

$r=11'-6''=138''$, $b=3'-6''=42''$, $a=89.''453$,

$\alpha=0.3092=17°-43'$, $\beta=\frac{\pi}{2}-\alpha=1.2616=72°-17'$

$\sin\alpha=0.304$, $\sin\beta=0.953$

$\cos\alpha=0.953$, $\cos\beta=0.304$

$I=56,200''^4$ （總斷面積ニテ）

$A_0=398\square''$, $A_1=A_2=138\square''$, $l_1=l_2=131$

ト假定スレハ次ノ諸項ヲ得

$\int_0^{\frac{\pi}{2}} d\theta=1.571$, $\int_\alpha^{\frac{\pi}{2}} sm^2 d\theta=0.226$, $\int_\beta^{\frac{\pi}{2}} md\theta=0.571$

$\int_\beta^{\frac{\pi}{2}} cm^2 d\theta=0.037$, $\int_\beta^{\frac{\pi}{2}} cd\theta=0.046$, $\int_0^{\frac{\pi}{2}} m^2 d\theta=0.260$

$\int_\alpha^{\frac{\pi}{2}} sd\theta=0.569$, $\int_0^\beta m\cos^2\theta\, d\theta=\int_0^\beta \cos^3\theta-\int_0^\beta \cos^4\theta=0.112$, $\int_0^{\frac{\pi}{2}} m^2 d\theta=0.356$

$\int_0^\beta \cos^2\theta\, d\theta=0.776$, $\int_\beta^{\frac{\pi}{2}} c^2 d\theta=0.009$, $\int_\alpha^{\frac{\pi}{2}} \sin^2\theta=0.776$

$\int_\beta^{\frac{\pi}{2}} s^2 d\theta=0.310$, $\int_\alpha^\beta \sin\theta\cos\theta\, d\theta=0.408$, $\int_\beta^{\frac{\pi}{2}} cmd\theta=0.041$

$\int_\alpha^{\frac{\pi}{2}} \cos^2\theta\sin\theta\, d\theta=0.289$, $\int_\beta^{\frac{\pi}{2}} scd\theta=0.033$, $\int_0^\beta \cos^4\theta\, d\theta=0.664$

$\int_\alpha^{\frac{\pi}{2}} smd\theta=0.327$

此等ノ諸項ヲ前揭式ニ代入シテ次ノ諸値ヲ得

$a_1=1.571$, $a_2=0.525r$, $a_3=-0.569r$, $k_1=0.132r^2$

$a_4=a_2$, $a_5=0.283r^2+386+110$, $a_6=-0.294r^2+58$, $k_2=0.079r^3+386r+15.8r$

$a_7=a_3$, $a_8=a_6$, $a_9=0.310r^2+386+110$, $k_3=-0.080r^3+16.8r$

之ニヨリテ

$1.571\,M_1+72.5\,T_1-78.5\,H_1=2,520$

$72.5\,M_1+5,890\,T_1-5,540\,H_1=263,000$

$-78.5\,M_1-5,540\,T_1+6,400\,H_1=-208,000$

ヲ得之ヲ書キ代フレハ

$M_1+46.1\,T_1-50.0\,H_1=1,600$

$M_1+81.2\,T_1-76.4\,H_1=3,630$

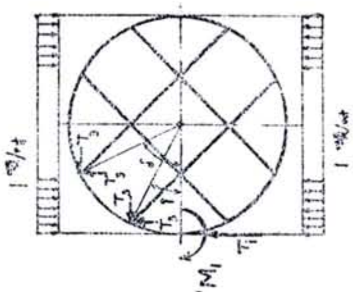
$M_1+70.5\,T_1-81.5\,H_1=2,650$

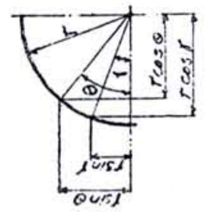
之ヲ解キテ

$M_1 = -630'''$,　　$T_1 = 78.6^t$,　　$T_2 = 59.4^t$,　　$H_1 = 27.6^t$

ヲ得之ニヨリテ各點ノ主應力ヲ求ムルコトヲ得

II. 四十五度ノ等布單位傾斜荷重ヲ受クル場合ノ主應力ノ計算





此ノ場合ニ於ケル井狀結構部材ノ應力ハ結構ノ形及荷重ノ對稱ナルカ爲メ總テ相等シク其値Sナリトシ便宜上外殼ニ對シテハT_3トシテ圖ニ示ス如ク作用スルモノト假定シ且一部材ノ全長ヲ$2l$トスレハ(I)ノ場合ト同樣ニ

$$\theta = (0 \text{ 乃至 } \gamma),\quad M' = M'_1 + rmT_1 - \tfrac{1}{2}r^2m^2 \cdots \cdots \quad (1)$$

$$\theta = (\gamma \text{ 乃至 } \delta),\quad M'' = M_1 + rmT_1 - \tfrac{1}{2}r^2m^2 + r(\sin\theta - \sin\gamma + \cos\gamma - \cos\theta)T_3 \cdots \quad (2)$$

$$\theta = (\delta \text{ 乃至 } \tfrac{\pi}{2}),\quad M''' = M_1 + rmT_1 - \tfrac{1}{2}r^2m^2 + r(2\sin\theta - \sin\gamma - \sin\delta + \cos\gamma + \cos\delta - 2\cos\theta)T_3 \cdots \quad (3)$$

但シ　$m = (1 - \cos\theta)$

然ルニ　$\sin\gamma = \cos\delta$,　　$\therefore\ \sin\gamma - \cos\delta = 0$

$\cos\gamma = \sin\delta$,　　$\therefore\ \cos\gamma - \sin\delta = 0$

$\therefore\ \sin\theta - \cos\theta = n$

$\cos\gamma - \sin\gamma = e$　ト置ケハ

$$M' = M_1 + rmT_1 - \tfrac{1}{2}r^2m^2 \cdots \cdots \quad (4)$$

$$M'' = M_1 + rmT_1 - \tfrac{1}{2}r^2m^2 + r(n+e)T_3 \cdots \cdots \quad (5)$$

$$M''' = M_1 + rmT - \tfrac{1}{2}r^2m^2 + 2rnT_3 \cdots \cdots \quad (6)$$

$$\theta = (0 \text{ 乃至 } \gamma),\quad T' = (T_1 - rm)\cos\theta \cdots \cdots \quad (7)$$

$$\theta = (\gamma \text{ 乃至 } \delta),\quad T''' = (T_1 - rm)\cos\theta + T_3(\cos\theta - \sin\theta) = (T_1 - rm)\cos\theta - nT_3 \cdots \quad (8)$$

$$\theta = (\delta \text{ 乃至 } \tfrac{\pi}{2}),\quad T'' = (T_1 - rm)\cos\theta + 2T_3(\cos\theta - \sin\theta) = (T_1 - rm)\cos\theta - 2nT_3 \cdots \quad (9)$$

以下(I)ノ場合ト同樣ノ解法ニヨリテM_1及T_1ヲ求ムルコト次ノ如シ

$$\omega = \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{M^2}{EI} ds + \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{T_2}{A_0E} ds + \frac{1}{2}\Sigma \frac{S^2}{EA_1} l + \varepsilon \cdots \cdots \quad (10)$$

$$\frac{\partial\omega}{\partial M_1} = 0 \cdots \cdots \quad (11)$$

$$\frac{\partial\omega}{\partial T_1} \cdots \cdots \quad (12)$$

然ルニ

$$\frac{1}{2}\Sigma \frac{S^2 l}{EA_1} = 2 \times \frac{1}{2} \cdot \frac{(\sqrt{2}T_3)^2 l}{EA_1} = \frac{2T_3^2 l}{EA_1} = K$$

$$\frac{\partial K}{\partial T_1} = \frac{4T_3 l}{EA_1} \cdot \frac{\partial T_3}{\partial T_1} = -\frac{2T_3 l}{EA_1} = \frac{-(r - T_1)l}{EA_1} \left(\because\ T_3 \equiv \frac{r - T_1}{2}\right)$$

$$\frac{\partial M'}{\partial M_1} = \frac{\partial M''}{\partial M_1} = \frac{\partial M'''}{\partial M_1} = 1, \qquad \frac{\partial T'}{\partial M_1} = \frac{\partial T''}{\partial M_1} = \frac{\partial T''}{\partial M_1} = 0$$

$$\frac{\partial M'}{\partial T_1} = rm, \quad \frac{\partial M''}{\partial T_1} = r\left(m - \frac{n+e}{2}\right), \quad \frac{\partial M'''}{\partial T_1} = r(m - n)$$

$$\frac{\partial T'}{\partial T_1} = \cos\theta, \quad \frac{\partial T''}{\partial T_1} = (\cos\theta + \tfrac{1}{2}n), \quad \frac{\partial T'''}{\partial T_1} = (\cos\theta + n) = \sin\theta$$

故ニ(11)式ヨリ

$$\int_0^{\frac{\pi}{2}}\left(M_1+rmT_1-\frac{r^2}{2}m^2\right)d\theta+\int_r^s r(n+e)T_3d\theta+\int_s^{\frac{\pi}{2}}2\,rnT_3d\theta=0 \quad \cdots\cdots \quad (13)$$

又(12)式ヨリ

$$\frac{1}{I}\Big[\int_0^r\Big(M_1+rmT_1-\frac{1}{2}r^2m^2\Big)r\,md\theta+\int_r^s\Big\{M_1+rmT_1-\frac{1}{2}r^2m^2+r(n+e)T_3\Big\}\Big\{rm-\frac{1}{2}(n+e)\Big\}rd\theta$$

$$+\int_s^{\frac{\pi}{2}}\Big\{M_1+rmT_1-\frac{1}{2}r^2m^2+2rnT_3\Big\}\Big\{m-n\Big\}rd\theta\Big]$$

$$+\frac{1}{A_1}\Big[\int_0^r(T_1-rm)\cos^2\theta\,d\theta+\int_r^s\Big\{(T_1-rm)\cos\theta-nT_3\Big\}\Big\{\cos\theta+\frac{n}{2}\Big\}d\theta$$

$$+\int_s^{\frac{\pi}{2}}\Big\{(T_1-rm)\cos\theta-2\,nT_3\Big\}\Big\{\sin\theta\Big\}d\theta\Big]-\frac{1}{r}\cdot\frac{2T_3l_1}{A_1}=0 \quad \cdots\cdots \quad (14)$$

ヲ得

今(13)及(14)式ヲ

$$a_1M_1+a_2T_1=k_1 \quad \cdots\cdots \quad (15)$$

$$a_3M_1+a_4T_1=k_2 \quad \cdots\cdots \quad (16)$$

ト置ケバ

$$a_1=\int_0^{\frac{\pi}{2}}d\theta$$

$$a_2=r\Big\{\int_0^{\frac{\pi}{2}}md\theta-\frac{1}{2}\int_r^s(n+e)d\theta-\int_s^{\frac{\pi}{2}}nd\theta\Big\}-k_1=r^2\Big\{-\frac{1}{2}\int_0^{\frac{\pi}{2}}m^2d\theta+\frac{1}{2}\int_r^s(n+e)d\theta+\int_s^{\frac{\pi}{2}}nd\theta\Big\}$$

$$a_3=a_2$$

$$a_4=\frac{r_2}{I}\Big\{\int_0^r m^2d\theta+\int_r^s\Big\{m-\frac{1}{2}(n+e)\Big\}^2d\theta+\int_s^{\frac{\pi}{2}}(m-n)^2d\theta\Big]+\frac{1}{A}\Big[\int_0^r\cos^2\theta d\theta+\int_r^s\Big(\cos\theta+\frac{n}{2}\Big)^2d\theta$$

$$+\int_s^{\frac{\pi}{2}}\sin^2\theta\,d\theta\Big]+\frac{1}{r}\cdot\frac{l_1}{A_1}$$

$$-k_2=\frac{r^3}{I}\Big[-\frac{1}{2}\int_0^r m^3d\theta-\frac{1}{4}\int_r^s\Big\{m^2-(n+e)\Big\}\Big\{2\,m-(n+e)\Big\}d\theta+\int_s^{\frac{\pi}{2}}\Big(-\frac{m^2}{2}+n\Big)\Big(m-n\Big)d\theta\Big]$$

$$+\frac{r}{A_0}\Big[\int_0^r-m\cos^2\theta\,d\theta+\int_r^s\Big(-m\cos\theta-\frac{n}{2}\Big)\Big(\cos\theta+\frac{n}{2}\Big)d\theta+\int_s^{\frac{\pi}{2}}(-m\cos\theta-n)\sin\theta\,d\theta\Big]-\frac{l_1}{A_1}$$

$$\therefore\quad a_4=\frac{r^2}{I}\Big[\int_0^r m^2d\theta\int_r^s\Big\{m^2-mn-em+\frac{1}{4}\Big(n^2+2en+e^2\Big)\Big\}d\theta+\int_s^{\frac{\pi}{2}}(m^2-2\,mn+n^2)d\theta\Big]$$

$$+\frac{1}{A_0}\Big[\int_0^r\cos^2\theta\,d\theta+\int_r^s\Big\{\cos^2\theta+n\cos\theta+\frac{n^2}{4}\Big\}d\theta+\int_s^{\frac{\pi}{2}}\sin^2\theta\,d\theta\Big]+\frac{l_1}{r}A_1$$

$$-k_2=\frac{r^3}{I}\Big[-\frac{1}{2}\int_0^r m^3d\theta-\frac{1}{4}\int_r^s\Big\{2m^3-2\,mn-m^2n-(2m+m^2)e+n^2+2\,ne+e^2\Big\}d\theta+\int_s^{\frac{\pi}{2}}\Big\{-\frac{m^2}{2}+nm+\frac{nm^2}{2}-n^2\Big\}d\theta\Big]$$

$$+\frac{r}{A_0}\Big[\int_0^r-m\cos^2\theta d\theta+\int_r^s\Big\{-m\cos^2\theta-\frac{n}{2}\cos\theta-\frac{nm}{2}\cos\theta-\frac{n^2}{4}\Big\}d\theta+\int_s^{\frac{\pi}{2}}(-m\cos\theta\sin\theta-n\sin\theta)d\theta\Big]-\frac{l_1}{A}$$

然ルニ

$$\int d\theta=\theta,\qquad \int md\theta=\theta-\sin\theta,\qquad \int nd\theta=-\cos\theta-\sin\theta$$

$$\int m^2d\theta=\frac{3}{2}\theta-2\sin\theta+\frac{1}{4}\sin 2\,\theta$$

$$\int mnd\theta = -\cos\theta - \sin\theta - \frac{1}{2}\sin^2\theta + \frac{1}{4}\sin 2\theta + \frac{1}{2}\theta$$

$$\int n^2 d\theta = \theta - \sin^2\theta$$

$$\int m^3 d\theta = \frac{5}{2}\theta - 4\sin\theta + \frac{3}{4}\sin 2\theta + \frac{1}{3}\sin^3\theta$$

$$\int m^2 n d\theta = \theta - \frac{4}{3}\cos\theta - \frac{5}{3}\sin\theta - \sin^2\theta + \frac{1}{2}\sin 2\theta + \frac{\cos\theta\sin^2\theta}{3} - \frac{\sin\theta\cos^2\theta}{3}$$

$$\int \cos^2\theta d\theta = \frac{1}{4}\sin 2\theta + \frac{1}{2}\theta$$

$$\int n\cos\theta\, d\theta = \frac{1}{2}\sin^2\theta - \frac{1}{4}\sin 2\theta - \frac{\theta}{2}$$

$$\int \sin^2\theta\, d\theta = -\frac{1}{4}\sin 2\theta + \frac{1}{2}\theta$$

$$\int m\cos^2\theta\, d\theta = \frac{1}{4}\sin 2\theta + \frac{\theta}{2} - \frac{\sin\theta\cos^2\theta}{3} - \frac{2}{3}\sin\theta$$

$$\int nm\cos\theta\, d\theta = \frac{\sin^2\theta}{2} - \left(\frac{1}{4}\sin 2\theta + \frac{\theta}{2}\right) + \cos\theta - \left(\frac{\cos\theta\sin^2\theta}{3} + \frac{2}{3}\cos\theta\right) + \left(\frac{\sin\theta\cos^2\theta}{3} + \frac{2}{3}\sin\theta\right)$$

$$\int(-m\cos\theta - n)\sin\theta\, d\theta = -\cos\theta + \left(\frac{\cos\theta\sin^2\theta}{3} + \frac{2}{3}\cos\theta\right) + \left(\frac{1}{4}\sin 2\theta - \frac{\theta}{2}\right)$$

$$\int m\cos\theta\sin\theta = \int(\cos\theta\sin\theta - \cos^2\theta\sin\theta)d\theta = \int(\cos\theta\sin\theta - \sin\theta + \sin^3\theta)\, d\theta = \frac{1}{2}\sin^2\theta + \cos\theta - \frac{\sin^2\theta\cos\theta}{3} - \frac{2}{3}\cos\theta$$

$$\int n\sin\theta\, d\theta = \int(\sin^2\theta - \cos\theta\sin\theta)d\theta = -\frac{1}{4}\sin 2\theta + \frac{\theta}{2} - \frac{1}{2}\sin^2\theta$$

$$\therefore\quad -\int m\cos\theta\sin\theta\, d\theta - \int n\cos\theta\, d\theta = -\cos\theta + \frac{\sin^2\theta\cos\theta}{3} + \frac{2}{3}\cos\theta + \frac{1}{4}\sin 2\theta - \frac{\theta}{2}$$

然ルニ折渡隧道用盾構ノ場合ニ在リテハ

$\alpha = 27° - 17' = 0.476,\qquad \beta = 62° - 43' = 1.0946,\qquad \sin\alpha = \cos\beta = 0.458,\qquad \sin\beta = \cos\alpha = 0.889$

依テ次ノ數値ヲ得

$\int_0^{\frac{\pi}{2}} d\theta = 1.571$	$-\int_\gamma^\delta mnd\theta = -0.020$	
$\int_\gamma^\delta nd\theta = 0$	$-\int_\delta^{\frac{\pi}{2}} mnd\theta = -0.276$	
$\int_\gamma^\delta d\theta = 0.619$	$\int_\gamma^\delta md\theta = 0.188$	
$\int_0^{\frac{\pi}{2}} m^2 d\theta = 0.357$	$\int_\gamma^\delta n^2 d\theta = 0.039$	$\int_\delta^{\frac{\pi}{2}} nd\theta = 0.347$
$\int_\delta^{\frac{\pi}{2}} n^2 d\theta = 0.266$	$\int_\gamma^\delta d\theta = 0.619$	$-\int_\delta^{\frac{\pi}{2}} \frac{m^3}{2} d\theta = -0.130$
$\int_0^\delta \cos^2\theta d\theta = 0.752$	$\int_\gamma^\delta m^2 d\theta = 0.066$	$\int_\gamma^\delta n\cos\theta\, d\theta = -0.020$
$\int_\gamma^\delta -mnd\theta = -0.020$	$\int_\delta^{\frac{\pi}{2}} \sin^2\theta\, d\theta = 0.442$	$\int_\gamma^\delta md\theta = 0.188$
$\int_\gamma^\delta n\cos\theta d\theta = -0.020$	$\int_\gamma^\delta n^2 d\theta = 0.039$	$\int_\gamma^\delta nm\cos\theta\, d\theta = 0.007$