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均等斷面積ヲ有スル雙鉸楕圓拱

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第一章　緒言

均等斷面積ヲ有スル楕圓拱(Elliptic arch with uniform cross section)ノ理論ハ楕圓積分(Elliptic integral)ヲ伴ヒ其計算頗面倒ニシテ實際使用セラルヽコト極メテ稀ナレトモ次ニ之カ設計ニ際シ必要ナル算式ノ一般ヲ示サントス

楕圓積分ノ理論ニ關シテハ數多ノ著書アレトモ本論文ニ於テハ主トシテ次ノ書籍ヲ參照セリ

Baker..................Elliptic Functions.

Martin Krause......Theorie der Elliptischen Funktionen.

Lugien Lévy.........Précis Élémentaire de la Théorie des Fonctions Elliptiques.

今之等ノ書籍中ヨリ本論文ノ研究ニ必要ナル公式ノ二三ヲ記セハ

(一)　第一類楕圓積分

$$F(k,\varphi)=\int_0^\varphi \frac{d\varphi}{\sqrt{1-k^2\sin^2\varphi}} \qquad k<1 \quad \cdots\cdots(1)$$

(二)　第二類楕圓積分

$$E(k,\varphi)=\int_0^{\varphi}\sqrt{1-k^2\sin^2\varphi}\,d\varphi \qquad k<1 \quad\cdots\cdots\cdots\cdots(2)$$

(三) $x=\sin\varphi$ トスレバ

$$\int_0^{x}\frac{dx}{\sqrt{(1-x^2)(1-k^2x^2)}}=\text{第一類楕圓積分}\cdots\cdots\cdots\cdots(3)$$

$$\int_0^{x}\frac{\sqrt{1-k^2x^2}}{\sqrt{1-x^2}}dx=\text{第二類楕圓積分}\cdots\cdots\cdots\cdots(4)$$

(四) 第一類楕圓積分ヲ無限級數ノ形ニナシテ計算スレバ

$$F(k,\varphi)=\varphi+\frac{1}{2}k^2J_2+\frac{1.3}{2.4}k^4J_4+\cdots\cdots+\frac{1.3\cdots\cdots(2n-1)}{2.4\cdots\cdots2n}k^{2n}J_{2n}+\cdots\cdots\cdots\cdots(5)$$

玆ニ

$$J_{2n}=\int_0^{\varphi}\sin^{2n}\varphi\,d\varphi$$

$$=-\frac{\cos\varphi}{2n}F+\frac{1.3\cdots\cdots(2n-3)(2n-1)}{2.4\cdots\cdots(2n-2)2n}\varphi$$

$$F=\sin^{2n-1}\varphi+\frac{2n-1}{2n-2}\sin^{2n-3}\varphi+\cdots\cdots+\frac{3.5\cdots\cdots(2n-3)(2n-1)}{2.4\cdots\cdots(2n-4)(2n-2)}\sin\varphi$$

$\varphi=\dfrac{\pi}{2}$ ノ時ハ

$$J_{2n}=\frac{1.3\cdots\cdots(2n-1)}{2.4\cdots\cdots2n}\cdot\frac{\pi}{2}$$

(五) 第二類楕圓積分ヲ無限級數ノ形ニナシテ計算スレバ

$$E(k,\varphi)=\varphi-\frac{\frac{1}{2}}{1}k^2J_2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{\lfloor 2}k^4J_4-\cdots\cdots+(-1)^n\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\cdots\cdots\left(\frac{1}{2}-n+1\right)}{\lfloor n}k^{2n}J_{2n}+ \quad\cdots(6)$$

$J_2\ J_4\cdots\cdots J_{2n}$ 等ノ値ハ第二類楕圓積分ノ時ト同一トス

(四)(五)ニ就テハ Martin Krause—Theorie der Elliptischen Funktionen ヲ參照セラレタシ

(六) $R=\sqrt{a+bx^2+cx^4}$ ($a\,b\,c$=定數)トスレハ

$$\int\frac{A+BR}{C+DR}dx=\int\frac{AC-BDR^2}{C^2-D^2R^2}dx-\int\frac{(AD-CB)R^2}{C^2-D^2R^2}\cdot\frac{1}{R}dx \quad\cdots\cdots(7)$$

茲ニ $A\ B\ C\ D$ ハ x ノ函數ニアラサル定數トス

(七) $R=\sqrt{a+bx^2+cx^4}$ ($a\,b\,c$=定數)トスレハ

$$Rx^{2n-3}=(2n-3)a\int\frac{x^{2n-4}}{R}dx+(2n-2)b\int\frac{x^{2n-2}}{R}dx+(2n-1)c\int\frac{x^{2n}}{R}dx \quad\cdots\cdots(8)$$

$n=2$ トスレハ

$$Rx=a\int\frac{1}{R}dx+2b\int\frac{x^2}{R}dx+3c\int\frac{x^4}{R}dx \quad\cdots\cdots(9)$$

(八) $\Delta=\sqrt{1-k^2\sin^2\varphi}$, $E=E(k,\varphi)$, $F=F(k,\varphi)$ トスレハ

$$\int_0^{\varphi}\frac{\sin^2\varphi}{\Delta}d\varphi=\frac{F-E}{k^2} \quad\cdots\cdots(10)$$

$$\int_0^{\varphi}\frac{\cos^2\varphi}{\Delta}d\varphi=\frac{E-(1-k^2)F}{k^2} \quad\cdots\cdots(11)$$

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$$\int_0^{\varphi}\frac{\tan^2\varphi}{\Delta}d\varphi=\frac{\Delta\tan\varphi-E}{1-k^2}\quad\cdots\cdots(12)$$

$$\int_0^{\varphi}\frac{\sec^2\varphi}{\Delta}d\varphi=\frac{\Delta\tan\varphi+(1-k^2)F-E}{1-k^2}\quad\cdots\cdots(13)$$

$$\int_0^{\varphi}\frac{1}{\Delta^3}d\varphi=\frac{1}{1-k^2}\left(E-\frac{k^2\sin\varphi\cos\varphi}{\Delta}\right)\quad\cdots\cdots(14)$$

$$\int_0^{\varphi}\frac{\sin^2\varphi}{\Delta^3}d\varphi=\frac{1}{1-k^2}\left\{\frac{E-(1-k^2)F}{k^2}-\frac{\sin\varphi\cos\varphi}{\Delta}\right\}\quad\cdots\cdots(15)$$

$$\int_0^{\varphi}\frac{\cos^2\varphi}{\Delta^3}d\varphi=\frac{F-E}{k^2}+\frac{\sin\varphi\cos\varphi}{\Delta}\quad\cdots\cdots(16)$$

(六)(七)(八)ニ就テハ Baker—Elliptic Functions ヲ參照セラレタシ

楕圓積分ノ表ニ就テハ未タ完全ナルモノナケレトモ Lugien Lévy—Précis Élémentaire de la Theorie des Fonctions Elliptiques ニ所載ノモノハ稍々參考トナル(キニヨリ附錄トシテ本論文末尾ニ揭ケアルヲ以テ參照セラレタシ (本表ハ編輯ノ都合ニヨリテ省略ス)

第二章　總論

第一圖及第二圖ニ於テ a 及 b ヲ楕圓ノ長軸 (Major axis) 及短軸 (Minor axis) トシ h 及 l ヲ拱橋ノ拱矢 (Rise) 及徑間 (Span) トス

H=支承 A 及 C ニ於テ一個ノ W ノタメニ生セル水平反力 (Horizontal reaction)

M=拱ノ任意點ニ於ケル彎曲率 (Bending moment)

但拱ノ上部纖維 (Upper fiber) ニ壓力ヲ與フル彎曲率ヲ正トシ張力ヲ與フルモノヲ負トス

N=拱ノ任意點ニ於ケル軸應力(Axial stress or normal stress)

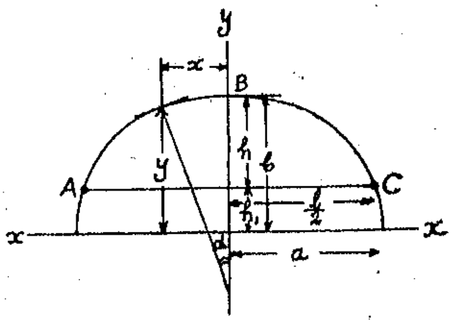
但拱ニ張力ヲ與フル軸應力ヲ正トシ壓力ヲ與フルモノヲ負トス

d=拱軸ニ沿フテBヨリDニ至ル距離

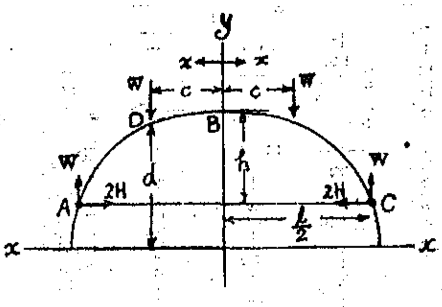
$\frac{l}{2}$=拱軸ニ沿フテBヨリAニ至ル距離

α=曲線上ノ任意點ニ於ケル法線(Normal)カy軸ト爲ス角

E=彈率(Modulus of elasticity)



第一圖



第二圖

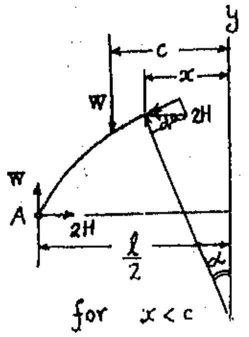
I=拱ノ任意點ニ於ケル惰率(Moment of inertia)

A=拱ノ任意點ニ於ケル斷面積(Cross sectional area)

第一圖及第二圖ヨリ拱ノ任意點ニ於ケル彎曲率ハ次ノ如シ

for $x<c$ $\quad M=W\left(\frac{l}{2}-c\right)-2H(y-h_1),\quad \frac{dM}{dH}=-2(y-h_1)$

for $x>c$ $\quad M=W\left(\frac{l}{2}-x\right)-2H(y-h_1),\quad \frac{dM}{dH}=-2(y-h_1)$



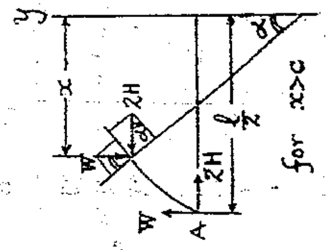
第三圖

第三圖及第四圖ヨリ拱ノ任意點ニ於ケル軸應力ハ次ノ如シ

for $x<c$ $\quad N=-2H\cos\alpha,\quad \frac{dN}{dH}=-2\cos\alpha$

for $x>c$ 　　$N=-2H\cos\alpha-W\sin\alpha,$ 　　$\frac{dN}{dH}=-2\cos\alpha$

應剪力 (Tangential stress) ノ影響ハ頗ル小ナルヲ以テ之ヲ無視スルトキハ拱ノ內働 (Internal work) ノ總和ハ次ノ如シ



第四圖

$$\omega=2\int_0^{c'}\frac{M^2}{2EI}ds+2\left(\int_0^{\frac{v}{2}}\frac{M^2}{2EI}ds-\int_0^{c'}\frac{M^2}{2EI}ds\right)$$

$$+2\int_0^{c'}\frac{N^2}{2EA}ds+2\left(\int_0^{\frac{v}{2}}\frac{N^2}{2EA}ds-\int_0^{c'}\frac{N^2}{2EA}ds\right)$$

Castigliano ノ第二定理ニヨリ

$$\frac{d\omega}{dH}=2\int_0^{c'}\frac{M}{EI}\left(\frac{dM}{dH}\right)ds+2\left\{\int_0^{\frac{v}{2}}\frac{M}{EI}\left(\frac{dM}{dH}\right)ds-\int_0^{c'}\frac{M}{EI}\left(\frac{dM}{dH}\right)ds\right\}$$

$$+2\int_0^{c'}\frac{N}{EA}\left(\frac{dN}{dH}\right)ds+2\left\{\int_0^{\frac{v}{2}}\frac{N}{EA}\left(\frac{dN}{dH}\right)ds-\int_0^{c'}\frac{N}{EA}\left(\frac{dN}{dH}\right)ds\right\}=0$$

$$\therefore\quad -2\int_0^{c'}\frac{1}{I}\left\{W\left(\frac{l}{2}-c\right)-2H(y-h_1)\right\}(y-h_1)ds$$

$$+\left[-2\int_0^{\frac{v}{2}}\frac{1}{I}\left\{W\left(\frac{l}{2}-x\right)-2H(y-h_1)\right\}(y-h_1)ds\right.$$

$$\left.+2\int_0^{c'}\frac{1}{I}\left\{W\left(\frac{l}{2}-x\right)-2H(y-h_1)\right\}(y-h_1)\,ds\right]$$

$$-2\int_0^{c'}\frac{1}{A}(-2H\cos\alpha)\cos\alpha\,ds$$

$$+\left[-2\int_0^{\frac{l}{2}}\frac{1}{A}(-2H\cos\alpha-W\sin\alpha)\cos\alpha\,ds\right.$$
$$\left.+2\int_0^{c'}\frac{1}{A}(-2H\cos\alpha-W\sin\alpha)\cos\alpha\,ds\right]=0$$

$ds=\dfrac{dx}{\cos\alpha}=\sec\alpha\,dx$ ナルヲ以テ

$$-\int_0^{c'}\frac{1}{I}\left\{W\left(\frac{l}{2}-c\right)-2H(y-h_1)\right\}(y-h_1)\,ds$$
$$-\int_0^{\frac{l'}{2}}\frac{1}{I}\left\{W\left(\frac{l}{2}-x\right)-2H(y-h_1)\right\}(y-h_1)\,ds$$
$$+\int_0^{c'}\frac{1}{I}\left\{W\left(\frac{l}{2}-x\right)-2H(y-h_1)\right\}(y-h_1)\,ds$$
$$+2\int_0^{c}\frac{1}{A}H\cos\alpha\,dx+\int_0^{\frac{l}{2}}\frac{1}{A}(2H\cos\alpha+W\sin\alpha)\,dx$$
$$-\int_0^{c}\frac{1}{A}(2H\cos\alpha+W\sin\alpha)dx=0$$

$$\therefore\quad \frac{W}{I}\int_0^{\frac{l'}{2}}\left(\frac{l}{2}-x\right)(y-h_1)ds-\frac{2H}{I}\int_0^{\frac{l'}{2}}(y-h_1)^2\,ds$$
$$-\frac{W}{I}\int_0^{c'}(c-x)(y-h_1)\,ds-\frac{2H}{A}\int_0^{\frac{l}{2}}\cos\alpha\,dx$$
$$-\frac{W}{A}\int_0^{\frac{l}{2}}\sin\alpha\,dx+\frac{W}{A}\int_0^{c}\sin\alpha\,dx=0$$

上ノ方程式ヨリ

$$H=\frac{1}{2}\cdot\frac{\left\{\int_0^{\frac{v}{2}}\left(\frac{l}{2}-x\right)(y-h_1)\,ds-\int_0^{c'}(c-x)(y-h_1)\,ds-\frac{I}{A}\int_0^{\frac{l}{2}}\sin\alpha\,dx+\frac{I}{A}\int_0^{c}\sin\alpha\,dx\right\}}{\int_0^{\frac{v}{2}}(y-h_1)^2\,ds+\frac{I}{A}\int_0^{\frac{l}{2}}\cos\alpha\,dx}W \quad\cdots\cdots(17)$$

軸應力ノ影響ハ彎曲率ノ影響ニ比シ比較的小ナルヲ以テ之ヲ無視スルトキハ

$$H=\frac{1}{2}\cdot\frac{\int_0^{\frac{v}{2}}\left(\frac{l}{2}-x\right)(y-h_1)\,ds-\int_0^{c'}(c-x)(y-h_1)\,ds}{\int_0^{\frac{v}{2}}(y-h_1)^2\,ds}W \quad\cdots\cdots(18)$$

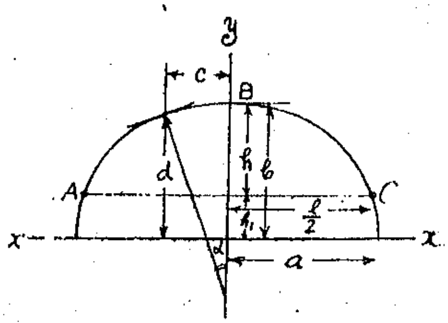
第三章　楕圓ノ方程式ト計算上必要ナル算式

第五圖ニ於テ　及　ヲ拱橋ノ拱矢及徑間トシ a 及 b ヲ楕圓ノ長軸及短軸トスレハ h 及 l ト共ニ a 或ハ b ヲ適當ニ假定スルトキハ容易ニ拱ノ形狀ヲ決定シ得ヘシ

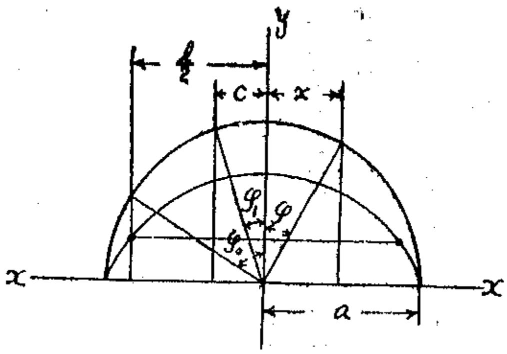
楕圓ノ方程式ハ正座標 (Rectangular coordinate) ヲ用フルトキハ

$$\left.\begin{array}{l}\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{or} \quad y=\frac{b}{a}\sqrt{a^2-x^2}\\ k=\sqrt{\frac{a^2-b^2}{a^2}}<1 \quad (k=\text{Eccentricity 偏心率})\text{トス}\end{array}\right\}\quad\cdots\cdots(19)$$

h 及 l ト共ニ a カ與ヘラレタル場合ニハ



第五圖



第六圖

$$\left.\begin{aligned} b&=\frac{ah}{a-\sqrt{a^2-\frac{l^2}{4}}}, \qquad h_1=b-h=\frac{\sqrt{a^2-\frac{l^2}{4}}}{a-\sqrt{a^2-\frac{l^2}{4}}} \\ d&=\frac{b}{a}\sqrt{a^2-c^2} \end{aligned}\right\}\cdots\cdots(20)$$

h 及 l ト共ニ b ガ與ヘラレタル場合ニハ

$$\left.\begin{aligned} a&=\frac{bl}{2\sqrt{2bh-h^2}}, \qquad h_1=b-h \\ d&=\frac{b}{a}\sqrt{a^2-c^2} \end{aligned}\right\}\cdots\cdots(21)$$

尙楕圓ノ方程式ハ極座標 (Polar coordinate) ヲ用フルトキハ

$$\left.\begin{aligned} &x=a\sin\varphi \\ &y=b\cos\varphi \end{aligned}\right.$$

φ＝偏心餘角 (Complement of eccentric angle)

$\varphi_1\varphi_0$ ハ $x=c$, $x=\frac{l}{2}$ ノ時ノ φ ノ値ヲ表ハスモノトス(第六圖參照) $\cdots\cdots(22)$

尙計算上必要ナル算式ヲ示セハ

$$\left.\begin{aligned} &\tan\alpha=\frac{b}{a}\cdot\frac{x}{\sqrt{a^2-x^2}}\left(\frac{dy}{dx}=-\frac{b}{a}\cdot\frac{x}{\sqrt{a^2-x^2}} \text{ ナルニヨリ}\right) \\ &\sin\alpha=\frac{bx}{a^2\sqrt{1-\frac{k^2}{a^2}x^2}} \end{aligned}\right\}$$

$$\left.\begin{aligned}\cos\alpha &= \frac{\sqrt{a^2-x^2}}{a\sqrt{1-\frac{k^2}{a^2}x^2}}\\ ds &= \frac{dx}{\cos\alpha} = \frac{a\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}\,dx\end{aligned}\right\}\quad\cdots\cdots(23)$$

第四章　集荷重ニ對スル水平反力

集荷重Wニ對スル水平反力Hヲ見出サンカタメ算式(17)ノ各項ニ橢圓ノ方程式(19)(22)(23)ヲ導入シ積分ヲ行ヘハ

$$(A)\quad \int_0^{\frac{l}{2}}\left(\frac{l}{2}-x\right)(y-h_1)\,ds$$

$$=\sqrt{1-\frac{k^2l^2}{4a^2}}\left\{\frac{bl^2}{24}+a^2\left(\frac{b}{3k^2}-\frac{h_1^2}{2b}\right)\right\}+\frac{al}{2}\left\{\frac{b}{2k}\sin^{-1}\frac{kl}{2a}-h_1E(k\,\varphi_0)\right\}$$

$$-a^2\left(\frac{b}{3k^2}-\frac{h_1}{2}\right)-\frac{b^2h_1}{4k}\left[\log\left\{\frac{b^2}{2k^2}+\frac{a^2h_1}{b}\left(\frac{h_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2l^2}{4a^2}}\right)\right\}-\log\frac{a^2(1+k)^2}{2k^2}\right]$$

解　$$(A)=a\int_0^{\frac{l}{2}}\left(\frac{l}{2}-x\right)\left(\frac{b}{a}\sqrt{a^2-x^2}-h_1\right)\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}\,dx$$

$$=\frac{bl}{2}\int_0^{\frac{l}{2}}\sqrt{1-\frac{k^2}{a^2}x^2}\,dx-b\int_0^{\frac{l}{2}}x\sqrt{1-\frac{k^2}{a^2}x^2}\,dx$$

$$-\frac{ah_1l}{2}\int_0^{\frac{l}{2}}\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}\,dx+ah_1\int_0^{\frac{l}{2}}x\,\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}\,dx$$

$$\int_0^{\frac{l}{2}} \sqrt{1-\frac{k^2}{a^2}x^2}\ dx = \frac{1}{2}\left(\frac{l}{2}\sqrt{1-\frac{k^2l^2}{4a^2}}+\frac{a}{k}\sin^{-1}\frac{kl}{2a}\right)$$

$$\int_0^{\frac{l}{2}} x\sqrt{1-\frac{k^2}{a^2}x^2}\ dx = -\frac{a^2}{3k^2}\left\{\left(1-\frac{k^2l^2}{4a^2}\right)^{\frac{3}{2}}-1\right\}$$

$$\int_0^{\frac{l}{2}} \frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx\ (x=a\sin\varphi \text{ ト置ケハ}) = \int_0^{\varphi_0}\sqrt{1-k^2\sin^2\varphi}\ d\varphi = E(k\,\varphi_0)$$

$\varphi_0=\sin^{-1}\frac{l}{2a}$ トス

$$\int_0^{\frac{l}{2}} x\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx = -\frac{a}{2}\left[\frac{h_1}{b}\sqrt{1-\frac{k^2l^2}{4a^2}}\right.$$
$$\left.-1+\frac{1-k^2}{2k}\left\{\log\left(\frac{b^2}{2k^2}+\frac{a^2h_1}{b}\left(\frac{h_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2l^2}{4a^2}}\right)\right)-\log\frac{a^2(1+k)^2}{2k^2}\right\}\right]$$

(B) $\int_0^{c'}(c-x)(y-h_1)\,ds$

$$=\sqrt{1-\frac{k^2c^2}{a^2}}\left\{\frac{bc^2}{6}+a^2\left(\frac{b}{3k^2}-\frac{h_1d}{2b}\right)\right\}+ac\left\{\frac{b}{2k}\sin^{-1}\frac{kc}{a}-h_1E(k\,\varphi_1)\right\}-a^2\left(\frac{b}{3k^2}-\frac{h_1}{2}\right)$$
$$-\frac{b^2h_1}{4k}\left[\log\left\{\frac{b^2}{2k^2}+\frac{a^2d}{b}\left(\frac{d}{b}+\frac{1}{k}\sqrt{1-\frac{k^2c^2}{a^2}}\right)\right\}-\log\frac{a^2(1+k)^2}{2k^2}\right]$$

解 $(B)=a\int_0^{c}(c-x)\left(\frac{b}{a}\sqrt{a^2-x^2}-h_1\right)\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx$

$$=bc\int_0^c\sqrt{1-\frac{k^2}{a^2}x^2}\,dx-b\int_0^c x\sqrt{1-\frac{k^2}{a^2}x^2}\,dx-ah_1c\int_0^c\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx+ah_1\int_0^c x\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx$$

$$\int_0^c\sqrt{1-\frac{k^2}{a^2}x^2}\,dx=\frac{1}{2}\left(c\sqrt{1-\frac{k^2c^2}{a^2}}+\frac{a}{k}\sin^{-1}\frac{kc}{a}\right)$$

$$\int_0^c x\sqrt{1-\frac{k^2}{a^2}x^2}\,dx=-\frac{a^2}{3k^2}\left\{\left(1-\frac{k^2c^2}{a^2}\right)^{\frac{3}{2}}-1\right\}$$

$$\int_0^c\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx\ (x=a\sin\varphi\ \text{ト置ケハ})=\int_0^{\varphi_1}\sqrt{1-k^2\sin^2\varphi}\,d\varphi$$

$$=E(k\,\varphi_1)\qquad \text{但}\ \varphi_1=\sin^{-1}\frac{c}{a}\ \text{トス}$$

$$\int_0^c x\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx=-\frac{a}{2}\left[\frac{d}{b}\sqrt{1-\frac{k^2c^2}{a^2}}\right.$$

$$\left.-1+\frac{1-k^2}{2k}\left\{\log\left(\frac{b^2}{2k^2}+\frac{a^2d}{b}\left(\frac{d}{b}+\frac{1}{k}\sqrt{1-\frac{k^2c^2}{a^2}}\right)\right)-\log\frac{a^2(1+k)^2}{2k^2}\right\}\right]$$

(C) $\displaystyle\int_0^{\frac{l}{2}}\sin\alpha\,dx=-\frac{b}{k^2}\left(\sqrt{1-\frac{k^2l^2}{4a^2}}-1\right)$

(D) $\displaystyle\int_0^{c}\sin\alpha\,dx=-\frac{b}{k^2}\left(\sqrt{1-\frac{k^2c^2}{a^2}}-1\right)$

$$(E)\quad \int_0^{\frac{l'}{2}} (y-h_1)^2\, ds = a\left[\frac{b^2}{3k^2}\left\{(1+k^2)\,E(k\,\varphi_0)-(1-k^2)\,F(k\,\varphi_0)\right\}\right.$$

$$\left.+h_1^2\,E(k\,\varphi_0)-\frac{bh_1}{a}\left(\frac{l}{3}\sqrt{1-\frac{k^2l^2}{4a^2}}+\frac{a}{k}\sin^{-1}\frac{kl}{2a}\right)\right]$$

解 $$(E)=a\int_0^{\frac{l}{2}}\left(\frac{b}{a}\sqrt{a^2-x^2}-h_1\right)^2\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx=a\,(b^2+h_1^2)\int_0^{\frac{l}{2}}\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx$$

$$-\frac{b^2}{a}\int_0^{\frac{l}{2}}x^2\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx-2bh_1\int_0^{\frac{l}{2}}\sqrt{1-\frac{k^2}{a^2}x^2}\,dx$$

$$\int_0^{\frac{l}{2}}\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx=E(k\,\varphi_0)$$

$$*\quad \int_0^{\frac{l}{2}}x^2\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx=\frac{a^2}{3k^2}\left\{(1-k^2)\,F(k\,\varphi_0)-(1-2k^2)\,E(k\,\varphi_0)\right\}-\frac{ah_1l}{6b}\sqrt{1-\frac{k^2l^2}{4a^2}}$$

＊ハ算式 (9) ヲ應用シテ計算ス

$$\int_0^{\frac{l}{2}}\sqrt{1-\frac{k^2}{a^2}x^2}\,dx=\frac{1}{2}\left\{\frac{l}{2}\sqrt{1-\frac{k^2l^2}{4a^2}}+\frac{a}{k}\sin^{-1}\frac{kl}{2a}\right\}$$

故ニ次ノ結果ヲ得ヘシ

$$(F)\quad \int_0^{\frac{l}{2}}\cos\alpha\,dx=\frac{a}{k^2}\left\{E(k\,\varphi_0)-(1-k^2)\,F(k\,\varphi_0)\right\}$$

$$H=\frac{1}{2}\cdot\frac{\left\{\begin{aligned}&\sqrt{1-\frac{k^2l^2}{4a^2}}\left\{\frac{bl^2}{24}+a^2\left(\frac{b}{3k^2}-\frac{h_1^2}{2b}\right)\right\}-\sqrt{1-\frac{k^2c^2}{a^2}}\left\{\frac{bc^2}{6}+a^2\left(\frac{b}{3k^2}-\frac{h_1d}{2b}\right)\right\}\\&+\frac{ab}{2k}\left(\frac{l}{2}\sin^{-1}\frac{kl}{2a}-c\sin^{-1}\frac{kc}{a}\right)-ah_1\left\{\frac{l}{2}E(k\varphi_0)-cE(k\varphi_1)\right\}\\&+\frac{b^2h_1}{4k}\log\frac{\frac{b^2}{2k^2}+\frac{a^2d}{b}\left(\frac{d}{b}+\frac{1}{k}\sqrt{1-\frac{k^2c^2}{a^2}}\right)}{\frac{b^2}{2k^2}+\frac{a^2h_1}{b}\left(\frac{h_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2l^2}{4a^2}}\right)}\\&-\frac{I}{A}\cdot\frac{b}{k^2}\left(\sqrt{1-\frac{k^2c^2}{a^2}}-\sqrt{1-\frac{k^2l^2}{4a^2}}\right)\end{aligned}\right\}}{\left\{\begin{aligned}&a\left[\frac{b^2}{3k^2}\left\{(1+k^2)E(k\varphi_0)-(1-k^2)F(k\varphi_0)\right\}+h_1^2E(k\varphi_0)\right.\\&\left.-\frac{bh_1}{a}\left(\frac{l}{3}\sqrt{1-\frac{k^2l^2}{4a^2}}+\frac{a}{k}\sin^{-1}\frac{kl}{2a}\right)\right]+\frac{I}{A}\cdot\frac{a}{k^2}\left\{E(k\varphi_0)-(1-k^2)F(k\varphi_0)\right\}\end{aligned}\right\}}W\quad\cdots\quad(24)$$

軸應力ノ影響ヲ無視スルトキハ

$$H=\frac{1}{2}\cdot\frac{\left\{\begin{aligned}&\sqrt{1-\frac{k^2l^2}{4a^2}}\left\{\frac{bl^2}{24}+a^2\left(\frac{b}{3k^2}-\frac{h_1^2}{2b}\right)\right\}-\sqrt{1-\frac{k^2c^2}{a^2}}\left\{\frac{bc^2}{6}+a^2\left(\frac{b}{3k^2}-\frac{h_1d}{2b}\right)\right\}\\&+\frac{ab}{2k}\left(\frac{l}{2}\sin^{-1}\frac{kl}{2a}-c\sin^{-1}\frac{kc}{a}\right)-ah_1\left\{\frac{l}{2}E(k\varphi_0)-cE(k\varphi_1)\right\}\\&+\frac{b^2h_1}{4k}\log\frac{\frac{b^2}{2k^2}+\frac{a^2d}{b}\left(\frac{d}{b}+\frac{1}{k}\sqrt{1-\frac{k^2c^2}{a^2}}\right)}{\frac{b^2}{2k^2}+\frac{a^2h_1}{b}\left(\frac{h_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2l^2}{4a^2}}\right)}\end{aligned}\right\}}{}W\quad\cdots\quad(25)$$

$$\left\{ a\left[\frac{b^2}{3k^2}\left\{(1+k^2)\,E(k\,\varphi_0)-(1-k^2)\,F(k\,\varphi_0)\right\} + h_1^{\,2}E(k\,\varphi_0)-\frac{bh_1}{a}\left(\frac{l}{3}\sqrt{1-\frac{k^2l^2}{4a^2}}+\frac{a}{k}\sin^{-1}\frac{kl}{2a}\right)\right] \right\}$$

例題 今 $h=45'$ $l=256'$ $b=175'$ ナル楕圓拱アリテ $c=32'$ ノ所ニWナル集荷重カ乘リシトキ算式(25)ヲ用ヒ水平反力Hヲ求メントス吾人ハ容易ニ次ノ値ヲ計算シ得ヘシ

$$h_1=b-h \qquad =130.'00$$

$$a=\frac{bl}{2\sqrt{b^2-h_1^2}}=191.'20$$

$$d=\frac{b}{a}\sqrt{a^2-c^2}=172.'52$$

$$k=\sqrt{\frac{a^2-b^2}{a^2}}=0.40284$$

$$\sin^{-1}\frac{kl}{2a} \qquad =0.27307$$

$$\sin^{-1}\frac{kc}{a} \qquad =0.06747$$

$$\varphi_1=\sin^{-1}\frac{c}{a}=0.16816$$

$$\varphi_0=\sin^{-1}\frac{l}{2a}=0.73348$$

$$E(k\,\varphi_0) \qquad =0.72379$$

$$E(k\,\varphi_1) \qquad =0.16803$$

$$F(k\,\varphi_0) \qquad =0.74317$$

尙(25)ノ各項ヲ計算スレハ

$$\sqrt{1-\frac{k^2l^2}{4a^2}}\left\{\frac{bl^2}{24}+a^2\left(\frac{b}{3k^2}-\frac{h_1^{\,2}}{2b}\right)\right\} \qquad =11,414,438.$$

$$\sqrt{1-\frac{k^2c^2}{a^2}}\left\{\frac{bc^2}{6}+a^2\left(\frac{b}{3k^2}-\frac{h_1d}{2b}\right)\right\} \qquad =10,749,230.$$

$$\frac{ab}{2k}\left\{\frac{l}{2}\sin^{-1}\frac{kl}{2a}-c\sin^{-1}\frac{kc}{a}\right\} = 1,185,925.$$

$$ah_1\left\{\frac{l}{2}E(k\varphi_0)-cE(k\varphi_1)\right\} = 2,169,130.$$

$$\frac{b^2h_1}{4k}\log\frac{\dfrac{b^2}{2k^2}+\dfrac{a^2d}{b}\left(\dfrac{d}{b}+\dfrac{1}{k}\sqrt{1-\dfrac{k^2c^2}{a^2}}\right)}{\dfrac{b^2}{2k^2}+\dfrac{a^2h_1}{b}\left(\dfrac{h_1}{b}+\dfrac{1}{k}\sqrt{1-\dfrac{kl^2}{4a^2}}\right)} = 494,399.$$

$$\frac{b^2}{3k^2}\left\{(1+k^2)E(k\varphi_0)-(1-k^2)F(k\varphi_0)\right\} = 13,756.$$

$$h_1^2E(k\varphi_0) = 12,232.$$

$$\frac{bh_1}{a}\left\{\frac{l}{3}\sqrt{1-\frac{k^2l^2}{4a^2}}+\frac{a}{k}\sin^{-1}\frac{kl}{2a}\right\} = 25,198.$$

故ニ次ノ結果ヲ得ヘシ

$$H=\frac{1}{2}\cdot\frac{11,414,438-10,749,230+1,185,925-2,169,130+494,399}{191.2(13,756+12,232-25,198)}W$$

$$=\frac{1}{2}\cdot\frac{176,402}{151,041}W=0.58395\,W.$$

第五章(其二)　等布荷重ニ對スル水平反力

等布荷重カ第七圖ノ如ク乘リシトキHヲwc_1(全荷重ノ半分)ノタメニ起レル水平反力トス拱ノ任意點ニ於ケル彎曲率ハ次ノ如シ

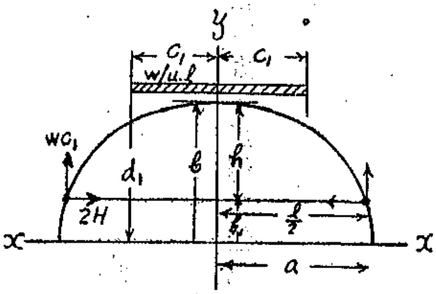
$$\text{for}\quad x<c_1\qquad M=wc_1\left(\frac{l}{2}-\frac{c_1}{2}\right)-\frac{w}{2}x^2-2H(y-h_1),\qquad \frac{dM}{dH}=-2(y-h_1)$$

for $x>c_1$　$M=wc_1\left(\frac{l}{2}-x\right)-2H(y-h_1)$,　$\frac{dM}{dH}=-2(y-h_1)$

又拱ノ任意點ニ於ケル軸應力ハ次ノ如シ

for $x<c_1$　$N=-2H\cos\alpha-wx\sin\alpha$,　$\frac{dN}{dH}=-2\cos\alpha$

for $x>c_1$　$N=-2H\cos\alpha-wc_1\sin\alpha$,　$\frac{dN}{dH}=-2\cos\alpha$



第七圖

拱ノ内働ノ總和ハ

$$\omega=2\int_0^{c'_1}\frac{M^2}{2EI}ds+2\left\{\int_0^{\frac{l'}{2}}\frac{M^2}{2EI}ds-\int_0^{c'_1}\frac{M^2}{2EI}ds\right\}$$

$$+2\int_0^{c'_1}\frac{N^2}{2EA}ds+2\left\{\int_0^{\frac{l'}{2}}\frac{N^2}{2EA}ds-\int_0^{c'_1}\frac{N^2}{2EA}ds\right\}$$

Castigliano ノ第二定理ニヨリ

$$\frac{d\omega}{dH}=2\int_0^{c'_1}\frac{M}{EI}\left(\frac{dM}{dH}\right)ds+2\left\{\int_0^{\frac{l'}{2}}\frac{M}{EI}\left(\frac{dM}{dH}\right)ds-\int_0^{c'_1}\frac{M}{EI}\left(\frac{dM}{dH}\right)ds\right\}$$

$$+2\int_0^{c'_1}\frac{N}{EA}\left(\frac{dN}{dH}\right)ds+2\left\{\int_0^{\frac{l'}{2}}\frac{N}{EA}\left(\frac{dN}{dH}\right)ds-\int_0^{c'_1}\frac{N}{EA}\left(\frac{dN}{dH}\right)ds\right\}=0$$

$$-2\int_0^{c'_1}\frac{1}{I}\left\{wc_1\left(\frac{l}{2}-\frac{c_1}{2}\right)-\frac{w}{2}x^2-2H(y-h_1)\right\}(y-h_1)ds$$

$$+\left[-2\int_0^{\frac{l'}{2}}\frac{1}{I}\left\{wc_1\left(\frac{l}{2}-x\right)-2H(y-h_1)\right\}(y-h_1)ds\right.$$

$$+2\int_0^{c'_1}\frac{1}{I}\left\{wc_1\left(\frac{l}{2}-x\right)-2H(y-h_1)\right\}(y-h_1)\,ds\Bigg]$$

$$-2\int_0^{c'_1}\frac{1}{A}(-2H\cos\alpha-wx\sin\alpha)\cos\alpha\,ds$$

$$+\left[-2\int_0^{\frac{l'}{2}}\frac{1}{A}(-2H\cos\alpha-wc_1\sin\alpha)\cos\alpha\,ds+2\int^{c'_1}\frac{1}{A}(-2H\cos\alpha-wc_1\sin\alpha)\cos\alpha\,ds\right]=0$$

上ノ方程式ヨリ次ノ結果ヲ得ヘシ

$$H=\frac{1}{2}\cdot\frac{\left\{\begin{array}{l}\int_0^{\frac{l'}{2}}\left(\frac{l}{2}-x\right)(y-h_1)\,ds-\frac{1}{2c_1}\int_0^{c'_1}(c_1-x)^2(y-h_1)ds\\ \quad-\frac{I}{A}\int_0^{\frac{l}{2}}\sin\alpha\,dx+\frac{I}{A}\cdot\frac{1}{c_1}\int_0^{c_1}(c_1-x)\sin\alpha\,dx\end{array}\right\}}{\int_0^{\frac{l'}{2}}(y-h_1)^2ds+\frac{I}{A}\int_0^{\frac{l}{2}}\cos\alpha\,dx}wc_1\qquad\cdots\cdots(26)$$

軸應力ノ影響ヲ無視スルトキハ

$$H=\frac{1}{2}\cdot\frac{\int_0^{\frac{l'}{2}}\left(\frac{l}{2}-x\right)(y-h_1)\,ds-\frac{1}{2c_1}\int_0^{c'_1}(c_1-x)^2(y-h_1)\,ds}{\int_0^{\frac{l'}{2}}(y-h_1)^2\,ds}wc_1\qquad\cdots\cdots(27)$$

水平反力Hヲ見出サンカタメ算式(26)ノ各項ニ楕圓ノ方程式(19)(22)(23)ヲ導入シ積分ヲ行ヘハ

$$(A)\quad\int_0^{\frac{l'}{2}}\left(\frac{l}{2}-x\right)(y-h_1)ds$$

$$=\sqrt{1-\frac{k^2l^2}{4a^2}}\left\{\frac{bl^2}{24}+a^2\left(\frac{b}{3k^2}-\frac{h_1^2}{2b}\right)\right\}+\frac{al}{2}\left\{\frac{b}{2k}\sin^{-1}\frac{kl}{2a}-h_1E(k\varphi_0)\right\}-a^2\left(\frac{b}{3k^2}-\frac{h_1}{2}\right)$$

$$-\frac{b^2h_1}{4k}\left[\log\left\{\frac{b^2}{2k^2}+\frac{a^2h_1}{b}\left(\frac{h_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2l^2}{4a^2}}\right)\right\}-\log\frac{a^2(1+k)^2}{2k^2}\right]$$

第四章(A)ノ値ニ等シ

(B) $\int_0^{c_1'}(c_1-x)^2(y-h_1)ds$

$$=\frac{ac_1}{3}\sqrt{1-\frac{k^2c_1^2}{a^2}}\left(\frac{bc_1^2}{4a}+\frac{13}{8}\cdot\frac{ab}{k^2}-\frac{2ad_1h_1}{b}\right)-a^2c_1\left(\frac{2b}{3k^2}-h_1\right)$$

$$+\frac{ab}{2k}\left(\frac{a^2}{4k^2}+c_1^2\right)\sin^{-1}\frac{kc_1}{a}+\frac{a^3h_1}{3k^2}\left\{\left(1-2k^2-\frac{3k^2c_1^2}{a^2}\right)E(k\varphi_1)-(1-k^2)F(k\varphi_1)\right\}$$

$$-\frac{a^2h_1c_1(1-k^2)}{2k}\left[\log\left\{\frac{b^2}{2k^2}+\frac{a^2d_1}{b}\left(\frac{d_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2c_1^2}{a^2}}\right)\right\}-\log\frac{a^2(1+k)^2}{2k^2}\right]$$

算式(9)(10)ヲ應用スレハ容易ニ右ノ結果ヲ得ヘシ

(C) $\int_0^{\frac{l}{2}}\sin\alpha\,dx=-\frac{b}{k^2}\left(\sqrt{1-\frac{k^2l^2}{4a^2}}-1\right)$

第四章(C)ノ値ニ等シ

(D) $\int_0^{c_1}(c_1-x)\sin\alpha\,dx=\frac{bc_1}{k^2}\left\{1-\frac{1}{2}\sqrt{1-\frac{k^2c_1^2}{a^2}}\right\}-\frac{ab}{2k^3}\sin^{-1}\frac{kc_1}{a}$

(E) $\int_0^{\frac{l'}{2}}(y-h_1)^2ds=a\left[\frac{b^2}{3k^2}\left\{(1+k^2)E(k\varphi_0)-(1-k^2)F(k\varphi_0)\right\}\right.$

$$+h_1^2\,E(k\,\varphi_0)-\frac{bh_1}{a}\left(\frac{l}{3}\sqrt{1-\frac{k^2l^2}{4a^2}}+\frac{a}{k}\sin^{-1}\frac{kl}{2a}\right)\Bigg]$$

第四章(E)ノ値ニ等シ

$$(F)\quad \int_0^{\frac{l}{2}}\cos\alpha\,dx=\frac{a}{k^2}\left\{E(k\,\varphi_0)-(1-k^2)\,F(k\,\varphi_0)\right\}$$

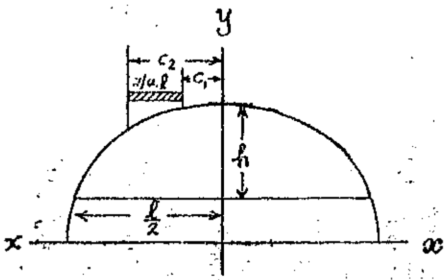
第四章(F)ノ値ニ等シ

以上ノ計算ニヨリ次ノ結果ヲ得ヘシ

$$H=\frac{1}{2}\cdot\frac{\left\{\begin{aligned}&\sqrt{1-\frac{k^2l^2}{4a^2}}\left\{\frac{bl^2}{24}+a^2\left(\frac{b}{3k^2}-\frac{h_1^2}{2b}\right)\right\}\\&-\frac{a^2}{6}\sqrt{1-\frac{k^2c_1^2}{a^2}}\left(\frac{bc_1^2}{4a^2}+\frac{13}{8}\cdot\frac{b}{k^2}-\frac{2h_1d_1}{b}\right)\\&+\frac{ab}{4k}\left\{l\sin^{-1}\frac{kl}{2a}-\frac{1}{c_1}\left(\frac{a^2}{4k^2}+c_1^2\right)\sin^{-1}\frac{kc_1}{a}\right\}\\&+\frac{a^3h_1}{6k^2c_1}\left\{(1-k^2)\,F(k\,\varphi_1)-\left(1-2k^2-\frac{3k^2c_1^2}{a^2}\right)E(k\,\varphi_1)\right\}-\frac{ah_1l}{2}E(k\,\varphi_0)\\&+\frac{b^2h_1}{4k}\log\frac{\frac{b^2}{2k^2}+\frac{a^2d_1}{b}\left(\frac{d_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2c_1^2}{a^2}}\right)}{\frac{b^2}{2k^2}+\frac{a^2h_1}{b}\left(\frac{h_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2l^2}{4a^2}}\right)}\\&+\frac{I}{A}\cdot\frac{b}{k^2}\left(\sqrt{1-\frac{k^2l^2}{4a^2}}-\frac{1}{2}\sqrt{1-\frac{k^2c_1^2}{a^2}}\right)-\frac{I}{A}\cdot\frac{ab}{2k^3c_1}\sin^{-1}\frac{kc_1}{a}\end{aligned}\right\}}{\text{Denominator Eq. (24)}}\,wc_1\quad\cdots\quad(28)$$

軸應力ノ影響ヲ無視スルトキハ

$$H=\frac{1}{2}\cdot\frac{\left\{\begin{aligned}&\sqrt{1-\frac{k^2l^2}{4a^2}}\left\{\frac{bl^2}{24}+a^2\left(\frac{b}{3k^2}-\frac{h_1^2}{2b}\right)\right\}\\&-\frac{a^2}{6}\sqrt{1-\frac{k^2c_1^2}{a^2}}\left\{\frac{bc_1^2}{4a^2}+\frac{13}{8}\cdot\frac{b}{k^2}-\frac{2h_1d_1}{b}\right\}\\&+\frac{ab}{4k}\left\{l\sin^{-1}\frac{kl}{2a}-\frac{1}{c_1}\left(\frac{a^2}{4k^2}+c_1^2\right)\sin^{-1}\frac{kc_1}{a}\right\}\\&+\frac{a^3h_1}{6k^2c_1}\left\{(1-k^2)F(k\varphi_1)-\left(1-2k^2-\frac{3k^2c_1^2}{a^2}\right)E(k\varphi_1)\right\}-\frac{ah_1l}{2}E(k\varphi_0)\\&+\frac{b^2h_1}{4k}\log\frac{\frac{b^2}{2k^2}+\frac{a^2d_1}{b}\left(\frac{d_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2c_1^2}{a^2}}\right)}{\frac{b^2}{2k^2}+\frac{a^2h_1}{b}\left(\frac{h_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2l^2}{4a^2}}\right)}\end{aligned}\right\}}{\text{Denominator Eq. (25)}}\,wc_1\quad\cdots\quad(29)$$



第八圖

等布荷重カ第八圖ノ如ク乘リタル場合ニハ先 c_1 マテ全部等布荷重カ乘レルモノト假定シテ其レニ對スル水平反力ヲ求メ次ニ c_1 マテ等布荷重カ乘リシトキノ水平反力ヲ求メ前者ヨリ後者ヲ減スレハ之即求ムル所ノ水平反力ナリ

第五章(其二) 等布荷重ニ對スル水平反力

又等布荷重 wc_1 ニ對スル水平反力 H ハ算式(24)ニ於テ $W=w\,dx$ $c=x$ $d=\frac{b}{a}\sqrt{a^2-x^2}$ ニ置キ代ヘ各項ヲ0ヨリ c_1 マテ積分スレハ算式(28)ト同一ノ結果ヲ得ヘシ但シ $d_1=\frac{b}{a}\sqrt{a^2-c_1^2}$ トス

$$(A)\quad \int_0^{c_1}\sqrt{1-\frac{k^2l^2}{4a^2}}\left\{\frac{bl^2}{24}+a^2\left(\frac{b}{3k^2}-\frac{h_1^2}{2b}\right)\right\}w\,dx=\sqrt{1-\frac{k^2l^2}{4a^2}}\left\{\frac{bl^2}{24}+a^2\left(\frac{b}{3k^2}-\frac{h_1^2}{2b}\right)\right\}wc_1$$

$$(B)\quad \int_0^{c_1}\sqrt{1-\frac{k^2x^2}{a^2}}\left\{\frac{bx^2}{6}+a^2\left(\frac{b}{3k^2}-\frac{h_1}{2a}\sqrt{a^2-x^2}\right)\right\}w\,dx$$

$$=\frac{wc_1}{6}\sqrt{1-\frac{k^2c_1^2}{a^2}}\left\{\frac{bc_1^2}{4}+\frac{7a^2b}{8k^2}-\frac{a^2h_1d_1}{b}\right\}+\frac{3}{16}\cdot\frac{a^3b}{k^3}w\sin^{-1}\frac{kc_1}{a}$$

$$-\frac{a^3h_1}{2}w\left\{\frac{1}{3k^2}(1+k^2)E(k\,\varphi_1)-\frac{1}{3k^2}(1-k^2)F(k\,\varphi_1)\right\}$$

$$(C)\quad \int_0^{c_1}\frac{ab}{2k}\left(\frac{l}{2}\sin^{-1}\frac{kl}{2a}-x\sin^{-1}\frac{kx}{a}\right)w\,dx$$

$$=\frac{ab}{2k}w\left(\frac{lc_1}{2}\sin^{-1}\frac{kl}{2a}-\frac{ac_1}{4k}\sqrt{1-\frac{k^2c_1^2}{a^2}}+\frac{a^2-2k^2c_1^2}{4k^2}\sin^{-1}\frac{kc_1}{a}\right)$$

$$(D)\quad \int_0^{c_1}ah_1\left\{x\,E(k\,\varphi_1)-\frac{l}{2}E(k\,\varphi_0)\right\}w\,dx$$

$$=ah_1w\left[\frac{a^2}{6k^2}\left\{\left(1-2k^2+\frac{3k^2c_1^2}{a^2}\right)E(k\,\varphi_1)-(1-k^2)F(k\,\varphi_1)\right\}+\frac{ac_1d_1}{6b}\sqrt{1-\frac{k^2c_1^2}{a^2}}-\frac{lc_1}{2}E(k\,\varphi_0)\right]$$

$E(k\,\varphi_1)=\int_0^x\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx$ ナルヲ以テ

解　$(D)=\int_0^{c_1}ah_1\left\{x\int_0^x\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx-\frac{l}{2}E(k\,\varphi_0)\right\}w\,dx$

$$=ah_1w\left\{\int_0^{c_1}x\int_0^x\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx\,dx-\frac{l}{2}E(k\,\varphi_0)\int_0^{c_1}dx\right\}$$

今次ノ如キ積分ヲ考フ

$$\int\frac{x^2}{2}\cdot\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx=\int\frac{x^2}{2}\cdot\frac{d}{dx}\left(\int\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx\right)dx$$

$$=\frac{x^2}{2}\int\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx-\int x\int\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx\,dx$$

上式ヲ書キ直セハ

$$\int_0^{c_1}x\int_0^x\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx\,dx=\left[\frac{x^2}{2}\int\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx\right]_0^{c_1}-\int_0^{c_1}\frac{x^2}{2}\frac{\sqrt{1-\frac{k^2}{a^2}x^2}}{\sqrt{a^2-x^2}}dx$$

$$=\frac{a^2}{6k^2}\left\{\left(1-2k^2+\frac{3k^2c_1^2}{a^2}\right)E(k\,\varphi_1)-(1-k^2)F(k\,\varphi_1)\right\}+\frac{ac_1d_1}{6b}\sqrt{1-\frac{k^2c_1^2}{a^2}}$$

$$(E)\quad\int_0^{c_1}\frac{b^2h_1}{4k}\log\frac{\frac{b^2}{2k^2}+a\sqrt{a^2-x^2}\left(\frac{1}{a}\sqrt{a^2-x^2}+\frac{1}{k}\sqrt{1-\frac{k^2}{a^2}x^2}\right)}{\frac{b^2}{2k^2}+\frac{a^2h_1}{b}\left(\frac{h_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2l^2}{4a^2}}\right)}w\,dx$$

$$=\frac{b^2h_1}{4k}wc_1\log\frac{\frac{b^2}{2k^2}+\frac{a^2d_1}{b}\left(\frac{d_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2c_1^2}{a^2}}\right)}{\frac{b^2}{2k^2}+\frac{a^2h_1}{b}\left(\frac{h_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2l^2}{4a^2}}\right)}+\frac{ab^2h_1}{2k^2}w\left\{F(k\,\varphi_1)-E(k\,\varphi_1)\right\}$$

(F)ノ積分ニ於テ對數ノ分母ハ定數ナルコトニ注意シ算式(7)ヲ應用スレハ上ノ如キ結果ヲ得ヘシ

$$(F)\quad \int_0^{c_1}\frac{I}{A}\cdot\frac{b}{k^2}\left\{\sqrt{1-\frac{k^2x^2}{a^2}}-\sqrt{1-\frac{k^2l^2}{4a^2}}\right\}w\,dx$$

$$=\frac{I}{A}\cdot\frac{b}{k^2}\,wc_1\left(\frac{1}{2}\sqrt{1-\frac{k^2c_1^2}{a^2}}-\sqrt{1-\frac{k^2l^2}{4a^2}}\right)+\frac{I}{A}\cdot\frac{ab}{2k^3}w\sin^{-1}\frac{kc_1}{a}$$

故ニ次ノ如キ結果ヲ得ヘシ

$$H=\frac{1}{2}\cdot\frac{\left\{\begin{aligned}&\sqrt{1-\frac{k^2l^2}{4a^2}}\left\{\frac{bl^2}{24}+a^2\left(\frac{b}{3k^2}-\frac{h_1^2}{2b}\right)\right\}\\&-\frac{a^2}{6}\sqrt{1-\frac{k^2c_1^2}{a^2}}\left(\frac{bc_1^2}{4a^2}+\frac{13}{8}\cdot\frac{b}{k^2}-\frac{2h_1d_1}{b}\right)\\&+\frac{ab}{4k}\left\{l\sin^{-1}\frac{kl}{2a}-\frac{1}{c_1}\left(\frac{a^2}{4k^2}+c_1^2\right)\sin^{-1}\frac{kc_1}{a}\right\}\\&+\frac{a^3h_1}{6k^2c_1}\left\{(1-k^2)F(k\,\varphi_1)-\left(-2k^2-\frac{3k^2c_1^2}{a^2}\right)E(k\,\varphi_1)\right\}-\frac{ah_1l}{2}E(k\,\varphi_0)\\&+\frac{b^2h_1}{4k}\log\frac{\frac{b^2}{2k^2}+\frac{a^2d_1}{b}\left(\frac{d_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2c_1^2}{a^2}}\right)}{\frac{b^2}{2k^2}+\frac{a^2h_1}{b}\left(\frac{h_1}{b}+\frac{1}{k}\sqrt{1-\frac{k^2l^2}{4a^2}}\right)}\\&+\frac{I}{A}\cdot\frac{b}{k^2}\left\{\sqrt{1-\frac{k^2l^2}{4a^2}}-\frac{1}{2}\sqrt{1-\frac{k^2c_1^2}{a^2}}\right\}-\frac{I}{A}\cdot\frac{ab}{2k^3c_1}\sin^{-1}\frac{kc_1}{a}\end{aligned}\right.}{\text{Denominator Eq. (24)}}wc_1\quad\cdots\quad(30)$$

算式(30)ハ算式(28)ニ等シク尙算式(30)ニ於テ軸應力ノ影響ヲ無視スルトキハ算式(29)ト同一ノ結果ヲ

得ヘシ

第六章　半楕圓拱

半楕圓拱 (Semi-elliptic arch) ノ場合ニハ算式(24)(28)等ニ於テ $a=\frac{l}{2}$, $b=h$, $h_1=0$; $\varphi_0=\frac{\pi}{2}$ ト置ケハ宜シ

集荷重 W ニ對スル水平反力 H ハ算式(24)ニヨリ次ノ如シ

$$H=\frac{\left\{\begin{aligned}&\frac{hl}{6}\left(\frac{1}{2}+\frac{1}{k^2}\right)-\frac{1}{6}\sqrt{1-\frac{4k^2c^2}{l^2}}\left(c^2+\frac{l^2}{2k^2}\right)\\&+\frac{l}{4k}\left(\frac{l}{2}\sin^{-1}k-c\sin^{-1}\frac{2kc}{l}\right)-\frac{I}{A}\cdot\frac{1}{k^2}\left(\sqrt{1-\frac{4k^2c^2}{l^2}}-\frac{2h}{l}\right)\end{aligned}\right.}{\frac{h^2l}{3k^2}\left\{\left(1+k^2\right)E\left(k\frac{\pi}{2}\right)-\left(1-k^2\right)F\left(k\frac{\pi}{2}\right)\right\}+\frac{I}{A}\cdot\frac{l}{k^2}\left\{E\left(k\frac{\pi}{2}\right)-\left(1-k^2\right)F\left(k\frac{\pi}{2}\right)\right\}}hW \quad \cdots\cdots(31)$$

軸應力ノ影響ヲ無視スルトキハ

$$H=\frac{\frac{hl}{6}\left(\frac{1}{2}+\frac{1}{k^2}\right)-\frac{1}{6}\sqrt{1-\frac{4k^2c^2}{l^2}}\left(c^2+\frac{l^2}{2k^2}\right)+\frac{l}{4k}\left(\frac{l}{2}\sin^{-1}k-c\sin^{-1}\frac{2kc}{l}\right)}{\frac{h^2l}{3k^2}\left\{\left(1+k^2\right)E\left(k\frac{\pi}{2}\right)-\left(1-k^2\right)F\left(k\frac{\pi}{2}\right)\right\}}hW \quad \cdots\cdots(32)$$

等布荷重 w_{c_1} ニ對スル水平反力 H ハ算式(28)ニヨリ次ノ如シ

$$\left\{\begin{aligned}&\frac{hl}{6}\left(\frac{1}{2}+\frac{1}{k^2}\right)-\frac{1}{24}\sqrt{1-\frac{4k^2c_1^2}{l^2}}\left(c_1^2+\frac{13}{8}\cdot\frac{l^2}{k^2}\right)\\&+\frac{l}{8k}\left\{l\sin^{-1}k-\frac{1}{c_1}\left(\frac{l^2}{16k^2}+c_1^2\right)\sin^{-1}\frac{2kc_1}{l}\right\}\end{aligned}\right.$$

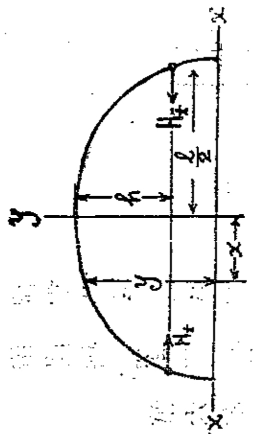
$$H=\frac{\left(+\frac{I}{A}\cdot\frac{1}{k^2}\left\{\frac{2h}{l}-\frac{1}{2}\sqrt{1-\frac{4k^2c_1^2}{l^2}}\right\}-\frac{I}{A}\cdot\frac{l}{4k^3c_1}\sin^{-1}\cdot\frac{2kc_1}{l}\right.}{\text{Denominator Eq. (31)}}hwc_1 \quad \cdots\cdots\cdots(33)$$

軸應力ヲ無視スルトキハ

$$H=\frac{\left\{\begin{array}{l}\frac{hl}{6}\left(\frac{1}{2}+\frac{1}{k^2}\right)-\frac{1}{24}\sqrt{1-\frac{4k^2c_1^2}{l^2}}\left(c_1^2+\frac{13}{8}\cdot\frac{l^2}{k^2}\right)\\ +\frac{l}{8k}\left\{l\sin^{-1}k-\frac{1}{c_1}\left(\frac{l^2}{16k^2}+c_1^2\right)\sin^{-1}\frac{2kc_1}{l}\right\}\end{array}\right.}{\text{Denominator Eq. (32)}}hwc_1 \quad \cdots\cdots\cdots(34)$$

第七章　熱應力 (Temperature stress)

第一　一般ノ楕圓拱



第九圖

雙鉸拱ハ溫度ノ昇降ニ依ッテ應力ヲ受クルモノニシテ次ニ之カ計算法ヲ示サントス

θ＝伸縮率 (Coefficient of expansion and contraction)

t＝度數ニ於テ表ハサレタル溫度ノ變化

H_t＝溫度ノ變化ノタメニ起レル水平反力

但溫度昇リシトキヲ正トシ溫度降リシトキヲ負トス

Bending moment　$M=-H_t(y-h_1)$,　$\frac{dM}{dH_t}=-(y-h_1)$

Axial stress　$N=-H_t\cos\alpha$,　$\frac{dN}{dH_t}=-\cos\alpha$

應剪力ノ影響ヲ無視スルトキハ拱ノ內働ノ總和ハ次ノ如シ

$$\omega=2\int_0^{\frac{l}{2}}\frac{M^2}{2EI}ds+2\int_0^{\frac{l}{2}}\frac{N^2}{2EA}ds$$

若シ拱橋ノ支承カ自由ニ移動シ得ルモノト假定セハ溫度カt度丈ケ昇降スルニ從ツテ徑間長ニモ$\pm t\theta l$丈ケノ變化ヲ生スヘキ理ナリ然シ拱橋ノ支承カ溫度ノ變化ニ係ラス何等移動セサルモノナルトキハ Castigliano ノ第一定理ノ應用ニヨリ次ノ關係式ヲ得ヘシ

$$\pm t\theta l=\frac{d\omega}{dH_t}=2\int_0^{\frac{l}{2}}\frac{M}{EI}\left(\frac{dM}{dH_t}\right)ds+2\int_0^{\frac{l}{2}}\frac{N}{EA}\left(\frac{dN}{dH_t}\right)ds$$

$$=2H_t\int_0^{\frac{l}{2}}\frac{1}{EI}(y-h_1)^2ds+2H_t\int_0^{\frac{l}{2}}\frac{1}{EA}\cos\alpha\,dx$$

$$\therefore\quad H_t=\pm\frac{1}{2}\cdot\frac{t\theta l\,EI}{\int_0^{\frac{l}{2}}(y-h_1)^2ds+\frac{I}{A}\int_0^{\frac{l}{2}}\cos\alpha\,dx}\qquad\cdots(35)$$

$$H_t=\pm\frac{1}{2}\cdot\frac{t\theta l\,EI}{\text{Denominator Eq. (24)}}\qquad\cdots(36)$$

軸應力ヲ無視スルトキハ

$$H_t=\pm\frac{1}{2}\cdot\frac{t\theta l\,EI}{\text{Denominator Eq. (25)}}\qquad\cdots(37)$$

溫度昇リシトキハ正號ヲ用ヒ溫度降リシトキハ負號ヲ用フ

第二　半楕圓拱

半楕圓拱ノ場合ニハ算式(36)(37)ニ於テ$a=\frac{l}{2}, b=h, h_1=0, \varphi_0=\frac{\pi}{2}$ニ置ケハ宜シ

$$H_t=\pm\frac{t\theta l\,EI}{\text{Denominator Eq. (31)}}\quad\cdots\cdots(38)$$

軸應力ヲ無視スルトキハ

$$H_t=\pm\frac{t\theta l\,EI}{\text{Denominator Eq. (32)}}\quad\cdots\cdots(39)$$

第八章　支承ノ移動ニ歸因スル應力 (Stress due to displacement of supports)

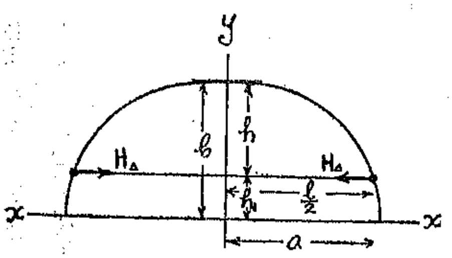
第一　一般ノ楕圓拱

若シ或原因ノタメ基礎地盤ニ弛ミヲ生シ支承ニ移動ヲ起ストキハ從ッテ徑間長ニモ變化ヲ生シ拱ハ應力ヲ受クルニ至ルモノニシテ恰モ温度ノ變化ヲ受クルト同様ノ影響ヲ蒙ルモノナリ

Δl＝支承移動ノタメニ生セル拱ノ徑間長ノ變化

但徑間長ノ減セシトキヲ正トシ徑間長ノ増セシトキヲ負トス

H_Δ＝徑間長ノ變化ノタメニ起レル水平反力



第十圖

Bending moment　$M=-H_\Delta(y-h_1)$,　$\dfrac{dM}{dH_\Delta}=-(y-h_1)$

Axial stress　$N=-H_\Delta\cos\alpha$,　$\dfrac{dN}{dH_\Delta}=-\cos\alpha$

應剪力ノ影響ヲ無視スルトキハ

$$\omega=2\int_0^{\frac{l}{2}}\frac{M^2}{2EI}ds+2\int_0^{\frac{l}{2}}\frac{N^2}{2EA}ds$$

Castigliano ノ第一定理ニヨリ

$$\pm\Delta l=\frac{d\omega}{dH_\Delta}=2\int_0^{\frac{l}{2}}\frac{M}{EI}\left(\frac{dM}{dH_\Delta}\right)ds+2\int_0^{\frac{l}{2}}\frac{N}{EA}\left(\frac{dN}{dH_\Delta}\right)ds$$

$$=2H_\Delta\int_0^{\frac{l}{2}}\frac{1}{EI}(y-h_1)^2ds+2H_\Delta\int_0^{\frac{l}{2}}\frac{1}{EA}\cos\alpha\,dx$$

$$\therefore\quad H_\Delta=\pm\frac{1}{2}\cdot\frac{\Delta l\,EI}{\int_0^{\frac{l}{2}}(y-h_1)^2ds+\frac{I}{A}\int_0^{\frac{l}{2}}\cos\alpha\,dx}\quad\cdots\cdots(40)$$

$$H_\Delta=\pm\frac{1}{2}\cdot\frac{\Delta l\,EI}{\text{Denominator Eq. (24)}}\quad\cdots\cdots(41)$$

軸應力ヲ無視スルトキハ

$$H_\Delta=\pm\frac{1}{2}\cdot\frac{\Delta l\,EI}{\text{Denominator Eq. (25)}}\quad\cdots\cdots(42)$$

正號ハ徑間長減セシトキ適用シ負號ハ之ニ反スル時適用ス

第二　半楕圓拱

此場合ニハ

$$H_\Delta=\pm\frac{\Delta l\,EI}{\text{Denominator Eq. (31)}}\quad\cdots\cdots(43)$$

軸應力ヲ無視スルトキハ

$$H_\Delta=\pm\frac{\Delta l\,EI}{\text{Denominator Eq. (32)}}\quad\cdots\cdots(44)$$

雙鉸拱ハ支承ノ高サニ移動ヲ生スルモ其影響頗小ナルヲ以テ普通考フルニ及ハス

備考、本論文中ノ諸式ニ於ケル對數ハ總テ Naperian logarithm ヲ示スモノニシテ之ヲ求ムルニハ Common logarithm ニテ計算セル結果ヲ 0.434294448…… ニテ除セハ宜シ

尚本論文中ノ諸式ノ計算ニハ Chambers' Mathematical Tables ヲ用フルヲ便トス

第九章　附錄　楕圓積分表

既ニ述ヘタル如ク楕圓積分表ニハ未タ完全ナルモノナケレトモ次ニ Kiepert—Integral Rechnung 及ヒ Lugien Lévy—Précis Élémentaire de la Théorie des Fonctions Elliptiques 所載ノモノヲ掲ケ參考ノ資ニ供セントス

但半楕圓拱ノ表ニ就テハ Kiepert 氏著書ニ依リ一般ノ楕圓拱ノ表ニ就テハ Lévy 氏ノ著書ニ依レルモノトス（本表ハ編輯上ノ都合ニヨリテ省略ス）（完）