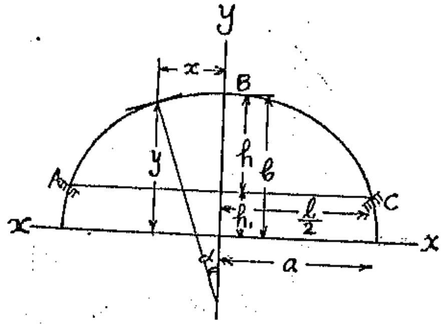
論說報告　土木學會誌　第三卷第五號　大正六年十月

變斷面積ヲ有スル無鉸楕圓拱

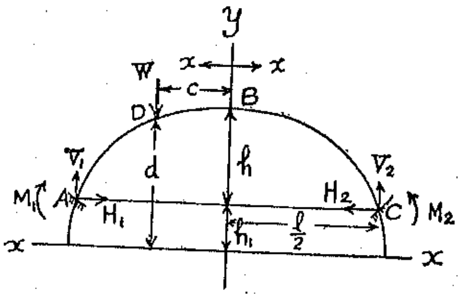
工學士　野口寅之助

第一章　總論

第一圖及第二圖ニ於テ a 及 b ヲ楕圓ノ長軸及短軸トシ h 及 l ヲ變斷面積ヲ有スル無鉸楕圓拱 (No-hinged elliptic arch with varying cross section) ノ拱矢及徑間トス



第一圖



第二圖

H_1, H_2＝支承 A 及 C ニ於ケル水平反力

V_1, V_2＝支承 A 及 C ニ於ケル垂直反力

M_1, M_2＝支承 A 及 C ニ於ケル力率

M＝拱ノ任意點ニ於ケル彎曲率

　但拱ノ上部纖維ニ壓力ヲ與フル彎曲率ヲ正トシ張力ヲ與フルモノヲ負トス

N＝拱ノ任意點ニ於ケル軸應力

　但拱ニ張力ヲ與フル軸應力ヲ正トシ壓力ヲ與フルモノヲ負トス

c'＝拱軸ニ沿フテ B ヨリ D ニ至ル距離

$\frac{l}{2}$＝拱軸ニ沿フテBヨリAニ至ル距離

α＝曲線上ノ任意點ニ於ケル法線カy軸ト爲ス角

E＝彈率 (Modulus of elasticity)

I＝拱ノ任意點ニ於ケル惰率

A＝拱ノ任意點ニ於ケル斷面積

初等力學ニヨリ容易ニ次ノ諸式ヲ得ヘシ

$$\left.\begin{aligned} V_2 &= W - V_1 \\ H_2 &= H_1 \\ M_2 &= M_1 + V_1 l - W\left(\frac{l}{2}+c\right) \end{aligned}\right\} \quad \cdots\cdots\cdots \quad (1)$$

拱ノy軸ヨリ左ノ部分ニ於ケル彎曲率ハ次ノ如シ

$$M = M_1 + V_1\left(\frac{l}{2}-x\right) - H_1(y-h_1) - W(c-x) \qquad \text{for } x<c$$

$$M = M_1 + V_1\left(\frac{l}{2}-x\right) - H_1(y-h_1) \qquad \text{for } x>c$$

又拱ノy軸ヨリ右ノ部分ニ於ケル彎曲率ハ次ノ如シ

$$M = M_2 + V_2\left(\frac{l}{2}-x\right) - H_2(y-h_1) = M_1 + V_1\left(\frac{l}{2}+x\right) - H_1(y-h_1) - W(c+x)$$

拱ノy軸ヨリ左ノ部分ニ於ケル軸應力ハ次ノ如シ

$$N = -H_1\cos\alpha + (W - V_1)\sin\alpha \qquad \text{for } x<c$$

$$N = -H_1 \cos\alpha - V_1 \sin\alpha \qquad \text{for } x > c$$

又拱ノ y 軸ヨリ右ノ部分ニ於ケル軸應力ハ次ノ如シ

$$N = -H_2 \cos\alpha - V_2 \sin\alpha$$
$$= -H_1 \cos\alpha - (W - V_1) \sin\alpha$$

應剪力ヲ無視スルトキ拱ノ内働ノ總和ハ次ノ如シ

$$\omega = \int_0^{c'} \frac{1}{2EI}\left\{M_1 + V_1\left(\frac{l}{2} - x\right) - H_1(y - h_1) - W(c - x)\right\}^2 ds + \int_{c'}^{\frac{l'}{2}} \frac{1}{2EI}\left\{M_1 + V_1\left(\frac{l}{2} - x\right) - H_1(y - h_1)\right\}^2 ds$$
$$+ \int_0^{\frac{l'}{2}} \frac{1}{2EI}\left\{M_1 + V_1\left(\frac{l}{2} + x\right) - H_1(y - h_1) - W(c + x)\right\}^2 ds + \int_0^{c'} \frac{1}{2EA}\left\{-H_1 \cos\alpha + (W - V_1)\sin\alpha\right\}^2 ds$$
$$+ \int_{c'}^{\frac{l'}{2}} \frac{1}{2EA}\left(H_1 \cos\alpha + V_1 \sin\alpha\right)^2 ds + \int_0^{\frac{l'}{2}} \frac{1}{2EA}\left\{H_1 \cos\alpha + (W - V_1)\sin\alpha\right\}^2 ds$$

Castigliano ノ第二定理ニヨリ

$$\frac{\partial\omega}{\partial M_1} = 0, \qquad \frac{\partial\omega}{\partial V_1} = 0, \qquad \frac{\partial\omega}{\partial H_1} = 0$$

上ノ方程式ヨリ次ノ結果ヲ得ヘシ

$$2M_1 \int_0^{\frac{l'}{2}} \frac{1}{I} ds + V_1 l \int_0^{\frac{l'}{2}} \frac{1}{I} ds - 2H_1 \int_0^{\frac{l'}{2}} \frac{1}{I}(y - h_1) ds - Wc \int_{-c'}^{+\frac{l'}{2}} \frac{1}{I} ds - W \int_{c'}^{\frac{l'}{2}} \frac{1}{I} x\, ds = 0 \quad \cdots\ \cdots\ \cdots\ \cdots \quad (2)$$

$$M_1 l \int_0^{\frac{l'}{2}} \frac{1}{I} ds + 2V_1 \left\{\int_0^{\frac{l'}{2}} \frac{1}{I}\left(\frac{l^2}{4} + x^2\right) ds + \int_0^{\frac{l'}{2}} \frac{1}{A} \sin^2\alpha\, ds\right\} - H_1 l \int_0^{\frac{l'}{2}} \frac{1}{I}(y - h_1) ds$$
$$- W\left(\frac{l}{2} + c\right) \int_{c'}^{\frac{l'}{2}} \frac{1}{I} x\, ds - W \int_{-c'}^{+\frac{l'}{2}} \frac{1}{I}\left(\frac{cl}{2} + x^2\right) ds - W \int_{-c'}^{+\frac{l'}{2}} \frac{1}{A} \sin^2\alpha\, ds = 0 \quad \cdots\ \cdots\ \cdots \quad (3)$$

1265

$$2M_1\int_0^{\frac{l}{2}}\frac{1}{I}(y-h_1)ds+V_1 l\int_0^{\frac{l}{2}}\frac{1}{I}(y-h_1)ds-2H_1\left\{\int_0^{\frac{l}{2}}\frac{1}{I}(y-h_1)^2ds+\int_0^{\frac{l}{2}}\frac{1}{A}\cos^2\alpha\,ds\right\}$$

$$-W\int_0^{c'}\frac{1}{I}(c-x)(y-h_1)ds-W\int_0^{\frac{l}{2}}\frac{1}{I}(c+x)(y-h_1)ds-W\int_{c'}^{\frac{l}{2}}\frac{1}{A}\sin\alpha\cos\alpha\,ds=0 \quad \cdots\cdots(4)$$

第二章　集荷重ニ對スル H_1 V_1 M_1 ノ決定

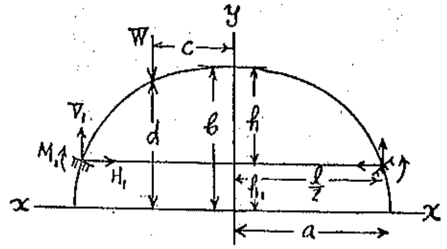
第三圖ニ於テ h 及 l ヲ拱橋ノ拱矢及徑間トシ a 及 b ヲ楕圓ノ長軸及短軸トスレハ h 及 l ト共ニ a 或ハ b ヲ適當ニ假定スルトキハ容易ニ拱ノ形狀ヲ決定シ得ヘシ

楕圓ノ方程式ハ

$$\left.\begin{aligned}&\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{or} \quad y=\frac{b}{a}\sqrt{a^2-x^2}\\&k=\sqrt{\frac{a^2-b^2}{a^2}}<1 \quad (k=\text{Eccentricity 偏心率})\text{トス}\end{aligned}\right\}\cdots\cdots(5)$$

h 及 l ト共ニ a カ與ヘラレタル場合ニハ

$$\left.\begin{aligned}&b=\frac{ah}{a-\sqrt{a^2-\frac{l^2}{4}}},\quad h_1=b-h=\frac{h\sqrt{a^2-\frac{l^2}{4}}}{a-\sqrt{a^2-\frac{l^2}{4}}}\\&d=\frac{b}{a}\sqrt{a^2-c^2}\end{aligned}\right\}\cdots\cdots(6)$$



第三圖

h 及 l ト共ニ b カ與ヘラレタル場合ニハ

又拱ノ斷面カ拱頂ヨリ兩端ニ向ツテ次ノ方程式ニヨリ增大スルモノト假定ス

$$\left.\begin{aligned} a &= \frac{bl}{2\sqrt{2bh-h^2}} \qquad h_1 = b-h \\ d &= \frac{b}{a}\sqrt{a^2-c^2} \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (7)$$

$$\left.\begin{aligned} I &= I_0 \sec\alpha \\ A &= A_0 \sec\alpha \end{aligned}\right\} \quad \cdots\cdots\cdots\cdots (8)$$

玆ニ I_0 及 A_0 ハ拱頂ニ於ケル惰率及斷面積ヲ示スモノトス

但 $\alpha=60^\circ$ ノ時ハ $\sec 60^\circ=2$ ナルヲ以テ尙ホ上ノ假定ヲ適用スルコトヲ得レトモ半楕圓拱 (Semi-elliptic arch) ノ場合ニハ $\alpha=90^\circ$ トナリ $\sec 90^\circ=\infty$ トナルヲ以テ上ノ假定ハ之ヲ適用スルコトヲ得サルモノトス

次ニ H_1 V_1 M_1 ノ値ヲ見出サンカタメニ算式(2)(3)(4)ノ各項ニ楕圓ノ方程式(5)ヲ導入シ積分ヲ行ヘハ

For equation (2)

$$\int_0^{\frac{l'}{2}} \frac{1}{I} ds = \frac{l}{2I_0}$$

$$\int_0^{\frac{l'}{2}} \frac{1}{I}(y-h_1) ds = \frac{1}{2I_0}\left(ab\sin^{-1}\frac{l}{2a} - \frac{h_1 l}{2}\right)$$

$$\int_{-c'}^{+\frac{l'}{2}} \frac{1}{I} ds = \frac{1}{I_0}\left(\frac{l}{2}+c\right)$$

$$\int_{c'}^{\frac{l'}{2}} \frac{1}{I} x\, ds = \frac{1}{2I_0}\left(\frac{l^2}{4}-c^2\right)$$

For equation (3)

$$\int_0^{\frac{l'}{2}} \frac{1}{I}\left(\frac{l^2}{4}+x^2\right)ds=\frac{l^3}{6I_0}$$

$$\int_0^{\frac{l'}{2}} \frac{1}{A}\sin^2\alpha\, ds=-\frac{b^2}{2A_0\,a^2\,k^2}\left(l-\frac{a}{k}\log\frac{2a+kl}{2a-kl}\right)$$

$$\int_{-c'}^{+\frac{l'}{2}} \frac{1}{I}\left(\frac{cl}{2}+x^2\right)ds=\frac{1}{3I_0}\left(\frac{l}{2}+c\right)^3$$

$$\int_{-c'}^{+\frac{l'}{2}} \frac{1}{A}\sin^2\alpha\, ds=-\frac{b^2}{A_0\,a^2k^2}\left\{\frac{l}{2}+c-\frac{a}{2k}\log\frac{(2a+kl)(a+ck)}{(2a-kl)(a-ck)}\right\}$$

For equation (4)

$$\int_0^{\frac{l'}{2}} \frac{1}{I}(y-h_1)^2\, ds=\frac{b}{I_0}\left\{\frac{bl}{2}\left(1-\frac{l^2}{12a^2}\right)-ah_1\sin^{-1}\frac{l}{2a}\right\}$$

$$\int_0^{\frac{l'}{2}} \frac{1}{A}\cos^2\alpha\, ds=\frac{1}{2A_0\,k^2}\left(l-\frac{b^2}{ak}\log\frac{2a+kl}{2a-kl}\right)$$

$$\int_{-c'}^{+\frac{l'}{2}} \frac{1}{I}(y-h_1)ds=\frac{1}{I_0}\left\{\frac{cd}{2}-h_1\left(\frac{l}{4}+c\right)+\frac{ab}{2}\left(\sin^{-1}\frac{l}{2a}+\sin^{-1}\frac{c}{a}\right)\right\}$$

$$\int_{c'}^{\frac{l'}{2}} \frac{1}{I}x(y-h_1)ds=\frac{1}{3I_0}(d-h_1)(a^2-c^2)-\frac{h_1}{6I_0}\left(\frac{l^2}{4}-c^2\right)$$

$$\int_{c'}^{\frac{l'}{2}} \frac{1}{A}\sin\alpha\cos\alpha\, ds=\frac{1}{A_0\,k^2}(d-h_1)-\frac{b^2}{A_0\,ak^3}\left(\tan^{-1}\frac{adk}{b^2}-\tan^{-1}\frac{ah_1\,k}{b^2}\right)$$

故ニ次ノ結果ヲ得ベシ

$$M_1 l+V_1\frac{l^2}{2}+H_1\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)-\frac{W}{2}\left(c+\frac{l}{2}\right)^2=0 \quad \cdots\cdots \quad (9)$$

$$M_1\frac{l^2}{2}+V_1\left\{\frac{l^3}{3}-\frac{I_0 b^2}{A_0 a^2k^2}\left(l-\frac{a}{k}\log\frac{2a+kl}{2a-kl}\right)\right\}+H_1\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)$$
$$-W\left[\frac{1}{6}\left(\frac{l}{2}+c\right)^2\left(\frac{5l}{2}-c\right)-\frac{I_0 b^2}{A_0 a^2k^2}\left\{\frac{l}{2}+c-\frac{a}{2k}\log\frac{(2a+kl)(a+ck)}{(2a-kl)(a-ck)}\right\}\right]=0 \quad \cdots\cdots \quad (10)$$

$$M_1\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+V_1\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+H_1\left[b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}\right.$$
$$\left.+\frac{I_0}{A_0 k_2}\left(l-\frac{b^2}{ak}\log\frac{2a+kl}{2a-kl}\right)\right]+W\left\{\frac{1}{3}\left(a^2+\frac{c^2}{2}\right)(d-h_1)-\frac{h_1}{3}\left(\frac{l}{4}+c\right)\left(\frac{l}{2}+c\right)\right.$$
$$\left.+\frac{abc}{2}\left(\sin^{-1}\frac{l}{2a}+\sin^{-1}\frac{c}{a}\right)+\frac{I_0}{A_0 k^2}(d-h_1)-\frac{I_0 b^2}{A_0 ak^3}\left(\tan^{-1}\frac{adk}{b^2}-\tan^{-1}\frac{ah_1 k}{b^2}\right)\right\}=0 \quad \cdots\cdots \quad (11)$$

算式(9)(10)(11)ニヨリH_1 V_1 M_1ノ値ヲ見出シ得ベシ

軸應力ノ影響ヲ無視スルトキハ(A_0ヲ含ム項ヲ省略スレバ宜シ)

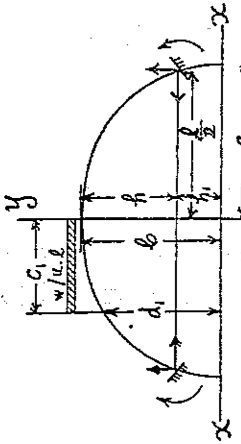
$$M_1 l+V_1\frac{l^2}{2}+H_1\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)-\frac{W}{2}\left(c+\frac{l}{2}\right)^2=0 \quad \cdots\cdots \quad (12)$$

$$M_1 l^2+V_1\frac{2l^3}{3}+H_1 l\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)-\frac{W}{3}\left(\frac{l}{2}+c\right)^2\left(\frac{5l}{2}-c\right)=0 \quad \cdots\cdots \quad (13)$$

$$M_1\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+V_1\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+2H_1 b\left\{\frac{bl}{2}\left(1-\frac{l^2}{12a^2}\right)\right.$$
$$\left.-ah_1\sin^{-1}\frac{l}{2a}\right\}+W\left\{\frac{1}{3}\left(a^2+\frac{c^2}{2}\right)(d-h_1)-\frac{h_1}{3}\left(\frac{l}{4}+c\right)\left(\frac{l}{2}+c\right)+\frac{abc}{2}\left(\sin^{-1}\frac{l}{2a}+\sin^{-1}\frac{c}{a}\right)\right\}=0 \quad \cdots\cdots \quad (14)$$

第三章　等布荷重ニ對スル H_1 V_1 M_1 ノ決定

第四圖ニ示スカ如キ等布荷重ニ對シ H_1 V_1 M_1 ノ値ヲ見出スニハ算式 (9) (10) (11) ニ於テ $W=w\,dx$, $c=x$, $d=\frac{b}{a}\sqrt{a^2-x^2}$ ニ置キ代ヘ各項ヲ 0 ヨリ c_1 マテ積分スレハ宜シ但シ $d_1=\frac{b}{a}\sqrt{a^2-c_1^2}$ トス



第四圖

For equation (9)

$$\int_0^{c_1}\frac{1}{2}\left(x+\frac{l}{2}\right)^2 w\,dx=\frac{1}{2}\left(\frac{l^2}{4}+\frac{lc_1}{2}+\frac{c_1^2}{3}\right)wc_1$$

For equation (10)

$$\int_0^{c_1}\frac{1}{6}\left(\frac{l}{2}+x\right)^2\left(\frac{5l}{2}-x\right)w\,dx=\frac{1}{12}\left\{l\left(\frac{5l}{4}+c_1\right)(l+c_1)-\frac{c_1^3}{2}\right\}wc_1$$

$$\int_0^{c_1}\frac{I_0 b^2}{A_0 a^2k^2}\left\{\frac{l}{2}+x-\frac{a}{2k}\log\frac{(2a+kl)(a+kx)}{(2a-kl)(a-kx)}\right\}w\,dx=\frac{I_0 b^2}{A_0 a^2k^2}\left[\frac{l}{2}+\frac{c_1}{2}-\frac{a}{2k}\left\{\log\frac{a+c_1k}{a-c_1k}+\frac{a}{c_1k}\log\left(1-\frac{c_1^2k^2}{a^2}\right)\right\}\right]wc$$

For equation (11)

$$\int_0^{c_1}\frac{1}{3}\left(a^2+\frac{x^2}{2}\right)(d-h_1)w\,dx=\frac{1}{24}\left\{d_1\left(\frac{7}{2}a^2+c_1^2\right)-8h_1\left(a^2+\frac{c_1^2}{6}\right)+\frac{9a^3b}{2c_1}\sin^{-1}\frac{c_1}{a}\right\}wc_1$$

$$\int_0^{c_1}\frac{h_1}{3}\left(\frac{l}{4}+x\right)\left(\frac{l}{2}+x\right)w\,dx=\frac{h_1}{24}\left(l^2+3lc_1+\frac{8}{3}c_1^2\right)wc_1$$

$$\int^{c_1}\frac{ab}{2}x\left(\sin^{-1}\frac{l}{2a}+\sin^{-1}\frac{x}{a}\right)w\,dx=\frac{ab}{4}\left(c_1\sin^{-1}\frac{l}{2a}+\frac{ad_1}{2b}-\frac{a^2-2c_1^2}{2c_1}\sin^{-1}\frac{c_1}{a}\right)wc_1$$

$$\int_0^{c_1}\frac{I_0}{A_0 k^2}(d-h_1)w\,dx=\frac{I_0}{A_0 k^2}\left(\frac{d_1}{2}-h_1+\frac{ab}{2c_1}\sin^{-1}\frac{c_1}{a}\right)wc_1$$

$$\int_0^{c_1}\frac{I_0 b^2}{A_0 ak^3}\left(\tan^{-1}\frac{adk}{b^2}-\tan^{-1}\frac{ah_1 k}{b^2}\right)w\,dx=\frac{I_0 b^2}{A_0 ak^3}\left\{\frac{\pi a}{4c_1 k}+\tan^{-1}\frac{ad_1 k}{b^2}\right.$$

$$\left.-\tan^{-1}\frac{ah_1 k}{b^2}-\frac{b}{c_1 k}\sin^{-1}\frac{c_1}{a}-\frac{a}{2c_1 k}\sin^{-1}\left(\frac{a^4 d_1^2-b^4 c_1^2}{a^4 d_1^2+b^4 c_1^2}\right)\right\}wc_1$$

故ニ次ノ結果ヲ得ベシ

$$M_1 l+V_1\frac{l^2}{2}+H_1\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)-\frac{wc_1}{2}\left(\frac{l^2}{4}+\frac{lc_1}{2}+\frac{c_1^2}{3}\right)=0 \quad \cdots\cdots\cdots \quad (15)$$

$$M_1\frac{l^2}{2}+V_1\left\{\frac{l^3}{3}-\frac{I_0 b^2}{A_0 a^2k^2}\left(l-\frac{a}{k}\log\frac{2a+kl}{2a-kl}\right)\right\}+H_1\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+wc_1\left(\frac{c_1^3}{24}-\frac{l}{12}\left(\frac{5l}{4}+c_1\right)(l+c_1)\right.$$

$$\left.+\frac{I_0 b^2}{A_0 a^2k^2}\left[\frac{l}{2}+\frac{c_1}{2}-\frac{a}{2k}\left\{\log\frac{a+c_1 k}{a-c_1 k}+\frac{a}{c_1 k}\log\left(1-\frac{c_1^2 k^2}{a^2}\right)\right\}\right]\right)=0 \quad \cdots \quad (16)$$

$$M_1\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+V_1\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+H_1\left[b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}+\frac{I_0}{A_0 k^2}\left(l-\frac{b^2}{ak}\log\frac{2a+kl}{2a-kl}\right)\right]$$

$$+wc_1\left[\frac{d_1}{24}\left(\frac{13}{2}a^2+c_1^2\right)-h_1\left(\frac{l^2}{24}+\frac{a^2}{3}+\frac{lc_1}{8}+\frac{c_1^2}{6}\right)+\frac{ab}{4}\left\{c_1\sin^{-1}\frac{l}{2a}+\left(\frac{a^2}{4c_1}+c_1\right)\sin^{-1}\frac{c_1}{a}\right\}-\frac{I_0}{A_0 k^2}\left(\frac{\pi b^2}{4c_1 k^2}+h_1-\frac{d_1}{2}\right)\right.$$

$$\left.+\frac{I_0 b^2}{A_0 ak^3}\left\{\tan^{-1}\frac{ah_1 k}{b^2}-\tan^{-1}\frac{ad_1 k}{b^2}+\frac{a^2+b^2}{2bc_1 k}\sin^{-1}\frac{c_1}{a}+\frac{a}{2c_1 k}\sin^{-1}\left(\frac{a^4 d_1^2-b^4 c_1^2}{a^4 d_1^2+b^4 c_1^2}\right)\right\}\right]=0 \cdots \quad (17$$

軸應力ノ影響ヲ無視スルトキハ

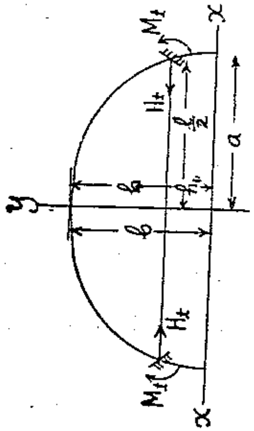
1271

$$M_1 l+V_1\frac{l^2}{2}+H_1\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)-\frac{wc_1}{2}\left(\frac{l^2}{4}+\frac{lc_1}{2}+\frac{c_1^2}{3}\right)=0 \cdots\cdots\cdots \quad (18)$$

$$M_1 l+V_1\frac{2l^2}{3}+H_1\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)-wc_1\left\{\frac{1}{6}\left(\frac{5l}{4}+c_1\right)(l+c_1)-\frac{c_1^3}{12l}\right\}=0 \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots \quad (19)$$

$$M_1\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+V_1\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+H_1 b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}$$
$$+wc_1\left[\frac{d_1}{24}\left(\frac{13}{2}a^2+c_1^2\right)-h_1\left(\frac{l^2}{24}+\frac{a^2}{3}+\frac{lc_1}{8}+\frac{c_1^2}{6}\right)+\frac{ab}{4}\left\{c_1\sin^{-1}\frac{l}{2a}+\left(\frac{a^2}{4c_1}+c_1\right)\sin^{-1}\frac{c_1}{a}\right\}\right]=0 \cdots \quad (20)$$

第四章 熱應力 (Temperature stress)



第五圖

θ=伸縮率 (Coefficient of expansion and contraction)

t=度數ニ於テ表ハサレタル温度ノ變化

H_t=温度ノ變化ノタメ支承ニ於テ生セル水平反力

M_t=温度ノ變化ノタメ支承ニ於テ生セル力率

Bending moment $\quad M=M_t-H_t(y-h_1),\quad \frac{\partial M}{\partial M_t}=1,\quad \frac{\partial M}{\partial H_t}=-(y-h_1)$

Axial stress $\quad N=-H_t\cos\alpha,\quad \frac{\partial N}{\partial M_t}=0,\quad \frac{\partial N}{\partial H_t}=-\cos\alpha$

應剪力ノ影響ヲ無視スルトキハ

$$\omega=2\int_0^{\frac{l'}{2}}\frac{M^2}{2EI}ds+2\int_0^{\frac{l'}{2}}\frac{N^2}{2EA}ds$$

Castigliano ノ第一定理ノ應用ニヨリ

$$\frac{\partial\omega}{\partial M_t}=2\int_0^{\frac{l'}{2}}\frac{M}{EI}\left(\frac{\partial M}{\partial M_t}\right)ds+2\int_0^{\frac{l'}{2}}\frac{N}{EA}\left(\frac{\partial N}{\partial M_t}\right)ds=0$$

$$\frac{\partial\omega}{\partial H_t}=\pm t\theta l=2\int_0^{\frac{l'}{2}}\frac{M}{EI}\left(\frac{\partial M}{\partial H_t}\right)ds+2\int_0^{\frac{l'}{2}}\frac{N}{EA}\left(\frac{\partial N}{\partial H_t}\right)ds$$

上ノ方程式ヨリ

$$M_t\int_0^{\frac{l'}{2}}\frac{1}{EI}\,ds-H_t\int_0^{\frac{l'}{2}}\frac{1}{EI}\,(y-h_1)\,ds=0$$

$$\pm t\theta l=-2M_t\int_0^{\frac{l'}{2}}\frac{1}{EI}\,(y-h_1)\,ds+2H_t\left\{\int_0^{\frac{l'}{2}}\frac{1}{EI}\,(y-h_1)^2\,ds+\int_0^{\frac{l'}{2}}\frac{1}{EA}\cos^2\alpha\,ds\right\}$$

上ノ二式ヨリ次ノ結果ヲ得ヘシ

$$H_t=\pm\frac{1}{2}\,\frac{t\theta l\,E}{\displaystyle\int_0^{\frac{l'}{2}}\frac{1}{I}\,(y-h_1)^2\,ds+\int_0^{\frac{l'}{2}}\frac{1}{A}\cos^2\alpha\,ds-\frac{\left(\int_0^{\frac{l'}{2}}\frac{1}{I}\,(y-h_1)\,ds\right)^2}{\int_0^{\frac{l'}{2}}\frac{1}{I}\,ds}}\quad\cdots\cdots\cdots\cdots(21)$$

$$M_t=\frac{\displaystyle\int_0^{\frac{l'}{2}}\frac{1}{I}\,(y-h_1)\,ds}{\displaystyle\int_0^{\frac{l'}{2}}\frac{1}{I}\,ds}\,H_t\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(22)$$

故ニ次ノ結果ヲ得ヘシ

$$H_t=\pm\frac{t\theta lEI_0}{b^2l\left(1-\frac{l^2}{12\,a^2}\right)-\frac{1}{l}\left(ab\sin^{-1}\frac{l}{2a}+\frac{h_1\,l}{2}\right)^2+\frac{I_0}{A_0\,k^2}\left(l-\frac{b^2}{ak}\log\frac{2a+kl}{2a-kl}\right)}\quad\cdots\cdots\cdots(23)$$

$$M_t=\frac{ab\sin^{-1}\dfrac{l}{2a}-\dfrac{h_1 l}{2}}{l}H_t\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\ (24)$$

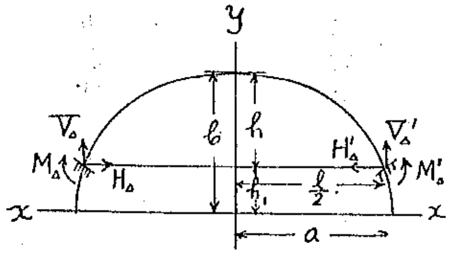
軸應力ノ影響ヲ無視スルトキハ

$$H_t=\pm\frac{t\theta l EI_0}{b^2 l\left(1-\dfrac{l^2}{12a^2}\right)-\dfrac{1}{l}\left(ab\sin^{-1}\dfrac{l}{2a}+\dfrac{h_1 l}{2}\right)^2}\ \cdots\cdots\cdots\cdots\cdots\cdots\ (25)$$

$$M_t=\frac{ab\sin^{-1}\dfrac{l}{2a}-\dfrac{h_1 l}{2}}{l}H_t\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\ (26)$$

溫度昇リシトキ正號ヲ用ヒ溫度降リシトキ負號ヲ用フ

第五章　支承ノ移動ニ歸因スル應力 (Stress due to displacement of support)



第六圖

若シ無鉸拱カ或ル原因ノ爲メ支承ニ移動ヲ生スルトキハ從ツテ徑間長 (Span length) 高サ (Relative height) 中心角 (Central angle) ニモ變化ヲ來タシ爲メニ拱ハ應力ヲ受クルニ至ルモノナリ

M_Δ, M_Δ'＝支承移動ノタメニ拱ノ兩端ニ於テ生セル力率・
V_Δ, V_Δ'＝支承移動ノタメニ拱ノ兩端ニ於テ生セル垂直反力
H_Δ, H_Δ'＝支承移動ノタメニ拱ノ兩端ニ於テ生セル水平反力

初等力學ニヨリ

$$H_\Delta'=H_\Delta,\qquad V_\Delta'=-V_\Delta,\qquad M_\Delta'=M_\Delta+V_\Delta l$$

拱ノ任意點ニ於ケル彎曲率ハ次ノ如シ

$$M=M_\Delta+V_\Delta\left(\frac{l}{2}-x\right)-H_\Delta(y-h_1)\qquad y\text{軸ヨリ左ノ部分ニ於テ}$$

$$M=M_\triangle'+V_\triangle'\left(\frac{l}{2}-x\right)-H_\triangle'(y-h_1)\qquad y\text{ 軸ヨリ右ノ部分ニ於テ}$$

$$=M_\triangle+V_\triangle\left(\frac{l}{2}+x\right)-H_\triangle(y-h_1)$$

又拱ノ任意點ニ於ケル軸應力ハ次ノ如シ

$$N=-H_\triangle\cos\alpha-V_\triangle\sin\alpha\qquad y\text{ 軸ヨリ左ノ部分ニ於テ}$$

$$N=-H_\triangle'\cos\alpha-V_\triangle'\sin\alpha\qquad y\text{ 軸ヨリ右ノ部分ニ於テ}$$

$$=-H_\triangle\cos\alpha+V_\triangle\sin\alpha$$

應剪力ノ影響ヲ無視スルトキハ拱ノ內働ノ總和ハ次ノ如シ

$$\omega=\int_0^{\frac{l'}{2}}\frac{1}{2EI}\left\{M_\triangle+V_\triangle\left(\frac{l}{2}-x\right)-H_\triangle(y-h_1)\right\}^2ds+\int_0^{\frac{l'}{2}}\frac{1}{2EI}\left\{M_\triangle+V_\triangle\left(\frac{l}{2}+x\right)-H_\triangle(y-h_1)\right\}^2ds$$

$$+\int_0^{\frac{l'}{2}}\frac{1}{2EA}\left(H_\triangle\cos\alpha+V_\triangle\sin\alpha\right)^2ds+\int_0^{\frac{l'}{2}}\frac{1}{2EA}\left(-H_\triangle\cos\alpha+V_\triangle\sin\alpha\right)^2ds$$

(第1) 支承ノ高サ (Relative height) ニ變化ヲ生シタル場合

Δy= 左端支承ノ高サ (Relative height) ニ於ケル變化

但上向ノ變化ヲ正トシ下向ヲ負トス

Castigliano ノ第1定理ニヨリ

$$\frac{\partial\omega}{\partial V_\triangle}=\pm\Delta y,\qquad\frac{\partial\omega}{\partial H_\triangle}=0,\qquad\frac{\partial\omega}{\partial M_\triangle}=0$$

上ノ方程式ヨリ

$$M_\triangle l\int_0^{\frac{l'}{2}}\frac{1}{I}\,ds+V_\triangle\left\{\frac{l^2}{2}\int_0^{\frac{l'}{2}}\frac{1}{I}\,ds+2\int_0^{\frac{l'}{2}}\frac{1}{I}x^2\,ds+2\int_0^{\frac{l'}{2}}\frac{1}{A}\sin^2\alpha\,ds\right\}-H_\triangle l\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds=\pm E\,\Delta y \quad\cdots\quad\cdots\quad (27)$$

$$M_\triangle\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds+V_\triangle\frac{l}{2}\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds-H_\triangle\left\{\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)^2\,ds+\int_0^{\frac{l'}{2}}\frac{1}{A}\cos^2\alpha\,ds\right\}=0 \quad\cdots\quad (28)$$

$$M_\triangle\int_0^{\frac{l'}{2}}\frac{1}{I}\,ds+V_\triangle\frac{l}{2}\int_0^{\frac{l'}{2}}\frac{1}{I}\,ds-H_\triangle\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds=0 \quad\cdots\quad (29)$$

故ニ次ノ結果ヲ得ヘシ

$$M_\triangle\frac{l^2}{2}+V_\triangle\left\{\frac{l^3}{3}-\frac{I_0 b^2}{A_0 a^2 k^2}\left(l-\frac{a}{k}\log\frac{2a+kl}{2a-kl}\right)\right\}+H_\triangle\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)=\pm I_0 E\,\Delta y \quad\cdots\quad (30)$$

$$M_\triangle\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+V_\triangle\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)$$
$$+H_\triangle\left[b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}+\frac{I_0}{A_0 k^2}\left(l-\frac{b^2}{ak}\log\frac{2a+kl}{2a-kl}\right)\right]=0 \quad\cdots\quad (31)$$

$$M_\triangle l+V_\triangle\frac{l^2}{2}+H_\triangle\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)=0 \quad\cdots\quad (32)$$

軸應力ノ影響ヲ無視スルトキハ

$$M_\triangle\frac{l^2}{2}+V_\triangle\frac{l^3}{3}+H_\triangle\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)=\pm I_0 E\,\Delta y \quad\cdots\quad (33)$$

$$M_\triangle\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+V_\triangle\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+H_\triangle b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}=0 \quad\cdots\quad (34)$$

$$M_\triangle l+V_\triangle\frac{l^2}{2}+H_\triangle\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)=0 \quad\cdots\quad (35)$$

（第三） 徑間長ニ變化ヲ生シタル場合

Δl＝左端支承ニ於テ計リタル徑間長ノ變化

但徑間長ノ減セシトキヲ正トシ增セシトキヲ負トス

Castigliano ノ第１定理ニヨリ

$$\frac{\partial\omega}{\partial H_\Delta}=\pm\Delta l,\qquad \frac{\partial\omega}{\partial V_\Delta}=0,\qquad \frac{\partial\omega}{\partial M_\Delta}=0$$

上ノ方程式ヨリ

$$-2M_\Delta\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds-V_\Delta l\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds+2H_\Delta\left\{\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)^2\,ds+\int_0^{\frac{l'}{2}}\frac{1}{A}\cos^2\alpha\,ds\right\}=\pm E\,\Delta l\cdots\quad(36)$$

$$M_\Delta\, l\int_0^{\frac{l'}{2}}\frac{1}{I}\,ds+V_\Delta\left\{\frac{l^2}{2}\int_0^{\frac{l'}{2}}\frac{1}{I}\,ds+2\int_0^{\frac{l'}{2}}\frac{1}{I}x^2\,ds+2\int_0^{\frac{l'}{2}}\frac{1}{A}\sin^2\alpha\,ds\right\}-H_\Delta l\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds=0\cdots\cdots\quad(37)$$

$$M_\Delta\int_0^{\frac{l'}{2}}\frac{1}{I}\,ds+V_\Delta\frac{l}{2}\int_0^{\frac{l'}{2}}\frac{1}{I}\,ds-H_\Delta\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds=0\cdots\cdots\quad(38)$$

上ノ方程式ヨリ次ノ結果ヲ得ヘシ

$$M_\Delta\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+V_\Delta\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)$$
$$+H_\Delta\left[b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}+\frac{I_0}{A_0k^2}\left(l-\frac{b^2}{ak}\log\frac{2a+kl}{2a-kl}\right)\right]=\pm I_0E\,\Delta l\quad\cdots\cdots\quad(39)$$

$$M_\Delta\frac{l^2}{2}+V\left\{\frac{l^3}{3}-\frac{I_0b^2}{A_0a^2k^2}\left(l-\frac{a}{k}\log\frac{2a+kl}{2a-kl}\right)\right\}+H_\Delta\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)=0\cdots\cdots\quad(40)$$

$$M_\triangle l + V_\triangle \frac{l^2}{2} + H_\triangle\left(\frac{h_1 l}{2} - ab\sin^{-1}\frac{l}{2a}\right) = 0 \qquad \cdots (41)$$

軸應力ノ影響ヲ無視スルトキハ

$$M_\triangle\left(\frac{h_1 l}{2} - ab\sin^{-1}\frac{l}{2a}\right) + V_\triangle\frac{l}{2}\left(\frac{h_1 l}{2} - ab\sin^{-1}\frac{l}{2a}\right) + H_\triangle b\left\{bl\left(1 - \frac{l^2}{12a^2}\right) - 2ah_1\sin^{-1}\frac{l}{2a}\right\} = \pm I_0 E\Delta l \cdots (42)$$

$$M_\triangle l + V_\triangle\frac{2l^2}{3} + H_\triangle\left(\frac{h_1 l}{2} - ab\sin^{-1}\frac{l}{2a}\right) = 0 \qquad \cdots (43)$$

$$M_\triangle l + V_\triangle\frac{l^2}{2} + H_\triangle\left(\frac{h_1 l}{2} - ab\sin^{-1}\frac{l}{2a}\right) = 0 \qquad \cdots (44)$$

(第三)　中心角ニ變化ヲ生シタル場合

$\Delta\alpha$＝左端ノ支承ニ於テ計リタル拱ノ中心角ノ變化

但中心角ノ減セシトキヲ正トシ增セシトキヲ負トス

Castigliano ノ第一定理ノ應用ニヨリ

$$\frac{\partial\omega}{\partial M_\triangle} = \pm\Delta\alpha, \qquad \frac{\partial\omega}{\partial H_\triangle} = 0, \qquad \frac{\partial\omega}{\partial V_\triangle} = 0$$

上ノ方程式ヨリ

$$2M_\triangle\int_0^{\frac{l'}{2}}\frac{1}{I}ds + V_\triangle l\int_0^{\frac{l'}{2}}\frac{1}{I}ds - 2H_\triangle\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds = \pm E\Delta\alpha \qquad \cdots (45)$$

$$M_\triangle\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds + V_\triangle\frac{l}{2}\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds - H_\triangle\left\{\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)^2 ds + \int_0^{\frac{l'}{2}}\frac{1}{A}\cos^2\alpha\,ds\right\} = 0 \qquad \cdots (46)$$

$$M_\triangle l\int_0^{\frac{l'}{2}}\frac{1}{I}ds + V_\triangle\left\{\frac{l^2}{2}\int_0^{\frac{l'}{2}}\frac{1}{I}ds + 2\int_0^{\frac{l'}{2}}\frac{1}{I}x^2 ds + 2\int_0^{\frac{l'}{2}}\frac{1}{A}\sin^2\alpha\,ds\right\} - H_\triangle l\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)\,ds = 0 \qquad \cdots (47)$$

上ノ方程式ヨリ次ノ結果ヲ得ヘシ

$$M_\Delta l+V_\Delta\frac{l^2}{2}+H_\Delta\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)=\pm\frac{1}{2}I_0 E\Delta a \quad \cdots\cdots \quad (48)$$

$$M_\Delta\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+V_\Delta\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)$$
$$+H_\Delta\left[b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}+\frac{I_0}{A_0k^2}\left(l-\frac{b^2}{ak}\log\frac{2a+kl}{2a-kl}\right)\right]=0 \quad \cdots \quad (49)$$

$$M_\Delta l^2+V_\Delta\left\{\frac{2l^3}{3}-\frac{2I_0b^2}{A_0a^2k^2}\left(l-\frac{a}{k}\log\frac{2a+kl}{2a-kl}\right)\right\}+H_\Delta l\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)=0 \quad \cdots\cdots \quad (50)$$

軸應力ノ影響ヲ無視スルトキハ

$$M_\Delta l+V_\Delta\frac{l^2}{2}+H_\Delta\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)=\pm\frac{1}{2}I_0 E\Delta a \quad \cdots\cdots \quad (51)$$

$$M_\Delta\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+V_\Delta\frac{l}{2}\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)+H_\Delta b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}=0 \quad \cdots \quad (52)$$

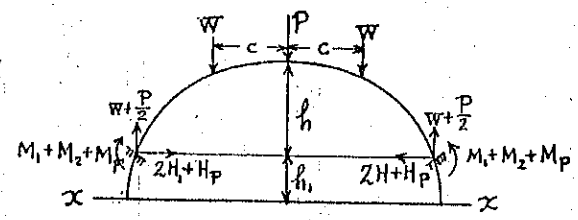
$$M_\Delta l+V_\Delta\frac{2l^2}{3}+H_\Delta\left(\frac{h_1 l}{2}-ab\sin^{-1}\frac{l}{2a}\right)=0 \quad \cdots\cdots \quad (53)$$

第六章　拱頂ニ於ケル撓度 (Deflection at crown)

撓度ニ對スル軸應力ノ影響ハ比較的小ナルヲ以テ以下撓度ノ計算ニ於テ之ヲ無視スルモ大差ナシ

第七圖ニ於テWヲ與ヘラレタル集荷重トシ又Pヲ補助荷重トシ拱ノ內働ノ總和ヲ求メ之ヲPニ付テ微分シ最後ニ$P=0$ニ置ケハ Castigliano ノ第一定理ニヨリ拱頂ニ於ケル撓度ヲ見出シ得ヘシ

1279



第七圖

第七圖ニ於テM_1 M_2ヲ一個ノWノタメニ左右兩支承ニ生セル力率トシH_1ヲ其ノ水平反力トス又H_P M_PヲPノタメニ生セル水平反力及力率トスレハM_1 M_2 H_1 M_P H_P等ノ値ハ算式(12)(13)(14)ニ依ツテ定ムルコトヲ得ヘク從ツテ$H_P=K_1P,\ M_P=K_2P$ト置ケハK_1 K_2ノ値モ容易ニ之ヲ決定シ得ヘシ但シK_1ハ數(Number)ニシテK_2ハ長サ(Length)ヲ表ス

Bending moment at any point

For $x<c$ $\quad M=W\left(\frac{l}{2}-c\right)+\frac{1}{2}P\left(\frac{l}{2}-x\right)-(2H_1+K_1P)(y-h_1)+M_1+M_2+K_2P$

$$\frac{\partial M}{\partial P}=\frac{1}{2}\left(\frac{l}{2}-x\right)-K_1(y-h_1)+K_2$$

For $x>c$ $\quad M=W\left(\frac{l}{2}-x\right)+\frac{1}{2}P\left(\frac{l}{2}-x\right)-(2H_1+K_1P)(y-h_1)+M_1+M_2+K_2P$

$$\frac{\partial M}{\partial P}=\frac{1}{2}\left(\frac{l}{2}-x\right)-K_1(y-h_1)+K_2$$

應剪力及軸應力ノ影響ヲ無視スルトキハ拱ノ內働ノ總和ハ次ノ如シ

$$\omega=2\int_0^{c'}\frac{M^2}{2EI}ds+2\int_{c'}^{\frac{l'}{2}}\frac{M^2}{2EI}ds$$

Castigliano ノ第一定理ノ應用ニヨリ一個ノWノタメニ起レル拱頂ニ於ケル撓度ハ次ノ如シ

$$\delta=\frac{1}{2}\cdot\frac{\partial\omega}{\partial P}=\int_0^{c'}\frac{M}{EI}\left(\frac{\partial M}{\partial P}\right)ds+\int_{c'}^{\frac{l'}{2}}\frac{M}{EI}\left(\frac{\partial M}{\partial P}\right)ds$$

$$=\left\{W\frac{l}{2}-2H_1(y-h_1)+M_1+M_2\right\}\int_0^{\frac{v}{2}}\frac{1}{EI}\left\{\frac{1}{2}\left(\frac{l}{2}-x\right)-K_1(y-h_1)+K_2\right\}ds$$

$$-Wc\int_0^{c'}\frac{1}{EI}\left\{\frac{1}{2}\left(\frac{l}{2}-x\right)-K_1(y-h_1)+K_2\right\}ds-W\int_{c'}^{\frac{v}{2}}\frac{1}{EI}x\left\{\frac{1}{2}\left(\frac{l}{2}-x\right)-K_1(y-h_1)+K_2\right\}ds\cdots\quad\cdots\quad(54)$$

上ノ方程式ヨリ次ノ結果ヲ得ヘシ

$$\delta=\frac{M_1+M_2}{2EI_0}\left\{l\left(\frac{K_1h_1}{2}+K_2+\frac{l}{8}\right)-K_1ab\sin^{-1}\frac{l}{2a}\right\}+\frac{H_1}{EI_0}\left\{\frac{a^2b}{3}+K_1b^2l\left(1-\frac{l^2}{12a^2}\right)+h_1l\left(\frac{K_2}{2}+\frac{l}{12}-\frac{a^2}{3l}\right)\right.$$

$$\left.-ab\left(2K_1h_1+K_2+\frac{l}{4}\right)\sin^{-1}\frac{l}{2a}\right\}+\frac{W}{EI_0}\left[\frac{K_1a^2}{3}(d-h_1)+\frac{l^2}{4}\left(\frac{K_1h_1}{3}+\frac{K_2}{2}+\frac{l}{12}\right)\right.$$

$$\left.-c^2\left\{\frac{K_1}{2}\left(h_1-\frac{d}{3}\right)+\frac{K_2}{2}+\frac{l}{8}-\frac{c}{12}\right\}-\frac{K_1ab}{2}\left(\frac{l}{2}\sin^{-1}\frac{l}{2a}-c\sin^{-1}\frac{c}{a}\right)\right]\quad\cdots\quad(55)$$

但上式ニ於ケル H_1 M_1 M_2 K_1 K_2 等ノ値ハ算式(12)(13)(14)ヨリ之ヲ見出シ得ヘキモノナリ
同様ニシテ等布荷重カ c_1 マテ乗リシトキニハ算式(55)ニ於テ $W=w\,dx$, $c=x$, $d=\frac{b}{a}\sqrt{a^2-x^2}$ ニ置キ代ヘ各項ヲ0ヨリ c_1 マテ積分スレハ次ノ結果ヲ得ヘシ

$$\delta=\frac{M_1+M_2}{2EI_0}\left\{l\left(\frac{K_1h_1}{2}+K_2+\frac{l}{8}\right)-K_1ab\sin^{-1}\frac{l}{2a}\right\}+\frac{H_1}{EI_0}\left\{\frac{a^2b}{3}+K_1b^2l\left(1-\frac{l^2}{12a^2}\right)+h_1l\left(\frac{K_2}{2}+\frac{l}{12}-\frac{a^2}{3l}\right)\right.$$

$$\left.-ab\left(2K_1h_1+K_2+\frac{l}{4}\right)\sin^{-1}\frac{l}{2a}\right\}+\frac{wc_1}{EI_0}\left[K_1a^2\left(\frac{5d_1}{16}-\frac{h_1}{3}\right)+\frac{l^2}{4}\left(\frac{K_1h_1}{3}+\frac{K_2}{2}+\frac{l}{12}\right)\right.$$

$$\left.-\frac{c_1^2}{6}\left\{K_1\left(h_1-\frac{d_1}{4}\right)+K_2+\frac{l}{4}-\frac{c_1}{8}\right\}+\frac{K_1ab}{4}\left\{\left(\frac{a^2}{12c_1}+c_1\right)\sin^{-1}\frac{c_1}{a}-l\sin^{-1}\frac{l}{2a}\right\}\right]\quad\cdots\quad(56)$$

1281

但上式ニ於ケルK_1 K_2ノ値ハ(55)式ニ於ケルモノト同一ナレトモH_1 M_1 M_2ノ値ハ等布荷重ニ對スル算式(18)(19)(20)ニヨリ之ヲ見出スコトヲ得ヘシ

本論文中ノ諸式ニ於ケル對數ハ總テ Naperian logarithm ヲ示スモノニシテ之等諸式中ニ於ケル對數正弦弧及正切弧等ノ計算ニハ Chambers's Mathematical Tables ヲ用フルヲ便トス(完)