論說報告

土木學會誌　第三卷第四號　大正六年八月

變斷面積ヲ有スル雙鉸楕圓拱

工學士　野口寅之助

第一章　總論

楕圓拱 (Elliptic arch) ハ分圓拱 (Circular arch) 及ヒ拋物線拱 (Parabolic arch) ニ比シ應力ノ計算頗面倒ナルヲ以テ實際使用セラルヽコト極メテ稀ナレトモ次ニ變斷面積ヲ有スル雙鉸楕圓拱 (Two-hinged elliptic arch with varying cross section) ノ設計ニ際シ必要ナル算式ノ一般ヲ示サントス

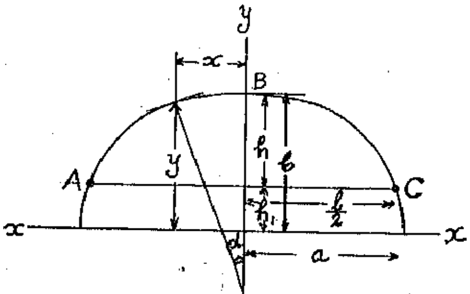
第一圖及第二圖ニ於テ a 及 b ヲ楕圓ノ長軸 (Major axis) 及短軸 (Minor axis) トシ h 及 l ヲ拱橋ノ拱矢 (Rise) 及徑間 (Span) トス

H＝支承 A 及 C ニ於テ一個ノ W ノタメニ生セル水平反力 (Horizontal reaction)

M＝拱ノ任意點ニ於ケル彎曲率 (Bending moment)

但拱ノ上部纖維 (Upper fiber) ニ壓力ヲ與フル彎曲率ヲ正トシ張力ヲ與フルモノヲ負トス

N＝拱ノ任意點ニ於ケル軸應力 (Axial stress or Normal stress)



第一圖

但拱ニ張力ヲ與フル軸應力ヲ正トシ壓力ヲ與フルモノヲ負トス

c'＝拱軸ニ沿フテBヨリDニ至ル距離

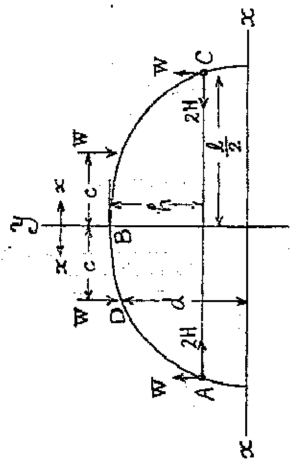
$\frac{l'}{2}$＝拱軸ニ沿フテBヨリAニ至ル距離

α＝曲線上ノ任意點ニ於ケル法線(Normal)カy軸ト爲ス角

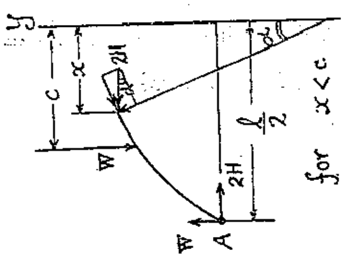
E＝彈率(Modulus of elasticity)

I＝拱ノ任意點ニ於ケル惰率(Moment of inertia)

A＝拱ノ任意點ニ於ケル斷面積(Cross sectional area)



第二圖



第三圖

第一圖及第二圖ヨリ拱ノ任意點ニ於ケル彎曲率ハ次ノ如シ

for $x<c$ $\quad M=W\left(\frac{l}{2}-c\right)-2H(y-h_1),\quad \frac{dM}{dH}=-2(y-h_1)$

for $x>c$ $\quad M=W\left(\frac{l}{2}-x\right)-2H(y-h_1),\quad \frac{dM}{dH}=-2(y-h_1)$

第三圖及第四圖ヨリ拱ノ任意點ニ於ケル軸應力ハ次ノ如シ

for $x<c$ $\quad N=-2H\cos\alpha,\quad \frac{dN}{dH}=-2\cos\alpha$

for $x>c$ $\quad N=-2H\cos\alpha-W\sin\alpha,\quad \frac{dN}{dH}=-2\cos\alpha$

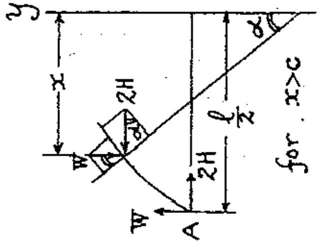
應剪力(Tangential stress)ノ影響ハ頗ル小ナルヲ以テ之ヲ無視スルトキハ拱ノ内働(Internal work)ノ總和ハ次ノ如シ

$$\omega=2\int_0^{c'}\frac{M^2}{2EI}ds+2\int_{c'}^{\frac{l'}{2}}\frac{M^2}{2EI}ds+2\int_0^{c'}\frac{N^2}{2EA}ds+2\int_{c'}^{\frac{l'}{2}}\frac{N^2}{2EA}ds$$

Castigliano ノ第二定理ニヨリ

$$\frac{d\omega}{dH}=2\int_0^{c'}\frac{M}{EI}\left(\frac{dM}{dH}\right)ds+2\int_{c'}^{\frac{l'}{2}}\frac{M}{EI}\left(\frac{dM}{dH}\right)ds$$

$$+2\int_0^{c'}\frac{N}{EA}\left(\frac{dN}{dH}\right)ds+2\int_{c'}^{\frac{l}{2}}\frac{N}{EA}\left(\frac{dN}{dH}\right)ds=0$$



第四圖

$ds=\dfrac{dx}{\cos\alpha}=\sec\alpha\,dx$ ナルヲ以テ

$$-2\int_0^{c'}\frac{1}{I}\left\{W\left(\frac{l}{2}-c\right)-2H(y-h_1)\right\}(y-h_1)ds$$

$$-2\int_{c'}^{\frac{l'}{2}}\frac{1}{I}\left\{W\left(\frac{l}{2}-x\right)-2H(y-h_1)\right\}(y-h_1)ds$$

$$+4\int_0^{c}\frac{1}{A}H\cos\alpha\,dx+2\int_c^{\frac{l}{2}}\frac{1}{A}(2H\cos\alpha+W\sin\alpha)dx=0$$

上ノ方程式ヨリ

$$-W\frac{l}{2}\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)ds+Wc\int_0^{c'}\frac{1}{I}(y-h_1)ds+W\int_{c'}^{\frac{l'}{2}}\frac{1}{I}x(y-h_1)ds$$

$$+2H\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)^2ds+2H\int_0^{\frac{l}{2}}\frac{1}{A}\cos\alpha\,dx+W\int_c^{\frac{l}{2}}\frac{1}{A}\sin\alpha\,dx=0$$

$$H=\frac{1}{2}\cdot\frac{\frac{l}{2}\int_0^{\frac{l}{2}}\frac{1}{I}(y-h_1)\,ds-c\int_0^{c}\frac{1}{I}(y-h_1)\,ds-\int_c^{\frac{l}{2}}\frac{1}{I}x(y-h_1)\,ds-\int_c^{\frac{l}{2}}\frac{1}{A}\sin\alpha\,dx}{\int_0^{\frac{l}{2}}\frac{1}{I}(y-h_1)^2ds+\int_0^{\frac{l}{2}}\frac{1}{A}\cos\alpha\,dx}W\quad\cdots\cdots\cdots(1)$$

軸應力ノ影響ハ彎曲率ノ影響ニ比シ比較的小ナルヲ以テ之ヲ無視スルトキハ（Aヲ含ム項ヲ省略スレハ宜シ）

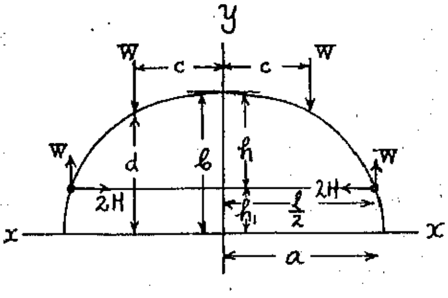
$$H=\frac{1}{2}\cdot\frac{\frac{l}{2}\int_0^{\frac{l}{2}}\frac{1}{I}(y-h_1)\,ds-c\int_0^{c}\frac{1}{I}(y-h_1)\,ds-\int_c^{\frac{l}{2}}\frac{1}{I}x(y-h_1)\,ds}{\int_0^{\frac{l}{2}}\frac{1}{I}(y-h_1)^2ds}W\quad\cdots\cdots\cdots(2)$$

第二章　集荷重ニ對スル水平反力

第五圖ニ於テh及lヲ拱橋ノ拱矢及徑間トシa及bヲ楕圓ノ長軸及短軸トスレハh及lト共ニa或ハbヲ適當ニ假定スルトキハ容易ニ拱ノ形狀ヲ決定シ得ヘシ

楕圓ノ方程式ハ

$$\left.\begin{array}{l}\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1\quad\text{or}\quad y=\dfrac{b}{a}\sqrt{a^2-x^2}\\[2ex] k=\sqrt{\dfrac{a^2-b^2}{a^2}}<1\quad(k=\text{Eccentricity 偏心率})\text{トス}\end{array}\right\}\quad\cdots\cdots\cdots(3)$$



第五圖

h及lト共ニaカ與ヘラレタル場合ニハ

h及l ト共ニbカ與ヘラレタル場合ニハ

$$\left.\begin{aligned} b&=\frac{ah}{a-\sqrt{a^2-\frac{l^2}{4}}}, \qquad h_1=b-h=\frac{h\sqrt{a^2-\frac{l^2}{4}}}{a-\sqrt{a^2-\frac{l^2}{4}}} \\ d&=\frac{b}{a}\sqrt{a^2-c^2} \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots(4)$$

$$\left.\begin{aligned} a&=\frac{bl}{2\sqrt{2bh-h^2}}, \qquad h_1=b-h \\ d&=\frac{b}{a}\sqrt{a^2-c^2} \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots(5)$$

又楕圓積分(Elliptic integral)ヲ避ケ計算ヲ容易ナラシメンカタメ拱ノ斷面積カ拱頂(Crown)ヨリ兩端ニ向ツテ次ノ方程式ニヨリ増大スルモノト假定ス

$$\left.\begin{aligned} I&=I_0\sec\alpha \\ A&=A_0\sec\alpha \end{aligned}\right\} \cdots\cdots\cdots\cdots\cdots(6)$$

茲ニI_0及A_0ハ拱頂ニ於ケル惰率及斷面積ヲ示スモノトス

但$\alpha=60°$ノ時ハ$\sec 60°=2$ナルヲ以テ尚ホ上ノ假定ヲ適用スルコトヲ得レトモ半楕圓拱(Semi-elliptic arch)ノ場合ニハ$\alpha=90°$トナリ$\sec 90°=\infty$トナルヲ以テ上ノ假定ハ之ヲ適用スルコトヲ得サルモノトス

水平反力Hヲ見出サンカタメ算式(1)ノ各項ニ楕圓ノ方程式(3)ヲ導入シ積分ヲ行ヘハ

$$\int_0^{\frac{l'}{2}} \frac{1}{I}(y-h_1)\,ds=\frac{1}{2I_0}\left(ab\sin^{-1}\frac{l}{2a}-\frac{h_1 l}{2}\right)$$

$$\int_0^{c'} \frac{1}{I}(y-h_1)\,ds=\frac{1}{2I_0}\left\{c(d-2h_1)+ab\sin^{-1}\frac{c}{a}\right\}$$

$$\int_{c'}^{\frac{l'}{2}} \frac{1}{I}x(y-h_1)ds=\frac{1}{3I_0}(d-h_1)(a^2-c^2)-\frac{h_1}{6I_0}\left(\frac{l^2}{4}-c^2\right)$$

$$\int_c^{\frac{l}{2}} \frac{1}{A}\sin\alpha\,dx=\frac{1}{A_0k^2}(d-h_1)-\frac{b^2}{A_0ak^3}\left(\tan^{-1}\frac{adk}{b^2}-\tan^{-1}\frac{ah_1k}{b^2}\right)$$

$$\int_0^{\frac{l'}{2}} \frac{1}{I}(y-h_1)^2ds=\frac{b}{I_0}\left\{\frac{bl}{2}\left(1-\frac{l^2}{12a^2}\right)-ah_1\sin^{-1}\frac{l}{2a}\right\}$$

$$\int_0^{\frac{l}{2}} \frac{1}{A}\cos\alpha\,dx=\frac{1}{2A_0k^2}\left(l-\frac{b^2}{ak}\log\frac{2a+kl}{2a-kl}\right)$$

故ニ次ノ結果ヲ得ヘシ

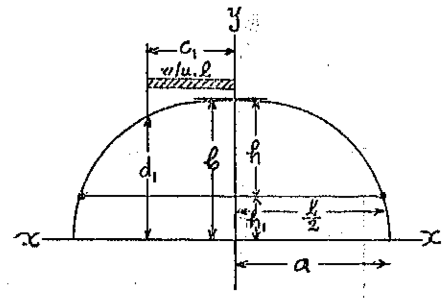
$$H=\frac{\left\{\begin{array}{l}\frac{h_1}{3}\left(a^2+\frac{3c^2}{2}-\frac{l^2}{4}\right)-\frac{d}{3}\left(a^2+\frac{c^2}{2}\right)+\frac{ab}{2}\left(\frac{l}{2}\sin^{-1}\frac{l}{2a}-c\sin^{-1}\frac{c}{a}\right)\\ -\frac{I_0}{A_0k^2}(d-h_1)+\frac{I_0b^2}{A_0ak^3}\left(\tan^{-1}\frac{adk}{b^2}-\tan^{-1}\frac{ah_1k}{b^2}\right)\end{array}\right\}}{b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}+\frac{I_0}{A_0k^2}\left(l-\frac{b^2}{ak}\log\frac{2a+kl}{2a-kl}\right)}W\quad\cdots\quad\cdots\quad\cdots(7)$$

軸應力ノ影響ヲ無視スルトキハ(A_0ヲ含ム項ヲ省略スレハ宜シ)

$$H=\frac{\dfrac{h_1}{3}\left(a^2+\dfrac{3c^2}{2}-\dfrac{l^2}{4}\right)-\dfrac{d}{3}\left(a^2+\dfrac{c^2}{2}\right)+\dfrac{ab}{2}\left(\dfrac{l}{2}\sin^{-1}\dfrac{l}{2a}-c\sin^{-1}\dfrac{c}{a}\right)}{b\left\{bl\left(1-\dfrac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\dfrac{l}{2a}\right\}}W \quad \cdots \quad \cdots \quad \cdots \quad \cdots(8)$$

第三章　等布荷重ニ對スル水平反力

第六圖ニ示スカ如キ等布荷重ニ對スル水平反力ヲ見出スニハ算式(7)ニ於テ　$W=w\,dx$,　$c=x$, $d=\dfrac{b}{a}\sqrt{a^2-x^2}$　ニ置キ代ヘ各項ヲ0ヨリc_1マテ積分スレハ宜シ但シ　$d_1=\dfrac{b}{a}\sqrt{a^2-c_1^2}$　トス



第六圖

$$\int_0^{c_1}\frac{h_1}{3}\left(a^2+\frac{3}{2}x^2-\frac{l^2}{4}\right)w\,dx=\frac{h_1}{3}\left(a^2+\frac{c_1^2}{2}-\frac{l^2}{4}\right)wc_1$$

$$\int_0^{c_1}\frac{d}{3}\left(a^2+\frac{x^2}{2}\right)w\,dx=\frac{1}{24}\left\{d_1\left(\frac{7}{2}a^2+c_1^2\right)+\frac{9a^3b}{2c_1}\sin^{-1}\frac{c_1}{a}\right\}wc_1$$

$$\int_0^{c_1}\frac{ab}{2}\left(\frac{l}{2}\sin^{-1}\frac{l}{2a}-x\sin^{-1}\frac{x}{a}\right)w\,dx=\frac{ab}{4}\left(-\frac{ad_1}{2b}+l\sin^{-1}\frac{l}{2a}+\frac{a^2-2c_1^2}{2c_1}\sin^{-1}\frac{c_1}{a}\right)wc_1$$

$$\int_0^{c_1}\frac{I_0}{A_0k^2}(d-h_1)w\,dx=\frac{I_0}{A_0k^2}\left(\frac{d_1}{2}-h_1+\frac{ab}{2c_1}\sin^{-1}\frac{c_1}{a}\right)wc_1$$

$$\int_0^{c_1}\frac{I_0b^2}{A_0ak^3}\left(\tan^{-1}\frac{adk}{b^2}-\tan^{-1}\frac{ah_1k}{b^2}\right)w\,dx=\frac{I_0b^2}{A_0ak^3}\left(\frac{\pi a}{4c_1k}+\tan^{-1}\frac{ad_1k}{b^2}\right.$$

$$\left.-\tan^{-1}\frac{ah_1k}{b^2}-\frac{b}{c_1k}\sin^{-1}\frac{c_1}{a}-\frac{a}{2c_1k}\sin^{-1}\frac{a^4d_1^2-b^4c_1^2}{a^4d_1^2+b^4c_1^2}\right)wc_1$$

故ニ次ノ結果ヲ得ヘシ

$$H=\frac{\left\{\begin{array}{l}\dfrac{h_1}{3}\left(a^2+\dfrac{c_1^2}{2}-\dfrac{l^2}{4}\right)-\dfrac{d_1^2}{24}\left(\dfrac{a^2}{2}+c_1^2\right)+\dfrac{ab}{4}\left(l\sin^{-1}\dfrac{l}{2a}-\dfrac{a^2+4c_1^2}{4c_1}\sin^{-1}\dfrac{c_1}{a}\right)\\ -\dfrac{I_0}{A_0k^2}\left(\dfrac{d_1}{2}-h_1-\dfrac{\pi b^2}{4c_1k^2}\right)+\dfrac{I_0b^2}{A_0ak^3}\left(\tan^{-1}\dfrac{ad_1k}{b^2}-\tan^{-1}\dfrac{ah_1k}{b^2}\right.\\ \qquad\left.-\dfrac{a^2+b^2}{2bc_1k}\sin^{-1}\dfrac{c_1}{a}-\dfrac{a}{2c_1k}\sin^{-1}\dfrac{a^4d_1^2-b^4c_1^2}{a^4d_1^2+b^4c_1^2}\right)\end{array}\right\}}{b\left\{bl\left(1-\dfrac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\dfrac{l}{2a}\right\}+\dfrac{I_0}{A_0k^2}\left(l-\dfrac{b^2}{ak}\log\dfrac{2a+kl}{2a-kl}\right)}wc_1 \quad \cdots\ \cdots\ \cdots(9)$$

軸應力ノ影響ヲ無視スルトキハ

$$H=\frac{\dfrac{h_1}{3}\left(a^2+\dfrac{c_1^2}{2}-\dfrac{l^2}{4}\right)-\dfrac{d_1}{24}\left(\dfrac{a^2}{2}+c_1^2\right)+\dfrac{ab}{4}\left(l\sin^{-1}\dfrac{l}{2a}-\dfrac{a^2+4c_1^2}{4c_1}\sin^{-1}\dfrac{c_1}{a}\right)}{b\left\{bl\left(1-\dfrac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\dfrac{l}{2a}\right\}}wc_1 \quad \cdots\ \cdots\ \cdots(10)$$

等布荷重カ第七圖ノ如ク乘リタル場合ニハ先 c_2 マテ全部等布荷重カ乘レルモノト假定シテ其レニ對スル水平反力ヲ求メ次ニ c_1 マテ等布荷重カ乘リシトキノ水平反力ヲ求メ前者ヨリ後者ヲ減スレハ之即求ムル所ノ水平反力ナリ

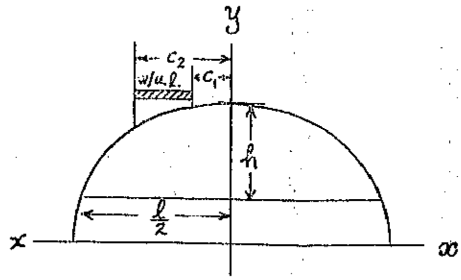
第四章　熱應力(Temperature stress)

雙鉸拱ハ溫度ノ昇降ニ依ツテ應力ヲ受クルモノニシテ次ニ之カ計算法ヲ示サントス

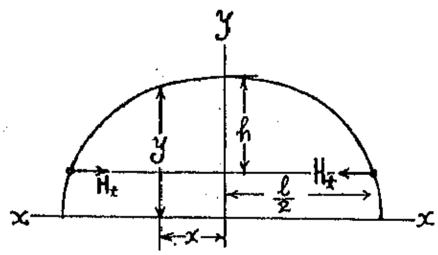
θ＝伸縮率 (Coefficient of expansion and contraction)

t＝度數ニ於テ表ハサレタル溫度ノ變化

H_t＝溫度ノ變化ノタメニ起レル水平反力



第七圖



第八圖

但溫度昇リシトキヲ正トシ溫度降リシトキヲ負トス

Bending moment　$M=-H_t(y-h_1)$,　$\dfrac{dM}{dH_t}=-(y-h_1)$

Axial stress　$N=-H_t\cos\alpha$,　$\dfrac{dN}{dH_t}=-\cos\alpha$

應剪力ノ影響ヲ無視スルトキハ拱ノ内働ノ總和ハ次ノ如シ

$$\omega=2\int_0^{\frac{l}{2}}\frac{M^2}{2EI}ds+2\int_0^{\frac{l}{2}}\frac{N^2}{2EA}ds$$

若シ拱橋ノ支承カ自由ニ移動シ得ルモノト假定セハ溫度カt度丈ケ昇降スルニ從ツテ徑間長ニモ$\pm t\theta l$丈ケノ變化ヲ生スヘキ理ナリ然シ拱橋ノ支承カ溫度ノ變化ニ係ラス何等移動セサルモノナルトキハ Castigliano ノ第一定理ノ應用ニヨリ次ノ關係式ヲ得ヘシ

$$\pm t\theta l=\frac{d\omega}{dH_t}=2\int_0^{\frac{l}{2}}\frac{M}{EI}\left(\frac{dM}{dH_t}\right)ds+2\int_0^{\frac{l}{2}}\frac{N}{EA}\left(\frac{dN}{dH_t}\right)ds$$

$$=2H\int_0^{\frac{l}{2}}\frac{1}{EI}(y-h_1)^2ds+2H_t\int_0^{\frac{l}{2}}\frac{1}{EA}\cos\alpha\,dx$$

$$\therefore\quad H_t=\pm\frac{1}{2}\cdot\frac{t\theta lE}{\int_0^{\frac{l}{2}}\frac{1}{I}(y-h_1)^2ds+\int_0^{\frac{l}{2}}\frac{1}{A}\cos\alpha\,dx}\quad\cdots\quad\cdots\quad\cdots\quad\cdots(11)$$

$$H_t=\pm\frac{t\theta lE}{b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}+\frac{I_0}{A_0l^2}\left(l-\frac{b^2}{ak}\log\frac{2a+kl}{2a-kl}\right)}\quad\cdots\quad\cdots(12)$$

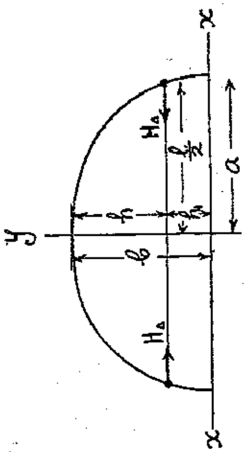
軸應力ノ影響ヲ無視スルトキハ

$$H_t=\pm\frac{t\theta lE}{b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(13)$$

溫度昇リシトキハ正號ヲ用ヒ溫度降リシトキハ負號ヲ用フ

第五章　支承ノ移動ニ歸因スル應力 (Stress due to displacement of support)

若シ或ル原因ノタメ基礎地盤ニ弛ヲ生シ支承ニ移動ヲ起ストキハ從ツテ徑間長ニモ變化ヲ生シ拱ハ應力ヲ受クルモノニシテ恰モ溫度ノ變化ヲ受クルト同様ノ影響ヲ蒙ムルモノナリ



第九圖

Δl＝拱ノ徑間長ノ變化

但徑間長ノ減セシトキヲ正トシ徑間長ノ增セシトキヲ負トス

H_Δ＝徑間長ノ變化ノタメニ起レル水平反力

Bending moment　$M=-H_\Delta(y-h_1)$,　$\frac{dM}{dH_\Delta}=-(y-h_1)$

Axial stress　$N=-H_\Delta\cos\alpha$,　$\frac{dN}{dH_\Delta}=-\cos\alpha$

應剪力ノ影響ヲ無視スルトキハ

$$\omega=2\int_0^{\frac{l}{2}}\frac{M^2}{2EI}ds+2\int_0^{\frac{l}{2}}\frac{N^2}{2EA}ds$$

Castigliano ノ第一定理ニヨリ

$$\pm\Delta l=\frac{d\omega}{dH_\Delta}=2\int_0^{\frac{l'}{2}}\frac{M}{EI}\left(\frac{dM}{dH_\Delta}\right)ds+2\int_0^{\frac{l'}{2}}\frac{N}{EA}\left(\frac{dN}{dH_\Delta}\right)ds$$

$$=2H_\Delta\int_0^{\frac{l'}{2}}\frac{1}{EI}(y-h_1)^2ds+2H_\Delta\int_0^{\frac{l}{2}}\frac{1}{EA}\cos\alpha\,dx$$

$$\therefore\quad H_\Delta=\pm\frac{1}{2}\cdot\frac{E\Delta l}{\int_0^{\frac{l'}{2}}\frac{1}{I}(y-h_1)^2ds+\int_0^{\frac{l}{2}}\frac{1}{A}\cos\alpha\,dx}\quad\cdots\cdots(14)$$

$$=\pm\frac{E\Delta l}{b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}+\frac{I_0}{A_0k^2}\left(l-\frac{b^2}{ak}\log\frac{2a+kl}{2a-kl}\right)}\quad\cdots\cdots(15)$$

軸應力ノ影響ヲ無視スルトキハ

$$H_\Delta=\pm\frac{E\Delta l}{b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}}\quad\cdots\cdots(16)$$

正號ハ徑間長減セシ時ニ適用シ負號ハ之ニ反スル時適用ス

雙鉸拱ハ支承ノ高サニ變化ヲ生スルモ其ノ影響頗小ナルヲ以テ普通考フルニ及ハス

第六章　拱頂ニ於ケル撓度 (Deflection at crown)

撓度ニ對スル軸應力ノ影響ハ比較的小ナルヲ以テ以下ノ計算ニ於テ之ヲ無視スルモノトス

今與ヘラレタル集荷重Wト共ニ補助荷重(Auxiliary load)Pヲ拱頂ニ掛ケ拱ノ內働ノ總和ヲ求メ之ヲPニ付テ微分シ最後ニ$P=0$ニ置ケハCastiglianoノ第一定理ニヨリ拱頂ニ於ケル撓度ヲ見出シ得ヘシ第十圖ニ於テHヲ一個ノWノタメニ起レル水平反力トシH_PヲPノタメニ起レル水平反力トスレハHノ値ハ算式(8)ニヨリ之ヲ見出シ得ヘクH_Pノ値ハ次ノ如シ

034

$$H_P=\frac{\frac{h_1}{3}\left(a^2-\frac{l^2}{4}\right)-ab\left(\frac{a}{3}-\frac{l}{4}\sin^{-1}\frac{l}{2a}\right)}{b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}}P$$

$$K=\frac{\frac{h_1}{3}\left(a^2-\frac{l^2}{4}\right)-ab\left(\frac{a}{3}-\frac{l}{4}\sin^{-1}\frac{l}{2a}\right)}{b\left\{bl\left(1-\frac{l^2}{12a^2}\right)-2ah_1\sin^{-1}\frac{l}{2a}\right\}}\quad\cdots\cdots\cdots(17)$$

トスレハ $H_P=KP$ トナル

拱ノ任意點ニ於ケル彎曲率ハ次ノ如シ

$$\text{for}\quad x<c\qquad M=W\left(\frac{l}{2}-c\right)+\frac{P}{2}\left(\frac{l}{2}-x\right)-(2H+KP)(y-h_1)$$

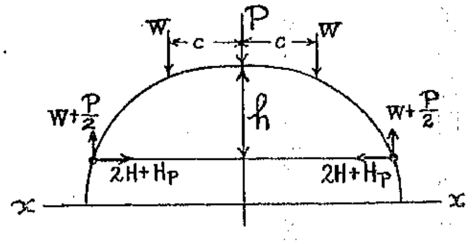
$$\frac{dM}{dP}=\frac{1}{2}\left(\frac{l}{2}-x\right)-K(y-h_1)$$

$$\text{for}\quad x>c\qquad M=W\left(\frac{l}{2}-x\right)+\frac{P}{2}\left(\frac{l}{2}-x\right)-(2H+KP)(y-h_1)$$

$$\frac{dM}{dP}=\frac{1}{2}\left(\frac{l}{2}-x\right)-K(y-h_1)$$

應剪力及軸應力ヲ無視スルトキハ拱ノ內働ノ總和ハ次ノ如シ

$$\omega=2\int_0^{c'}\frac{M^2}{2EI}ds+2\int_{c'}^{\frac{l}{2}}\frac{M^2}{2EI}ds$$



第十圖

Castigliano ノ第一定理ニヨリ一個ノ W ニ對スル拱頂ニ於ケル撓度ハ次ノ如シ

$$\delta=\frac{1}{2}\cdot\frac{d\omega}{dP}=\int_0^{c'}\frac{M}{EI}\left(\frac{dM}{dP}\right)ds+\int_{c'}^{\frac{l'}{2}}\frac{M}{EI}\left(\frac{dM}{dP}\right)ds$$

$$=\int_0^{c'}\frac{1}{EI}\left\{W\left(\frac{l}{2}-c\right)-2H(y-h_1)\right\}\left\{\frac{1}{2}\left(\frac{l}{2}-x\right)-K(y-h_1)\right\}ds$$

$$+\int_{c'}^{\frac{l'}{2}}\frac{1}{EI}\left\{W\left(\frac{l}{2}-x\right)-2H(y-h_1)\right\}\left\{\frac{1}{2}\left(\frac{l}{2}-x\right)-K(y-h_1)\right\}ds$$

上ノ式ヨリ

$$\delta=-2H\int_0^{\frac{l'}{2}}\frac{1}{EI}(y-h_1)\left\{\frac{1}{2}\left(\frac{l}{2}-x\right)-K(y-h_1)\right\}ds+W\frac{l}{2}\int_0^{\frac{l'}{2}}\frac{1}{EI}\left\{\frac{1}{2}\left(\frac{l}{2}-x\right)-K(y-h_1)\right\}ds$$

$$-Wc\int_0^{c'}\frac{1}{EI}\left\{\frac{1}{2}\left(\frac{l}{2}-x\right)-K(y-h_1)\right\}ds-W\int_{c'}^{\frac{l'}{2}}\frac{1}{EI}\left\{\frac{1}{2}\left(\frac{l}{2}-x\right)-K(y-h_1)\right\}ds \quad \cdots(18)$$

上ノ式ヨリ次ノ結果ヲ得ヘシ

$$\delta=\frac{2H}{EI_0}\left\{\frac{Kb^2l}{2}\left(1-\frac{l^2}{12a^2}\right)+\frac{1}{6}\left(a^2h+\frac{h_1l^2}{4}\right)-ab\left(Kh_1+\frac{l}{8}\right)\sin^{-1}\frac{l}{2a}\right\}$$

$$+\frac{W}{EI_0}\left\{\frac{1}{2}\left(Kh_1+\frac{l}{4}\right)\left(\frac{l^2}{6}-c^2\right)+\frac{Ka^2}{3}(d-h_1)+\frac{c^2}{6}\left(Kd+\frac{c}{2}\right)+\frac{Kab}{2}\left(c\sin^{-1}\frac{c}{a}-\frac{l}{2}\sin^{-1}\frac{l}{2a}\right)\right\}\cdots(19)$$

茲ニHノ値ハ算式(8)ニヨリテ之ヲ求ムルコトヲ得ヘシ

等布荷重カc_1マテ乘リシトキニハ算式(19)ニ於テ　$W=w\,dx$, $c=x$, $d=\frac{b}{a}\sqrt{a^2-x^2}$ ニ置キ代ヘ各項ヲ0ヨリc_1マテ積分スレハ次ノ結果ヲ得ヘシ

$$\delta=\frac{2H}{EI_0}\left\{\frac{Kb^2l}{2}\left(1-\frac{l^2}{12a^2}\right)+\frac{1}{6}\left(a^2h+\frac{h_1l^2}{4}\right)-ab\left(Kh_1+\frac{l}{8}\right)\sin^{-1}\frac{l}{2a}\right\}$$

$$+\frac{wc_1}{2EI_0}\left[\frac{1}{3}\left(Kh_1+\frac{l}{4}\right)\left(\frac{l^2}{2}-c_1{}^2\right)+\frac{Ka^2}{3}\left(\frac{13}{8}d_1-2h_1\right)+\frac{c_1{}^2}{12}\left(Kd_1+\frac{c_1}{2}\right)\right.$$

$$\left.+\frac{Kab}{2}\left\{\left(c_1+\frac{a^2}{4c_1}\right)\sin^{-1}\frac{c_1}{a}-l\sin^{-1}\frac{l}{2a}\right\}\right]\cdots\quad\cdots(20)$$

玆ニHノ値ハ算式(10)ニヨリテ之ヲ求ムルコトヲ得ヘシ

本論文中ノ諸式ニ於ケル對數ハ總テ Naperian logarithm ヲ示スモノニシテ之等諸式中ニ於ケル對數正弦弧及正切弧等ノ計算ニハ Chambers's Mathematical Tables ヲ用フルヲ便トス(完)