論説

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# 三徑間ニシテ單扶構ヲ有スル吊橋ノ略理論

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以下論スル所ノ吊橋ノ理論ハ Principle of least work ニ依ルモノニシテ廣井先生著 Statically-indeterminate stresses ニ負フ所頗ル大ナリ

## 第一章 吊橋ノ理論ニ關スル假定

(一) 荷重ノ位置ノ如何ニ係ラス各吊材(Hanger)ニ於ケル張力ヲ一様ナリト假定スルコト

各吊材ノ張力ハ實際ニ於テ多少異ルモノナレトモ徑間大ニシテ剛扶構(Rigid stiffening truss)ヲ有スル吊橋ニ於テハ此種假定モ眞ニ近キモノナリ

(二) 死荷重ハ扶構ニ何等應力ヲ與フルコトナク直チニ鍊條(Cable)ニ運ハルヽモノナリト假定スルコト

此ノ假定ハ吊橋ノ構造如何ニ依ルモノニシテ正規狀態 (Normal condition) ト正規温度 (Normal temperature) ニ於テ吊材ヲ適當ニ締メ付ケ活荷重ヲ受ケサル際ハ扶構ノ兩端カ支承ト相接觸スルノミニシテ少シモ反力ナキ如ク吊材ノ長サヲ調整スレハ此ノ假定ハ眞ナリ

(三) 鍊條ニ働ク死荷重ヲ水平毎呎一様ナリト假定スルコト

徑間大ニシテ剛扶構ヲ有シ死荷重大ナル吊橋ニ於テハ鍊條ノ自重ニ比シ扶構及床構等ノ重量

頗ル大ナルヲ以テ此ノ假定モ眞ニ近シ

(四)溫度ノ變化並ニ活荷重ノ爲メニ起ル鍊條ノ撓度ヲ無視スルコト

鍊條ハ溫度ノ變化並ニ活荷重ノ爲メ可ナリ大ナル撓度ヲ起スモノニシテ時トシテハ垂矢(Sag)ノ一〇%以上ニ及フコト稀ナリトセス然レトモ理論ヲ簡單ナラシメンカ爲メ此處ニハ之ヲ無視スルモノトス

是レ所謂略理論タル所以ニシテ吊橋ノ最モ正確ナル理論ヲ研究セントセハ鍊條ノ撓度ヲ考ニ取ラサルヘカラス

(五)扶構ヲ一本ノ桁(Beam)ト假定スルコト

普通扶構ハ高サ低ク徑間大ナルヲ以テ扶構ノ內働(Internal work)ノ計算ニ於テ之ヲ一本ノ桁ト假定スルモ大差ナシ

(六)塔上ニ於ケル摩擦抵抗ヲ無視スルコト

普通塔(Tower)ノ頂上ニハ轉子(Roller)カ設置セラレアルヲ以テ摩擦抵抗ハ非常ニ小ナリ故ニ塔ノ內働計算ニ於テ彎曲率(Bending moment)ヲ無視セルハ此ノ假定ニ依ルモノナリ

(七)鎭碇(Anchorage)ニ於ケル變形ヲ無視スルコト

鎭碇ニ於ケル變形(deformation)ハ比較的小ナルモノナルカ故ニ內働計算ノ際之ヲ無視スルモ大差ナシ

## 第二章　死荷重ニ對スル鍊條ノ形狀ト其ノ水平分力

前章假定(二)及ヒ(三)ニヨリ死荷重ハ扶構ニ何等應力ヲ與フルコトナク吊材ニヨリ直ニ鍊條ニ運ハルヽモノトシ且ツ水平每呎ニ一樣ナリト假定ス

今中央徑間ニ於ケル水平每呎ノ死荷重ヲ$w$トシ側徑間ニ於ケル水平每呎ノ死荷重ヲ$w_1$ト假定ス

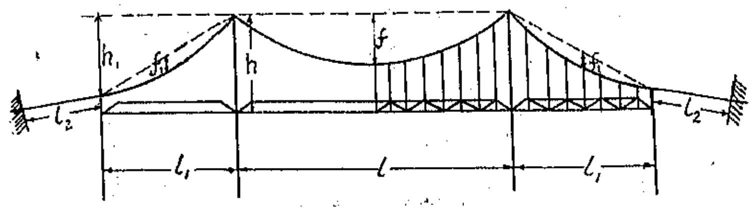
レハ鍊條應力ノ水平分力ハ各部分ニ於テ相等シク且ツ鍊條カ拋線狀ヲナスコトヲ證明セン

今鍊條ノ水平分力ヲ$H$トスレハ前章假定(六)ニヨリ塔上ノ摩擦抵抗ヲ無視スル時ハ塔上ニ於ケル反力垂直ナルヲ以テ鍊條應力ノ水平分力$H$ハ鍊條ノ何レノ部分ニ於テモ相等シキコト明ナリ

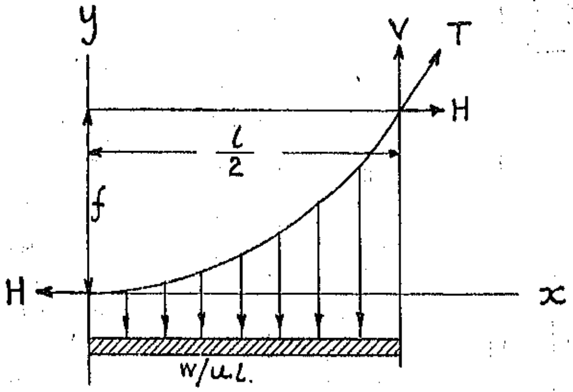
又鍊條カ拋線狀ヲナスコトハ次ノ如クシテ證明スルコトヲ得ヘシ

（第一）中央鍊條 (Center cable)

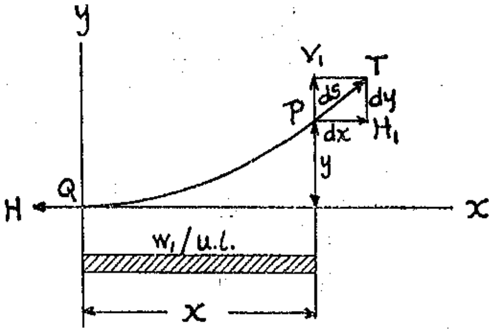
今第二圖ノ右端ニ於テ力率ヲ取レハ




第一圖




第二圖




第三圖

$$Hf-\frac{1}{2}w\left(\frac{l}{2}\right)^2=0$$

$$\therefore\quad H=\frac{wl^2}{8f}\quad\cdots\cdots(1)$$

第三圖ニ於テ$QP$ナル鍊條ノ一部分ヲ考フルニ$V_1=wx$ナルコト明ナリ

又$P$點ノ應力ヲ$H_1$ $V_1$ニ分解スレハ容易ニ次ノ關係式ヲ得ヘシ

$$\frac{dy}{dx}=\frac{V_1}{H_1}=\frac{wx}{H}$$

or $y=\int\frac{wx}{H}dx+C=\frac{w}{2H}x^2+C$

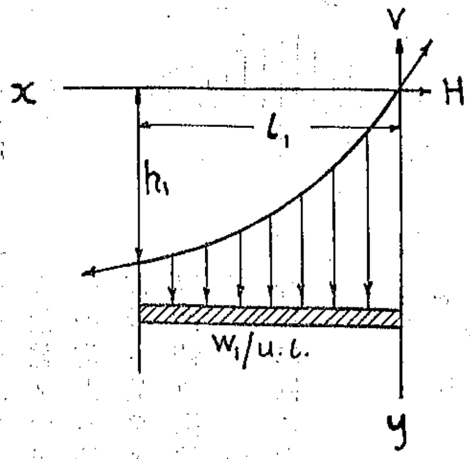
$x=0$ ナルトキ $y=0$ ナルヲ以テ $C=0$ ナリ

$$\therefore\quad y=\frac{w}{2H}x^2=\frac{4f}{l^2}x^2 \quad\cdots\cdots(2)$$

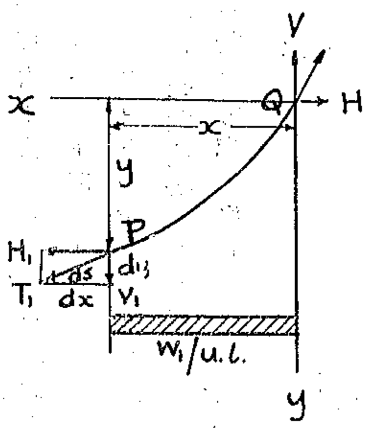
是レ拋線ノ方程式ニシテ鍊條カ拋線狀ヲナスコトヲ知ル

$\frac{dy}{dx}=\frac{8f}{l^2}x$ ナルヲ以テ中央鍊條ノ任意ノ點ニ於ケル張力 $T$ ハ次ノ如クシテ求ムルコトヲ得ヘシ

$$T=H\frac{ds}{dx}=H\frac{\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx}{dx}=H\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}} \quad\cdots\cdots(3)$$




第四圖




第五圖

(第二)　側鍊條 (Side cable)

第四圖左端ニ於テ力率ヲ取レハ

$$Hh_1-Vl_1+\frac{1}{2}w_1l_1^2=0$$

$$\therefore\quad V=\frac{1}{2}w_1l_1+H\frac{h_1}{l_1}$$

第五圖ニ於テ $PQ$ ナル鍊條ノ一部分ヲ考フレハ $H_1=H$ ニシテ $V_1=V-w_1x$ ナルコト明ナリ

又鍊條上ノ任意ノ一點 $P$ ニ

於ケル應力ヲ$H_1$ $V_1$ニ分解スレハ容易ニ次ノ關係式ヲ得ヘシ

$$\frac{dy}{dx}=\frac{V_1}{H_1}=\frac{V-w_1x}{H}=\frac{\frac{1}{2}w_1l_1+H\frac{h_1}{l_1}-w_1x}{H}$$

$$y=\int\left(\frac{\frac{1}{2}w_1l_1+H\frac{h_1}{l_1}-w_1x}{H}\right)dx+C=\frac{1}{H}\left(\frac{1}{2}w_1l_1+H\frac{h_1}{l_1}\right)x-\frac{1}{2H}w_1x^2+C$$

$x=0$ナルトキ$y=0$ナルヲ以テ$C=0$ナリ

又$x=\frac{l_1}{2}$ナルトキ$y=\frac{h_1}{2}+f_1$ナルヲ以テ

$$H=\frac{w_1l_1^2}{8f_1}\quad\cdots\cdots(4)$$

$H$ノ値ヲ$y$ノ式ニ代入スレハ

$$y=\frac{4f_1}{l_1^2}\left(l_1x-x^2\right)+\frac{h_1}{l_1}x\quad\cdots\cdots(5)$$

是レ拋線ノ方程式ニシテ側鏁條モ矢張拋線狀ヲナスコトヲ知ル

$\frac{dy}{dx}=\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x$ナルヲ以テ側鏁條ノ任意ノ點ニ於ケル張力$T_1$ハ次式ノ如シ

$$T_1=H\frac{ds}{dx}=H\frac{\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}{dx}=H\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}\quad\cdots\cdots(6)$$

算式(1)及ヒ(4)ヨリ次ノ關係式ヲ得ヘシ

$$\frac{f_1}{f}=\frac{w_1l_1^2}{wl^2}\quad\cdots\cdots(7)$$

即チ設計ノ初メニ當リ垂矢ノ決定ニ於テ中央徑間及ヒ側徑間ニ於ケル垂矢ハ上ノ關係式ヲ滿足スル如ク之ヲ決定セサルヘカラス

## 第三章　活荷重ニ對スル鏁條ノ形狀ト其ノ水平分力

第一章假定(一)ニヨリ或任意ノ位置ニ置カレタル活荷重ニ對シ鏁條ニ働ク吊材張力(Hanger pull)ヲ水平每呎ニ一樣ナリト假定スレハ前同樣ニ鏁條應力ノ水平分力$H$ハ各部分ニ於テ相等シク鏁條カ常ニ拋線狀ヲナスコトヲ容易ニ證明シ得ヘシ

今$W$ナル任意ノ活荷重ニ對スル水平每呎ノ吊材張力ヲ中央徑間ニテ$p$トシ側徑間ニテ$p_1$トシ鏁條應力ノ水平分力ヲ$H$トシ又中央鏁條及側鏁條ノ張力ヲ$T$及ヒ$T_1$トスレハ第二章同樣次ノ算式ヲ證明シ得ヘシ

$$H=\frac{pl^2}{8f} \quad \text{or} \quad p=\frac{8Hf}{l^2} \qquad \cdots\cdots(8)$$

$$H=\frac{p_1 l_1^2}{8f_1} \quad \text{or} \quad p_1=\frac{8Hf_1}{l_1^2} \qquad \cdots\cdots(9)$$

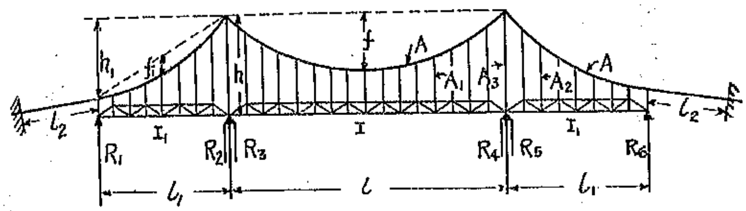
$$T=H\left\{1+\left(\frac{8f}{l^2}\right)^2 x^2\right\}^{\frac{1}{2}} \qquad \cdots\cdots(10)$$

$$T_1=H\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}} \qquad \cdots\cdots(11)$$

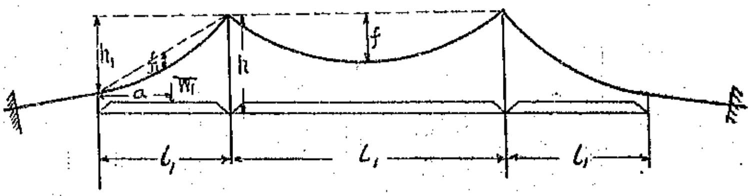
## 第四章　活荷重ヨリ起ル鏁條ノ水平分力

總テ扶構ハ下向反力ニ抗スル爲メ其ノ兩端ニ於テ鎮碇セラルヽモノトス但シ鉸結ニシテ力率ハ起ラサルモノトス

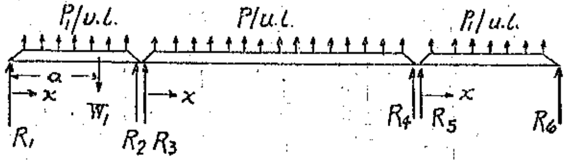
第七圖ニ於テ$W_1$ノ爲メニ起ル吊材ノ張力ヲ水平每呎ニ付キ中央徑間ニテ$p$トシ側徑間ニテ$p_1$ト




第六圖




第七圖




第八圖

スレハ算式(8)(9)ニヨリ

$$p=\frac{8Hf}{l^2}\qquad p_1=\frac{8Hf_1}{l_1^2}$$

(一) 扶構ノ内働ト其ノ微係數

各扶構ノ反力ハ次ノ如シ

$$R_1=\frac{1}{l_1}\left\{W_1(l_1-a)-4Hf_1\right\}\qquad R_2=\frac{1}{l_1}\left\{W_1a-4Hf_1\right\}$$

$$R_3=R_4=-\frac{4}{l}Hf\qquad R_5=R_6=-\frac{4}{l_1}Hf_1$$

左側扶構ノ彎曲率

$x<a$ $$M=\frac{1}{l_1}\left\{W_1(l_1-a)-4Hf_1\right\}x+\frac{4Hf_1}{l_1^2}x^2$$

$$\frac{dM}{dH}=\frac{4f_1}{l_1}\left(-x+\frac{x^2}{l_1}\right)$$

$x>a$ $$M=\frac{1}{l_1}\left\{W_1(l_1-a)-4Hf_1\right\}x+\frac{4Hf_1}{l_1^2}x^2-W_1(x-a)$$

$$\frac{dM}{dH}=\frac{4f_1}{l_1}\left(-x+\frac{x^2}{l_1}\right)$$

中央扶構ノ彎曲率

$$M=-\frac{4Hf}{l}x+\frac{4Hf}{l^2}x^2$$

$$\frac{dM}{dH}=\frac{4f}{l}\left(-x+\frac{x^2}{l}\right)$$

右側扶構ノ彎曲率

$$M=-\frac{4Hf_1}{l_1}x+\frac{4Hf_1}{l_1^2}x^2 \qquad \frac{dM}{dH}=\frac{4f_1}{l_1}\left(-x+\frac{x^2}{l_1}\right)$$

$E$=鋼ノ彈率 (Modulus of elasticity)

$I$=中央扶構ノ惰率 (Moment of inertia)

$I_1$=側扶構ノ惰率

トシ腹材ノ變形ヲ省略スル時ハ扶構ノ總內働ハ次ノ如シ

$$\omega=\int_0^a\frac{M^2}{2EI_1}dx+\int_a^{l_1}\frac{M^2}{2EI_1}dx+\int_0^{l}\frac{M^2}{2EI}dx+\int_0^{l_1}\frac{M^2}{2EI_1}dx$$

$$\frac{d\omega}{dH}=\int_0^a\frac{M}{EI_1}\left(\frac{dM}{dH}\right)dx+\int_a^{l_1}\frac{M}{EI_1}\left(\frac{dM}{dH}\right)dx+\int_0^{l}\frac{M}{EI}\left(\frac{dM}{dH}\right)dx+\int_0^{l_1}\frac{M}{EI_1}\left(\frac{dM}{dH}\right)dx$$

$$=\frac{4f_1}{EI_1l_1}\int_0^{l_1}\left[\frac{1}{l_1}\left\{W_1(l_1-a)-4Hf_1\right\}x+\frac{4Hf_1}{l_1^2}x^2\right]\left(-x+\frac{x^2}{l_1}\right)dx$$

$$-\frac{4W_1f_1}{EI_1l_1}\int_a^{l_1}(x-a)\left(-x+\frac{x^2}{l_1}\right)dx+\frac{16Hf^2}{EIl^2}\int_0^{l}\left(-x+\frac{x^2}{l}\right)^2dx$$

$$+\frac{16Hf_1^2}{EI_1l_1^2}\int_0^{l_1}\left(-x+\frac{x^2}{l_1}\right)^2dx$$

$$=\frac{8H}{15E}\left(\frac{f^2l}{I}+\frac{2f_1^2l_1}{I_1}\right)-\frac{af_1}{3EI_1}\left(l_1-\frac{2a^2}{l_1}+\frac{a^3}{l_1^2}\right)W_1\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(a)$$

(二)鏈條ノ內働ト其ノ微係數

算式(10)及(11)ニヨリ中央鏈條及側鏈條ノ張力$T$及ヒ$T_1$ハ次ノ如シ

$$T=H\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}} \qquad \frac{dT}{dH}=\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}$$

$$M=-\frac{4Hf_1}{l_1}x+\frac{4Hf_1}{l_1^2}x^2 \qquad \frac{dM}{dH}=\frac{4f_1}{l_1}\left(-x+\frac{x_2}{l_1}\right)$$

$E$=鋼ノ彈率 (Modulus of elasticity)

$I$=中央扶構ノ惰率 (Moment of inertia)

$I_1$=側扶構ノ惰率

トシ腹材ノ變形ヲ省略スル時ハ扶構ノ總內働ハ次ノ如シ

$$\omega=\int_0^a\frac{M^2}{2EI_1}dx+\int_a^{l_1}\frac{M^2}{2EI_1}dx+\int_0^l\frac{M^2}{2EI}dx+\int_0^{l_1}\frac{M^2}{2EI_1}dx$$

$$\frac{d\omega}{dH}=\int_0^a\frac{M}{EI_1}\left(\frac{dM}{dH}\right)dx+\int_a^{l_1}\frac{M}{EI_1}\left(\frac{dM}{dH}\right)dx+\int_0^l\frac{M}{EI}\left(\frac{dM}{dH}\right)dx+\int_0^{l_1}\frac{M}{EI_1}\left(\frac{dM}{dH}\right)dx$$

$$=\frac{4f_1}{EI_1l_1}\int_0^{l_1}\left[\frac{1}{l_1}\left\{W_1(l_1-a)-4Hf_1\right\}x+\frac{4Hf_1}{l_1^2}x^2\right]\left(-x+\frac{x^2}{l_1}\right)dx$$

$$-\frac{4W_1f_1}{EI_1l_1}\int_a^{l_1}(x-a)\left(-x+\frac{x^2}{l_1}\right)dx+\frac{16Hf^2}{EIl^2}\int_0^l\left(-x+\frac{x^2}{l}\right)^2dx$$

$$+\frac{16Hf_1^2}{EI_1l_1^2}\int_0^{l_1}\left(-x+\frac{x^2}{l_1}\right)^2dx$$

$$=\frac{8H}{15E}\left(\frac{f^2l}{I}+\frac{2f_1^2l_1}{I_1}\right)-\frac{af_1}{3EI_1}\left(l_1-\frac{2a^2}{l_1}+\frac{a^3}{l_1^2}\right)W_1\cdots\cdots\cdots(a)$$

(二)鍊條ノ內働ト其ノ微係數

算式(10)及(11)ニヨリ中央鍊條及側鍊條ノ張力$T$及ヒ$T_1$ハ次ノ如シ

$$T=H\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}} \qquad \frac{dT}{dH}=\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}$$

$$+l_2\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}\Bigg]$$

$$=\frac{2H}{EA}\left\{\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right\}\quad \text{nearly}\quad \cdots\cdots\cdots\cdots\cdots\cdots (b)$$

但シ $\frac{d\omega}{dH}$ ノ眞値ハ次ノ如キモノナレトモ(b)式ノ與フル略値ト大差ナキヲ以テ(b)式ヲ採用ス

$$\frac{d\omega}{dH}=\frac{2H}{EA}\Bigg[l\left(1+\frac{16f^2}{l^2}\right)^{\frac{1}{2}}\left(\frac{5}{16}+\frac{2f^2}{l^2}\right)+\frac{3}{64}\frac{l^2}{f}\log_e\left\{\left(1+\frac{16f^2}{l^2}\right)^{\frac{1}{2}}+\frac{4f}{l}\right\}\Bigg]$$

$$+\frac{2H}{EA}\Bigg[-\frac{l_1(h_1-4f_1)}{64f_1}\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}\left\{5+2\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}$$

$$+\frac{l_1(h_1+4f_1)}{64f_1}\left\{1+\left(\frac{h_1+4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}\left\{5+2\left(\frac{h_1+4f_1}{l_1}\right)^2\right\}$$

$$+\frac{3l_1^2}{64f_1}\log_e\frac{\left\{1+\left(\frac{h_1+4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}+\frac{h_1+4f_1}{l_1}}{\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}+\frac{h_1-4f_1}{l_1}}\Bigg]$$

$$+\frac{2H}{EA}\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}l_2$$

(三)吊材ノ內働ト其ノ微係數

(A)中央徑間ノ吊材

$n$ヲ中央扶構ノ構格數(Panel number)トスレハ $l/n$ ハ格間長(Panel length)トナル

$$y_1+y_2+\cdots+y_{\frac{n}{2}-1}=\frac{4f}{l^2}\left(\frac{l}{n}\right)^2+\frac{4f}{l^2}\left(2\frac{l}{n}\right)^2+\cdots\cdots+\frac{4f}{l^2}\left\{\left(\frac{n}{2}-1\right)\frac{l}{n}\right\}^2$$

$$=\frac{4f}{l^2}\left(\frac{l}{n}\right)^2\left\{1^2+2^2+\cdots\cdots+\left(\frac{n}{2}-1\right)^2\right\}$$

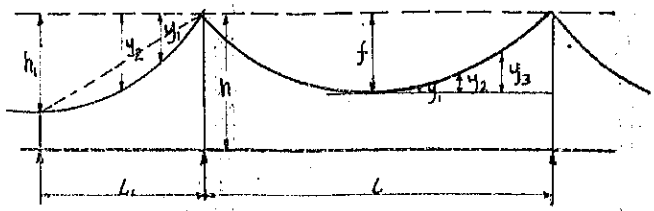
$$=\frac{1}{3}\frac{f}{n}\left(\frac{n}{2}-1\right)(n-1)$$

中央徑間ニ於ケル吊材ノ全長ヲ$L_1$トスレハ

$$L_1=(h-f)(n-1)+\frac{2}{3}\frac{f}{n}\left(\frac{n}{2}-1\right)\left(n-1\right)=n\left(h-\frac{2}{3}f\right) \quad \text{nearly}$$

今$S_1$ヲ中央徑間ニ於ケル各吊材ノ張力トスレハ

$$S_1=p\left(\frac{l}{n}\right)=\frac{8\,Hf}{nl} \qquad \frac{dS_1}{dH}=\frac{8f}{nl}$$




第九圖

(B) 側徑間ノ吊材

$n_1$ヲ側扶構ノ構格數トスレハ$\frac{l_1}{n_1}$ハ其ノ格間長トナル

$$y_1+y_2+\cdots\cdots+y_{n_1}=\frac{4f_1}{l_1^2}l_1\,\frac{l_1}{n_1}(1+2+\cdots\cdots+n_1)$$

$$-\frac{4f_1}{l_1^2}\left(\frac{l_1}{n_1}\right)^2\left\{1^2+2^2+\cdots\cdots\cdots+n_1^2\right\}$$

$$+\frac{h_1}{l_1}\,\frac{l_1}{n_1}(1+2+\cdots\cdots\cdots+n_1)$$

$$=(n_1+1)\left\{\frac{h_1}{2}+\frac{2}{3}f_1\left(1-\frac{1}{n_1}\right)\right\}=(n_1+1)\left(\frac{h_1}{2}+\frac{2}{3}f_1\right) \quad \text{nearly}$$

側徑間ニ於ケル吊材ノ全長ヲ$L_2$トスレハ

$$L_2=hn_1-(n_1+1)\left(\frac{h_1}{2}+\frac{2}{3}f_1\right)=n_1\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right) \quad \text{nearly}$$

今$S_2$ヲ側徑間ニ於ケル各吊材ノ張力トスレハ

$$S_2=p_1\left(\frac{l_1}{n_1}\right)=\frac{8\,Hf_1}{n_1 l_1}\qquad \frac{dS_2}{dH}=\frac{8f_1}{n_1 l_1}$$

今$A_1$及ヒ$A_2$ヲ中央徑間及側徑間ニ於ケル各吊材ノ斷面積トスレハ全吊材ニ於ケル内働ノ總和ハ次ノ如シ

$$\omega=\frac{S_1^2L_1}{2EA_1}+2\frac{S_2^2L_2}{2EA_2}$$

$$\frac{d\omega}{dH}=\frac{S_1L_1}{EA_1}\left(\frac{dS_1}{dH}\right)+2\frac{S_2L_2}{EA_2}\left(\frac{dS_2}{dH}\right)$$

$$=64\frac{H}{E}\left\{\frac{f^2}{A_1nl^2}\left(h-\frac{2}{3}f\right)+\frac{2f_1^2}{A_2n_1l_1^2}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)\right\}\quad\cdots\cdots\cdots\cdots\cdots\cdots(c)$$

(四)　塔ノ内働ト其ノ微係數

中央鐐條及ヒ側鐐條ノ方程式ハ算式(2)及ヒ(5)ニヨリ

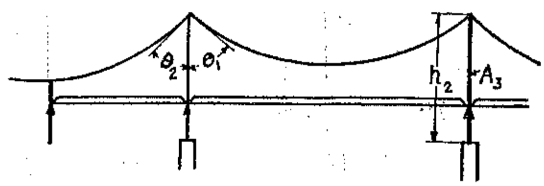
$$y=\frac{4f}{l^2}x^2\qquad \text{center cable}$$

$$y=\frac{4f_1}{l_1^2}\left(l_1x-x^2\right)+\frac{h_1}{l_1}x\qquad \text{side cable}$$

$$\frac{dy}{dx}=\frac{8f}{l^2}x\qquad \cot\theta_1=\frac{4f}{l}$$

$$\frac{dy}{dx}=\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\qquad \cot\theta_2=\frac{h_1+4f_1}{l_1}$$

塔上ニ於ケル反力垂直ナルヲ以テ塔ノ受クル全應力$S$ハ次ノ如シ




第十圖

$$S=H\cot\theta_1+H\cot\theta_2=H\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)\qquad \frac{dS}{dH}=\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)$$

第一章假定(六)ニヨリ塔上ノ摩擦抵抗ヲ無視スル時ハ塔ニ於ケル內働ノ總和ハ$h_2$ヲ塔ノ高サトシ$A_3$ヲ塔ノ平均斷面積トスレハ次ノ如シ

$$\omega=2\frac{S^2h_2}{2EA_3}\qquad \frac{d\omega}{dH}=2\frac{Sh_2}{EA_3}\left(\frac{dS}{dH}\right)$$

$$\frac{d\omega}{dH}=\frac{2H}{EA_3}\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)^2h_2\quad\cdots\cdots(d)$$

(五) 鍊條ノ水平分力$H$ノ決定

Castigliano ノ第二定理ニヨリ$H$ハ構造物ニ於ケル內働ノ總和ヲ最小ナラシムルモノナラサルヘカラス

$$\therefore\ \frac{d\omega}{dH}=\text{Equation }(a)+\text{Equation }(b)+\text{Equation }(c)+\text{Equation }(d)=0$$

上ノ方程式ヨリ次ノ結果ヲ得ヘシ

$$H=\frac{\dfrac{af_1}{3l_1}\left(l_1-\dfrac{2a^2}{l_1}+\dfrac{a^3}{l_1^2}\right)W_1}{\left[\begin{array}{l}\dfrac{8}{15}\left(\dfrac{f^2l}{I}+\dfrac{2f_1^2l_1}{I_1}\right)+\dfrac{2}{A}\left(\dfrac{l}{2}+l_1+l_2+l_3+\dfrac{4f^2}{l}+\dfrac{3h_1^2+16f_1^2}{2l_1}\right)\\+64\left\{\dfrac{f^2}{A_1nl^2}\left(h-\dfrac{2}{3}f\right)+\dfrac{2f_1^2}{A_2n_1l_1^2}\left(h-\dfrac{h_1}{2}-\dfrac{2}{3}f_1\right)\right\}\\+\dfrac{2}{A_3}\left(\dfrac{4f}{l}+\dfrac{h_1+4f_1}{l_1}\right)^2h_2\end{array}\right]}\quad\cdots\cdots(12)$$

吊材及塔ノ影響ハ頗ル小ナルヲ以テ之ヲ省略スル時ハ($A_1$ $A_2$ $A_3$ヲ含ム項ヲ省略スレハ宜シ)

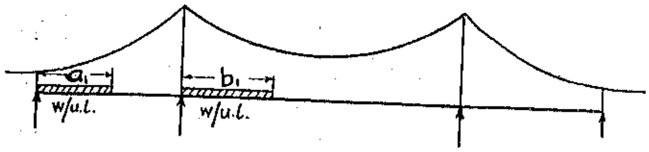
$$H=\frac{\frac{af_1}{3I_1}\left(l_1-\frac{2a^2}{l_1}+\frac{a^3}{l_1^2}\right)W_1}{\frac{8}{15}\left(\frac{f^2l}{I}+\frac{2f_1^2l_1}{I_1}\right)+\frac{2}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3l_1^2+16f_1^2}{2l_1}\right)}\quad\cdots\cdots\cdots(13)$$

$w$ナル等布荷重カ$a_1$マテ乘リシ場合ニハ算式(12)及ヒ(13)ノ分子ニ於テ$W_1$ノ代リニ$wdx$ヲ入レ$a$ノ代リニ$x$ヲ置キ0ヨリ$a_1$マヲ積分スレハ宜シ

$$H=\frac{\frac{a_1^2f_1}{3I_1}\left(\frac{l_1}{2}-\frac{a_1^2}{2l_1}+\frac{a_1^3}{5l_1^2}\right)w}{\text{Denominator eq. (12) or (13)}}\quad\cdots\cdots\cdots(14)$$

左側徑間カ全部等布荷重ヲ受ケタル場合ハ次ノ如シ

$$H=\frac{\frac{f_1l_1^3}{15I_1}w}{\text{Denominator eq. (12) or (13)}}\quad\cdots\cdots\cdots(15)$$



第十一圖

同樣ニシテ$W_2$ナル荷重ニ對スル鍊條ノ水平分力$H$ヲ求ムレハ次ノ如シ

$$H=\frac{\frac{bf}{3I}\left(l-\frac{2b^2}{l}+\frac{b^3}{l^2}\right)W_2}{\text{Denominator eq. (12) or (13)}}\quad\cdots\cdots\cdots(16)$$

$w$ナル等布荷重カ$b_1$マテ乘リシトキニハ

$$H=\frac{\frac{b_1^2f}{3I}\left(\frac{l}{2}-\frac{b_1^2}{2l}+\frac{b_1^3}{5l^2}\right)w}{\text{Denominator eq. (12) or (13)}}\quad\cdots\cdots\cdots(17)$$

中央徑間カ全部等布荷重ヲ受ケタルトキハ

$$H=\frac{\frac{fl^2}{15I}w}{\text{Denominator eq. (12) or (13)}} \quad \cdots\cdots\cdots\cdots\cdots\cdots(18)$$

## 第五章　各部材ノ最大應力

### (一) 鍊條ノ水平分力ノ最大ナル場合

死荷重ヨリ起ル鍊條ノ水平分力 $H_w$ ハ算式(1)或ハ(4)ニヨリ容易ニ之ヲ求ムルコトヲ得ヘシ

$$H_w=\frac{wl^2}{8f} \quad \text{or} \quad H_w=\frac{wl_1^2}{8f_1}$$

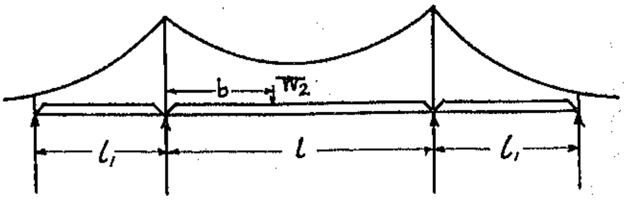
活荷重ノ爲メニ起ル鍊條ノ水平分力 $H_p$ ノ最大トナル場合ハ $H_p$ ノ感線 (influence line) ヲ畫ケハ明ニシテ其ノ大體ノ形狀ハ第十三圖ノ如シ

又死荷重及ヒ活荷重ヨリ起ル鍊條ノ張力ハ算式(3)(6)(10)(11)ニヨリ

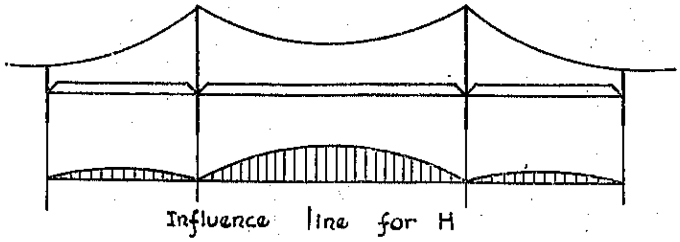
$$T=(H_w+H_p)\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}} \quad \text{center cable}$$

$$T_1=(H_w+H_p)\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}} \quad \text{side cable}$$

上ノ式ヨリ鍊條ノ張力ハ塔ノ頂上ニ於テ最大ナルコト明ニシテ其ノ値ハ次ノ如シ




第十二圖




第十三圖

$$T_{max}=(H_w+H_p)\left\{1+\left(\frac{4f}{l}\right)^2\right\}^{\frac{1}{2}}$$ center cable

$$T_{1max}=(H_w+H_p)\left\{1+\left(\frac{h_1+4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}$$ side cable

(二)吊材ニ於ケル最大ナル張力

$w$ or $w_1$=死荷重ヨリ起ル中央徑間及側徑間ニ於ケル水平每呎ノ吊材張力

$p$ or $p_1$=活荷重ヨリ起ル中央徑間及側徑間ニ於ケル最大ナル水平每呎ノ吊材張力

$\frac{l}{n}$ or $\frac{l_1}{n_1}$=中央扶構及側扶構ノ格間長

トス吊材ノ受クル最大ナル張力ハ次式ノ如シ

$$(w+p)\frac{l}{n}=\left(w+\frac{8H_pf}{l^2}\right)\frac{l}{n}$$ center span hanger

$$(w_1+p_1)\frac{l_1}{n_1}=\left(w_1+\frac{8H_pf_1}{l_1^2}\right)\frac{l_1}{n_1}$$ side span hanger

(三)側扶構ガ最大ナル應力ヲ受クル場合

一般ニ扶構ハ死荷重ヨリ少シモ應力ヲ受ケサルモノトス

側扶構ハ荷重ノ位置ノ如何ニヨリ ±Moment 及ヒ ±Shear ヲ受クルモノニシテ其ノ感線ノ大體ノ形狀ハ第十四圖及第十五圖ノ如シ

又側扶構ノ任意點 $x$ ニ於ケル Bending moment 及ヒ Shear ハ次ノ如シ

Bending moment at any point $x$

due to $W_1$

$$M=\frac{1}{l_1}\left\{W_1(l_1-a)-4Hf_1\right\}x+\frac{4Hf_1}{l_1^2}x^2 \qquad \text{for } x<a$$

$$M=\frac{1}{l_1}\left\{W_1(l_1-a)-4Hf_1\right\}x+\frac{4Hf_1}{l_1^2}x^2-W_1(x-a) \qquad \text{for } x>a$$

due to $W_2$ $\quad M=-\frac{4Hf_1}{l_1}x+\frac{4Hf_1}{l_1^2}x^2$

due to $W_3$ $\quad M=-\frac{4Hf_1}{l_1}x+\frac{4Hf_1}{l_1^2}x^2$

Shear at any point $x$

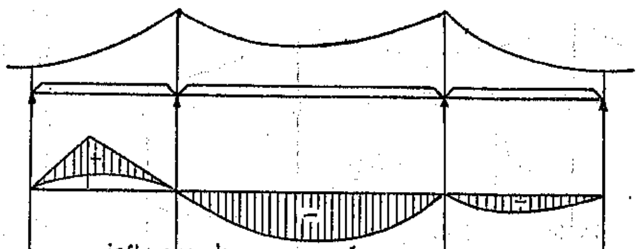
due to $W_1$

$$S=\frac{1}{l_1}\left\{W_1(l_1-a)-4Hf_1\right\}+\frac{8Hf_1}{l_1^2}x \qquad \text{for } x<a$$

$$S=\frac{1}{l_1}\left\{W_1(l_1-a)-4Hf_1\right\}+\frac{8Hf_1}{l_1^2}x-W_1 \qquad \text{for } x>a$$

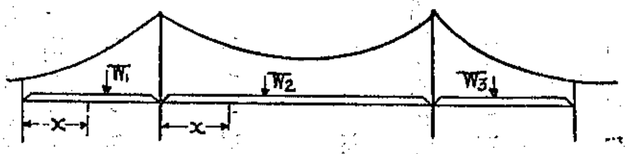
due to $W_2$ $\quad S=-\frac{4Hf_1}{l_1}+\frac{8Hf_1}{l_1^2}x$

due to $W_3$ $\quad S=-\frac{4Hf_1}{l_1}+\frac{8Hf_1}{l_1^2}x$




第十四圖

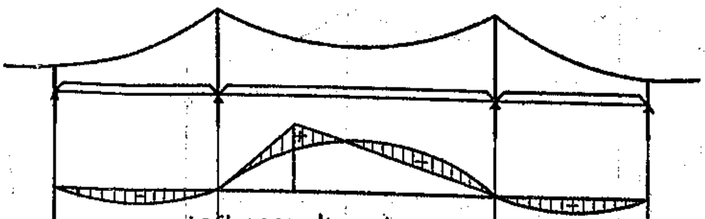

第十五圖




第十六圖

(四)中央扶構カ最大ナル應力ヲ受クル場合

中央扶構モ荷重ノ位置ノ如何ニヨリ＋Moment及ヒ＋Shearヲ受クルモノニシテ其ノ感線ハ第十七圖及ヒ第十八圖ニ示スカ如シ




第十七圖


第十八圖

又中央扶構ノ任意點$x$ニ於ケルBending moment及ヒShearハ次ノ如シ

Bending moment at any point $x$

due to $W_1$　$M=-\frac{4Hf}{l}x+\frac{4Hf}{l^2}x^2$

due to $W_2$　$\begin{cases} M=\frac{1}{l}\{W_2(l-b)-4Hf\}x+\frac{4Hf}{l^2}x^2 & \text{for } x<b \\ M=\frac{1}{l}\{W_2(l-b)-4Hf\}x+\frac{4Hf}{l^2}x^2-W_2(x-b) & \text{for } x>b \end{cases}$

due to $W_3$　$M=-\frac{4Hf}{l}x+\frac{4Hf}{l^2}x^2$

Shear at any point $x$

due to $W_1$　$S=-\frac{4Hf}{l}+\frac{8Hf}{l^2}x$

due to $W_2$　$\begin{cases} S=\frac{1}{l}\{W_2(l-b)-4Hf\}+\frac{8Hf}{l^2}x & \text{for } x<b \\ S=\frac{1}{l}\{W_2(l-b)-4Hf\}+\frac{8Hf}{l^2}x-W_2 & \text{for } x>b \end{cases}$

due to $W_3$　$S=-\frac{4Hf}{l}+\frac{8Hf}{l^2}x$

温度ノ變化ヨリ起ル熱應力 (Temperature stress) ハ扶構ノ剛性 (Stiffness) ニ基因スルモノニシテ次ノ二ノ解法ニ就テ述ヘントス

(A) 第一解法

$ds$ ヲ極小ナル曲線ノ長トスレハ

$$ds=\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}dx \quad \text{for center cable}$$

$$ds=\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\left\{1+\left(\frac{l_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}dx \quad \text{for side cable}$$

今鍊條ノ全長ヲ $L$ トスレハ

$$L=2l_2+2\int_0^{\frac{l}{2}}\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}dx+2\int_0^{l_1}\left\{1+\left(\frac{l_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}dx$$

$$=l+2l_1+2l_2+\frac{8f^2}{3l}+\frac{3l_1^2+16f_1^2}{3l_1} \quad \text{nearly}$$

温度カ正規温度 (Normal temperature) ヨリ $t°$ 丈ケ昇降スル時 $\theta$ ヲ伸率トシ $\Delta f$ $\Delta f_1$ ヲ垂矢ノ變化トスレハ

$$L'=\left(l+2l_1+2l_2+\frac{8f^2}{3l}+\frac{3l_1^2+16f_1^2}{3l_1}\right)(1\pm\theta t)$$

$$=l+2l_1+2l_2+\frac{8(f\pm\Delta f)^2}{3l}+\frac{3l_1^2+16(f_1\pm\Delta f_1)^2}{3l_1} \quad \cdots\cdots\cdots(a)$$

温度ノ昇降ニ從ッテ鍊條及ヒ扶構ハ共ニ下向或ハ上向ノ撓度ヲ起スモノニシテ其ノ撓度ノ大サハ兩者相等シカラスト雖モ其ノ差頗ル小ナルヲ以テ是等ヲ相等シト假定シ且ツ徑間長ニ變化ナ

シト假定スル時ハ容易ニ次ノ關係式ヲ得ヘシ

今溫度ノ變化ヨリ起ル扶構ノ撓度ヲ$\varDelta f$及ヒ$\varDelta f_1$トシ吊材ノ張力ヲ水平每呎ニ$p_1$及ヒ$p_1'$トスレハ撓度ノ理論ニヨリ

$$\varDelta f=\frac{5p_1l^4}{384EI} \quad \text{or} \quad p_1=\frac{384EI}{5l^4}\varDelta f \cdots\cdots\cdots\cdots (b)$$

$$\varDelta f_1=\frac{5p_1'l_1^4}{384EI_1} \quad \text{or} \quad p_1'=\frac{384EI_1}{5l_1^4}\varDelta f_1 \cdots\cdots\cdots\cdots (c)$$

又算式(7)ニヨリ

$$\frac{f_1\pm\varDelta f_1}{f\pm\varDelta f}=\frac{p_1'l_1^2}{p_1l^2}=\frac{p_1'l_1^2}{\frac{384EI}{5l^4}\varDelta f l^2}$$

$$\therefore \quad p_1'=\frac{f_1\pm\varDelta f_1}{f\pm\varDelta f}\times\frac{384EI}{5l^2l_1^2}\varDelta f=\frac{384f_1EI}{5fl^2l_1^2}\varDelta f \quad \text{nearly} \cdots\cdots\cdots\cdots (d)$$

(c)(d)二式ヨリ

$$\varDelta f_1=\frac{f_1l_1^2I}{fl^2I_1}\varDelta f \cdots\cdots\cdots\cdots (e)$$

(e)式ニ於ケル$\varDelta f_1$ノ値ヲ(a)式ニ入ルヽ時ハ次ノ式ヲ得ヘシ

$$\pm\left(l+2l_1+2l_2+\frac{8f^2}{3l}+\frac{3h_1^2+16f_1^2}{3l_1}\right)\theta t=\frac{8}{3l}\left\{\pm2f\varDelta f+(\varDelta f)^2\right\}+\frac{16}{3l_1}\left\{\pm2f_1\varDelta f_1+(\varDelta f_1)^2\right\}$$

$(\varDelta f)^2$及$(\varDelta f_1)^2$ナル項ヲ省略スルトキハ

$$\left(l+2l_1+2l_2+\frac{8f^2}{3l}+\frac{3h_1^2+16f_1^2}{3l_1}\right)\theta t=\frac{8}{3l}2f\varDelta f+\frac{16}{3l_1}2f_1\left(\frac{f_1l_1^2I}{fl^2I}\right)\varDelta f \quad \text{nearly}$$

$$\therefore\quad \Delta f=\frac{\left(l+2l_1+2l_2+\dfrac{8f^2}{3l}+\dfrac{3h_1^2+16f_1^2}{3l_1}\right)\theta t}{\dfrac{16}{3l}f+\dfrac{32}{3}\,\dfrac{f_1^2l_1I}{fl^2I_1}}\quad \text{nearly}\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots(19)$$

故ニ中央徑間及側徑間ノ吊材ノ水平毎呎ノ張力 $p_t$ 及ヒ $p_t'$ ハ次ノ如シ

$$p_t=\pm\frac{384EI}{5l^4}\Delta f \qquad p_t'=\pm\frac{384f_1EI}{5fl^2l_1^2}\Delta f$$

又温度ノ變化ヨリ起ル鍊條ノ水平分力 $H_t$ ハ次ノ如シ

$$H_t=\pm\frac{E\left(l+2l_1+2l_2+\dfrac{8f^2}{3l}+\dfrac{3h_1^2+16f_1^2}{3l_1}\right)\theta t}{\dfrac{5}{9}\left(\dfrac{f^2l}{I}+\dfrac{2f_1^2l_1}{I_1}\right)}\quad \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots\ \cdots(20)$$

但シ温度降リシ時ハ正號ヲ用ヒ温度昇リシ時ハ負號ヲ用フルモノトス

又吊材ノ熱應力ハ次ノ如シ

$$p_t\left(\frac{l}{n}\right)=\frac{8H_tf}{nl} \qquad p_t'\left(\frac{l_1}{n_1}\right)=\frac{8H_tf_1}{n_1l_1}$$

又扶構ノ Bending moment 及ヒ Shear ハ次ノ如シ

For side stiffening truss

$$M=-\frac{4H_tf_1}{l_1}x+\frac{4H_tf_1}{l_1^2}x^2$$

$$S=-\frac{4H_tf_1}{l_1}+\frac{8H_tf_1}{l_1^2}x$$

For central stiffening truss

$$M=-\frac{4H_tf}{l}x+\frac{4H_tf}{l^2}x^2$$

$$S=-\frac{4H_tf}{l}+\frac{8H_tf}{l^2}x$$



(B) 第二解法

温度ノ變化ヨリ起ル熱應力ハ又次ノ如クシテ其ノ略値ヲ求メ得ヘシ

今温度カ正規温度ヨリ$t^o$丈ケ降下シタル場合ヲ考ヘ其ノ時ニ起レル鏼條應力ノ水平分力ヲ$H_t$トス

又温度ノ變化ノ爲メニ起レル内働ノ總和ハ次ノ如クシテ其ノ略値ヲ見出シ得ヘシ

(一) 扶構ノ内働ト其ノ微係數

$$\omega=2\int_0^{l_1}\frac{M^2}{2EI_1}dx+\int_0^{l}\frac{M^2}{2EI}dx$$

$$\frac{d\omega}{dH_t}=2\int_0^{l_1}\frac{M}{EI_1}\left(\frac{dM}{dH_t}\right)dx+\int_0^{l}\frac{M}{EI}\left(\frac{dM}{dH_t}\right)dx$$

$$=\frac{32H_tf_1^2}{EI_1l_1^2}\int_0^{l_1}\left(-x+\frac{x^2}{l_1}\right)^2dx+\frac{16H_tf^2}{EIl^2}\int_0^{l}\left(-x+\frac{x^2}{l}\right)^2dx$$

$$=\frac{8H_t}{15E}\left\{\frac{f^2l}{I}+\frac{2f_1^2l_1}{I_1}\right\}\quad\cdots\cdots\cdots\cdots(a)$$

(二) 鏼條ノ内働ト其ノ微係數

温度ノ變化ノ爲メ鏼條ニ起ル内働ハ次ノ如シ

$$\omega=2\int_0^{\frac{l'}{2}}\frac{T^2ds}{2EA}+2\int_0^{l_1'}\frac{T_1^2ds}{2EA}+2\frac{T_2^2l_2}{2EA}-2\int_0^{\frac{l'}{2}}T(\theta t\,ds)-2\int_0^{l_1'}T_1(\theta t\,ds)-2T_2(\theta t\,l_2)$$

$$\frac{d\omega}{dH_t}=2\int_0^{\frac{l'}{2}}\frac{T}{EA}\left(\frac{dT}{dH_t}\right)ds+2\int_0^{l_1'}\frac{T_1}{EA}\left(\frac{dT_1}{dH_t}\right)ds+2\frac{T_2l_2}{EA}\left(\frac{dT_2}{dH_t}\right)$$

$$-2\int_0^{\frac{l'}{2}}\left(\frac{dT}{dH_t}\right)\theta t\,ds-2\int_0^{l_1'}\left(\frac{dT_1}{dH_t}\right)\theta t\,ds-2\left(\frac{dT_2}{dH_t}\right)\theta t\,l_2$$

$$=\frac{2H_t}{EA}\left[\int_0^{\frac{l}{2}}\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{3}{2}}dx+\int_0^{l_1}\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{3}{2}}dx+l_2\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}\right]$$

$$-2\theta t\left[\int_0^{\frac{l}{2}}\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}dx+\int_0^{l_1}\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}dx+l_2\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}\right]$$

$$=\frac{2H_t}{EA}\left\{\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right\}-2\theta t\left\{\frac{l}{2}+l_1+l_2+\frac{8f^2}{3l}+\frac{3h_1^2+16f_1^2}{3l_1}\right\}\quad\cdots\cdots\cdots\cdots(b)$$

### (三) 吊材ノ内働ト其ノ微係數

$$\omega=\frac{S_1^2L_1}{2EA_1}+2\frac{S_2^2L_2}{2EA_2}-S_1(\theta t\,L_1)-2S_2(\theta t\,L_2)$$

$$\frac{d\omega}{dH_t}=\frac{S_1L_1}{EA_1}\left(\frac{dS_1}{dH_t}\right)+2\frac{S_2L_2}{EA_2}\left(\frac{dS_2}{dH_t}\right)-\left(\frac{dS_1}{dH_t}\right)\theta t\,L_1-2\left(\frac{dS_2}{dH_t}\right)\theta t\,L_2$$

$$=\frac{64H_t}{E}\left\{\frac{f^2}{A_1nl^4}\left(h-\frac{2}{3}f\right)+\frac{2f_1^2}{A_2n_1l_1^2}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)\right\}$$

$$-2\theta t\left\{\frac{4f}{l}\left(h-\frac{2}{3}f\right)+\frac{8f_1}{l_1}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)\right\}\quad\cdots\cdots\cdots\cdots(c)$$

(四) 塔ノ内働ト其ノ微係數

$$\omega=2\frac{S^2h_2}{2EA_3}+2S(\theta th_2)$$

$$\frac{d\omega}{dH_t}=2\frac{Sh_2}{EA_3}\left(\frac{dS}{dH_t}\right)+2\theta th_2\left(\frac{dS}{dH_t}\right)$$

$$=\frac{2H_th_2}{EA_3}\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)^2+2\theta t\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)h_2 \quad \cdots\cdots (d)$$

(五) 水平分力 $H_t$ ノ決定

Castigliano ノ第二定理ニヨリ

$$\frac{d\omega}{dH_t}=\text{Equation }(a)+\text{Equation }(b)+\text{Equation }(c)+\text{Equation }(d)=0$$

上ノ方程式ヨリ次ノ結果ヲ得ヘシ

$$H_t=\pm\frac{2E\theta\left[\left(\frac{l}{2}+l_1+l_2+\frac{8f^2}{3l}+\frac{3h_1^2+16f_1^2}{3l_1}\right)+\frac{4f}{l}\left(h-\frac{2}{3}f\right)+\frac{8f_1}{l_1}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)-\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)h_2\right]t}{\text{Denominator eq. (12)}} \quad \cdots\cdots (21)$$

吊材及塔ノ影響頗ル小ナルヲ以テ之ヲ無視スルトキハ

$$H_t=\pm\frac{2E\theta\left(\frac{l}{2}+l_1+l_2+\frac{8f^2}{3l}+\frac{3h_1^2+16f_1^2}{3l_1}\right)t}{\text{Denominator eq. (13)}} \quad \cdots\cdots (22)$$

温度カ正規温度ヨリ降リシ時ハ正號ヲ用ヒ反對ニ温度カ昇リシ時ハ負號ヲ用フルモノトス

## 第七章　温度ノ變化並ニ活荷重ヨリ起ル鍊條ノ撓度ニ歸因スル應力ノ影響

温度ノ變化並ニ活荷重ノ爲メニ鍊條ハ比較的大ナル撓度ヲ起スモノニシテ時トシテハ垂矢(Sag)ノ10%以上ニ達スルコト稀ナリトセス

從ッテ是マテ計算セル諸式ハ垂矢ニ變化ナキモノト假定シテ之ヲ行ヘルカ故ニ斯クシテ求メタル鍊條ノ水平分力$H$ノ値ハ正確ナル値ニアラスシテ略値ナルコト明ナリ(勿論$H$ノ最モ正確ナル値ヲ求ムルニハ鍊條ノ撓度ニヨル影響ヲ考ニ入レサルヘカラス)

又吊材ノ張力及扶構ノ應力モ鍊條ノ撓度ニヨリ影響ヲ蒙ムルモノニシテ次ノ如キ更正ヲ施セハ稍正確ナル値ヲ得ヘシ

今$\Delta f$ $\Delta f_1$ヲ温度ノ變化又ハ活荷重ノ爲メニ起レル中央徑間及側徑間ニ於ケル鍊條ノ撓度トシ$H$ヲ第四章ニヨリ求メタル鍊條水平應力ノ略値トスレハ吊材ノ水平每呎ノ張力ハ次ノ如シ

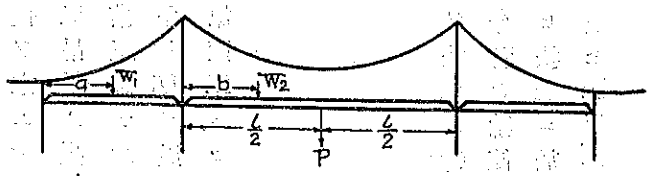
$$p=\frac{8H(f\pm\Delta f)}{l^2} \quad \text{for center span}$$

$$p_1=\frac{8H(f_1\pm\Delta f_1)}{l_1^2} \quad \text{for side span}$$

又扶構ノ彎曲率及ヒ剪力ヲ計算スルニモ上ノ更正セラレタル吊材張力ヲ用フレハ稍正確ナリ

今若シ鍊條ノ撓度ヲ考ニ入レ最モ正確ナル鍊條應力ノ水平分力$H_1$ヲ求メ得タリトスレハ吊材及ヒ扶構ニ於ケル最モ正確ナル應力ノ値ハ第五章ニ於テ$H$ノ代リニ$H_1$ヲ入レ$f$及$f_1$ノ代リニ$f+\Delta f$及ヒ$f_1+\Delta f_1$ヲ代入スレハ之ヲ求メ得ルコト明ナリ

## 第八章　活荷重ノ爲メニ受クル扶構ノ撓度




第十九圖

(第一)中央扶構ノ撓度

今$W_1$ナル活荷重ト同時ニ補助荷重(Auxiliary load) $P$ヲ中央扶構ノ中點ニ掛ケ是等ヨリ起ル内働ヲ計算シ之ヲ$P$ニ付テ微分シ最後ニ$P=0$ニ置ケハ Castigliano ノ第一定理ニヨリ中央扶構ノ中點ニ於ケル撓度ヲ見出シ得ヘシ

$W_1$ヨリ起ル鍊條ノ水平分力ヲ$H$トシ$P$ヨリ起ル鍊條ノ水平分力ヲ$H_p$トスレハ算式(18)ニヨリ

$$H_p=\frac{\frac{5fl^2}{48I}P}{\text{Denominator eq. (12) or (13)}}$$

$$K=\frac{\frac{5fl^2}{48I}}{\text{Denominator eq. (12) or (13)}}$$

ト置ケハ $H_p=KP$ トナル又是等ノ荷重ノ爲メニ起ル水平毎呎ノ吊材張力ヲ中央徑間及側徑間ニ於テ$p$及$p_1$トスレハ

$$p=\frac{8(H+KP)f}{l^2}\qquad p_1=\frac{8(H+KP)f_1}{l_1^2}$$

(一)扶構ノ内働ト其ノ微係數

左側扶構ノ彎曲率

$x<a$

$$M=\frac{1}{l_1}\left\{W_1(l_1-a)-4(H+KP)f_1\right\}x+\frac{4(H+KP)f_1}{l_1^2}x^2$$

$$\frac{dM}{dP}=\frac{4Kf_1}{l_1}\left(-x+\frac{x^2}{l_1}\right)$$

$$x>a \qquad M=\frac{1}{l_1}\left\{W_1(l_1-a)-4(H+KP)f_1\right\}x+\frac{4(H+KP)f_1}{l_1^2}x^2-W_1(x-a)$$

$$\frac{dM}{dP}=\frac{4Kf_1}{l_1}\left(-x+\frac{x^2}{l_1}\right)$$

中央扶構ノ彎曲率

$$M=\frac{1}{l}\left\{\frac{1}{2}Pl-4(H+KP)f\right\}x+\frac{4(H+KP)f}{l^2}x^2 \qquad \frac{dM}{dP}=\frac{1}{l}\left(\frac{1}{2}l-4Kf\right)x+\frac{4Kf}{l^2}x^2$$

右側扶構ノ彎曲率

$$M=-\frac{4}{l_1}(H+KP)f_1x+\frac{4(H+KP)f_1}{l_1^2}x^2 \qquad \frac{dM}{dP}=\frac{4Kf_1}{l_1}\left(-x+\frac{x^2}{l_1}\right)$$

扶構ニ於ケル總内働ハ次ノ如シ

$$\omega=\int_0^a\frac{M^2}{2EI_1}dx+\int_a^{l_1}\frac{M^2}{2EI_1}dx+2\int_0^{\frac{l}{2}}\frac{M^2}{2EI}dx+\int_0^{l_1}\frac{M^2}{2EI_1}dx$$

$$\frac{d\omega}{dP}=\int_0^a\frac{M}{EI_1}\left(\frac{dM}{dP}\right)dx+\int_a^{l_1}\frac{M}{EI_1}\left(\frac{dM}{dP}\right)dx+2\int_0^{\frac{l}{2}}\frac{M}{EI}\left(\frac{dM}{dP}\right)dx+\int_0^{l_1}\frac{M}{EI_1}\left(\frac{dM}{dP}\right)dx$$

$$=\frac{Hfl}{3EI}\left(\frac{8}{5}Kf-\frac{5}{16}l\right)+\frac{4HKf_1}{3EI_1}\left\{\frac{4}{5}f_1l_1-\frac{W_1a}{4H}\left(l_1-\frac{2a^2}{l_1}+\frac{a^3}{l_1^2}\right)\right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots(a)$$

(二)鋼條ノ内働ト其ノ微係數

中央鋼條及側鋼條ノ張力ハ次ノ如シ

$$T=(H+KP)\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}} \qquad \frac{dT}{dP}=K\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}$$

1311

$$T_1=(H+KP)\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}} \qquad \frac{dT_1}{dP}=K\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}$$

$$T_2=(H+KP)\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}} \qquad \frac{dT_2}{dP}=K\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}$$

$$\omega=2\int_0^{\frac{l'}{2}}\frac{T^2ds}{2EA}+2\int_0^{l_1'}\frac{T_1^2ds}{2EA}+2\frac{T_2^2l_2}{2EA}$$

$$\frac{d\omega}{dP}=2\int_0^{\frac{l'}{2}}\frac{T}{EA}\left(\frac{dT}{dP}\right)ds+2\int_0^{l_1'}\frac{T_1}{EA}\left(\frac{dT_1}{dP}\right)ds+2\frac{T_2l_2}{EA}\left(\frac{dT_2}{dP}\right)$$

$$ds=\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}dx$$

$$ds=\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}dx$$

$$\frac{d\omega}{dP}=\frac{2HK}{EA}\int_0^{\frac{l}{2}}\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{3}{2}}dx+\frac{2HK}{EA}\int_0^{l_1}\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{3}{2}}dx$$

$$+\frac{2HK}{EA}\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}l_2$$

$$=\frac{2HK}{EA}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right) \quad \text{nearly} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(b)$$

（三）吊材ノ内働ト其ノ微係數

中央徑間及側徑間ニ於ケル吊材ノ張力ヲ$S_1$及$S_2$トスレバ

$$S_1=p\left(\frac{l}{n}\right)=\frac{8(H+KP)f}{nl} \qquad \frac{dS_1}{dP}=\frac{8Kf}{nl}$$

$$S_2=p_1\left(\frac{l_1}{n_1}\right)=\frac{8(H+KP)f_1}{n_1l_1} \qquad \frac{dS_2}{dP}=\frac{8Kf_1}{n_1l_1}$$

吊材ニ於ケル總内働ハ次ノ如シ

$$\omega=\frac{S_1^2L_1}{2EA_1}+2\frac{S_2^2L_2}{2EA_2}$$

$$\frac{d\omega}{dP}=\frac{S_1L_1}{EA_1}\left(\frac{dS_1}{dP}\right)+2\frac{S_2L_2}{EA_2}\left(\frac{dS_2}{dP}\right)$$

$$=\frac{64HK}{E}\left\{\frac{f^2}{A_1nl^2}\left(h-\frac{2}{3}f\right)+\frac{2f_1^2}{A_2n_1l_1^2}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)\right\} \cdots\cdots\cdots\cdots\cdots\cdots(c)$$

(四)塔ノ内働ト其ノ微係數

塔ノ受クル全應力ハ次ノ如シ

$$S=(H+KP)\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right) \qquad \frac{dS}{dP}=K\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)$$

$$\omega=2\frac{S^2h_2}{2EA^3}$$

$$\frac{d\omega}{dP}=2\frac{Sh_2}{EA_3}\left(\frac{dS}{dP}\right)=\frac{2HK}{EA_3}\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)^2h_2 \cdots\cdots\cdots\cdots\cdots\cdots\cdots(d)$$

(五)中央扶構ノ撓度ノ決定

Castigliano ノ第一定理ニヨリ中央扶構ノ中點ニ於ケル撓度δハ次ノ如シ

$$\delta=\Sigma\frac{d\omega}{dP}=\text{Equation }(a)+\text{Equation }(b)+\text{Equation }(c)+\text{Equation }(d)$$

上ノ方程式ヨリ次ノ結果ヲ得ヘシ

$$\delta=\frac{H}{E}\Bigg[\frac{fl}{3I}\left(\frac{8}{5}Kf-\frac{5}{16}l\right)+\frac{4Kf_1}{3I_1}\left\{\frac{4}{5}f_1l_1-\frac{W_1a}{4H}\left(l_1-\frac{2a^2}{l_1}+\frac{a^3}{l_1^2}\right)\right\}$$
$$+\frac{2K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)$$
$$+64K\left\{\frac{f^2}{A_1nl^2}\left(h-\frac{2}{3}f\right)+\frac{2f_1^2}{A_2n_1l_1^2}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)\right\}+\frac{2K}{A_3}\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)^2h_2\Bigg]\cdots\cdots\cdots(23)$$

吊材及塔ノ變形ヨリ來タル影響ヲ省略スルトキハ($A_1$ $A_2$ $A_3$ヲ含ム項ヲ省ケハ宜シ)

$$\delta=\frac{H}{E}\Bigg[\frac{fl}{3I}\left(\frac{8}{5}Kf-\frac{5}{16}l\right)+\frac{4Kf_1}{3I_1}\left\{\frac{4}{5}f_1l_1-\frac{W_1a}{4H}\left(l_1-\frac{2a^2}{l_1}+\frac{a^3}{l_1^2}\right)\right\}$$
$$+\frac{2K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)\Bigg]\cdots\cdots\cdots(24)$$

撓度ニ對シ吊材及塔ノ變形ヨリ來ル影響ハ頗ル小ナルヲ以テ今後ハ常ニ之ヲ省略スルモノトス

$w$ナル等布荷重カ$a_1$マテ乘リシトキニハ(前式ニ於テ$W_1$ノ代リニ$wdx$ヲ入レ$a$ノ代リニ$x$ヲ置キ0ヨリ$a_1$マテ積分スレハ宜シ)

$$\delta=\frac{H}{E}\Bigg[\frac{fl}{3I}\left(\frac{8}{5}Kf-\frac{5}{16}l\right)+\frac{4Kf_1}{3I_1}\left\{\frac{4}{5}f_1l_1-\frac{wa_1^2}{4H}\left(\frac{l_1}{2}-\frac{a_1^2}{2l_1}+\frac{a_1^3}{5l_1^2}\right)\right\}$$
$$+\frac{2K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)\Bigg]\cdots\cdots\cdots(25)$$

同樣ニシテ$W_2$ニ對スル中央扶搆ノ撓度ヲ求ムレハ次ノ如シ

$$\delta=\frac{H}{E}\Bigg[\frac{16Kf_1^2l_1}{15I_1}+\frac{fl}{3I}\left(\frac{8}{5}Kf-\frac{5}{16}l\right)+\frac{W_2b}{HI}\left\{\left(\frac{l^2}{16}-\frac{b^2}{12}\right)-\frac{Kf}{3l}\left(l^2-2b^2+\frac{b^3}{l}\right)\right\}$$

$$+\frac{2K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3l_1^2+16f_1^2}{2l_1}\right)\Bigg]\quad\cdots\cdots(26)$$

茲ニ注意ヲ要スルハ$W_2$ナル荷重ニ於テ左端支承ヨリ計レル$b$カ$l/2$ヨリ大ナル時ハ$b$ハ右端支承ヨリ之ヲ計ラサルヘカラス

$w$ナル等布荷重カ$b_1$マテ乗リシ時ニハ

$$\delta=\frac{H}{E}\Bigg[\frac{16K_1f_1^2l_1}{15I_1}+\frac{fl}{3I}\left(\frac{8}{5}Kf-\frac{5}{16}\right)+\frac{wb_1^2}{HI}\Bigg\{\left(\frac{l^2}{32}-\frac{b_1^2}{48}\right)$$

$$-\frac{Kf}{3l}\left(\frac{l^2}{2}-\frac{b_1^2}{2}+\frac{b_1^3}{5l}\right)\Bigg\}+\frac{2K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3l_1^2+16f_1^2}{2l_1}\right)\Bigg]\quad\cdots\cdots(27)$$

又等布荷重ニ於テ左端支承ヨリ計レル$b_1$カ$l/2$ヨリ大ナル時ハ先ツ$l$ナル徑間ノ滿載荷重ニ對スル撓度ヲ求メ其レヨリ$l-b_1$ナル部分カ荷重ヲ受ケシ際ニ起ル撓度ヲ求メ之ヲ減スレハ宜シ

（第二）側扶構ノ撓度

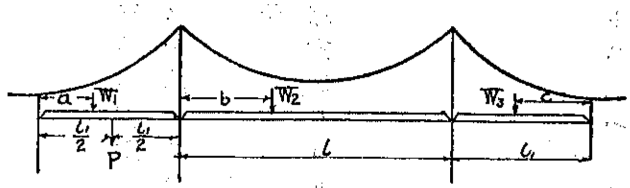
此ノ場合ニモ補助荷重$P$ヲ側扶構ノ中點ニ掛ケ$P$ノ爲メニ起レル鋼條ノ水平分力ヲ$H_p$トスレハ算式(12)ニヨリ

$$H_p=\frac{\frac{5fl_1^2}{48I_1}P}{\text{Denominator eq. (12) or (13)}}$$

$$K=\frac{\frac{5fl_1^2}{48I_1}}{\text{Denominator eq. (12) or (13)}}$$

トスレハ$H_p=KP$トナル$W_1$ノ爲メニ起ル側扶構ノ撓度ハ次ノ如シ

$$\delta=\frac{H}{E}\Bigg[\frac{8Kf^2l}{15I}+\frac{f_1l_1}{3I_1}\left(\frac{16}{5}Kf_1-\frac{5}{16}l_1\right)$$
$$+\frac{W_1a}{HI_1}\left\{\left(\frac{l_1^2}{16}-\frac{a^2}{12}\right)-\frac{Kf_1}{3l_1}\left(l_1^2-2a^2+\frac{a^3}{l_1}\right)\right\}$$
$$+\frac{2K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)\Bigg]\quad\cdots\cdots(28)$$




第二十圖

茲ニ注意ヲ要スルハ $W_1$ ナル荷重ニ於テ左端支承ヨリ計レル $a$ カ $\frac{l_1}{2}$ ヨリ大ナル時ハ $a$ ハ右端支承ヨリ之ヲ計ラサルヘカラス

$w$ ナル等布荷重カ $a_1$ マテ乘リシ時ニハ

$$\delta=\frac{H}{E}\Bigg[\frac{8Kf^2l}{15I}+\frac{f_1l_1}{3I_1}\left(\frac{16}{5}Kf_1-\frac{5}{16}l_1\right)+\frac{wa_1^2}{HI_1}\left\{\left(\frac{l_1^2}{32}-\frac{a_1^2}{48}\right)-\frac{Kf_1}{3l_1}\left(\frac{l_1^2}{2}-\frac{a_1^2}{2}+\frac{a_1^3}{5l_1}\right)\right\}$$
$$+\frac{2K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)\Bigg]\quad\cdots\cdots(29)$$

又等布荷重ニ於テ左端支承ヨリ計レル $a_1$ カ $\frac{l_1}{2}$ ヨリ大ナル時ハ先ツ $l_1$ ナル徑間ノ滿載荷重ニ對スル撓度ヲ求メ其レヨリ $l_1-a_1$ ナル部分カ荷重ヲ受ケシ際ニ起ル撓度ヲ求メ之ヲ減スレハ宜シ

$W_2$ ノ爲メニ起ル撓度ハ次ノ如シ

$$\delta=\frac{H}{E}\Bigg[\frac{f_1l_1}{3I_1}\left(\frac{16}{5}Kf_1-\frac{5}{16}l_1\right)+\frac{4Kf}{3I}\left\{\frac{2}{5}fl-\frac{W_2b}{4H}\left(l-\frac{2b^2}{l}+\frac{b^3}{l^2}\right)\right\}$$
$$+\frac{2K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)\Bigg]\quad\cdots\cdots(30)$$

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$w$ ナル等布荷重カ $b_1$ マテ乗リシトキハ

$$\delta=\frac{H}{E}\left[\frac{f_1l_1}{3I_1}\left(\frac{16}{5}Kf_1-\frac{5}{16}l_1\right)+\frac{4Kf}{3I}\left\{\frac{2}{5}fl-\frac{wb_1^2}{4H}\left(\frac{l}{2}-\frac{b_1^2}{2l}+\frac{b_1^3}{5l^2}\right)\right\}\right.$$
$$\left.+\frac{2K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)\right]\cdots\cdots\cdots(31)$$

$W_3$ ノタメニ起ル撓度ハ次ノ如シ

$$\delta=\frac{H}{E}\left[\frac{8Kf^2l}{15I}+\frac{f_1l_1}{3I_1}\left\{\frac{16}{5}Kf_1-\frac{5}{16}l_1-\frac{W_3Kc}{Hl_1}\left(l_1-\frac{2c^2}{l_1}+\frac{c^3}{l_1^2}\right)\right\}\right.$$
$$\left.+\frac{2K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)\right]\cdots\cdots\cdots(32)$$

$w$ ナル等布荷重カ $c_1$ マテ乗リシ時ニハ

$$\delta=\frac{H}{E}\left[\frac{8Kf^2l}{15I}+\frac{f_1l_1}{3I_1}\left\{\frac{16}{5}Kf_1-\frac{5}{16}l_1-\frac{wKc_1^2}{Hl_1}\left(\frac{l_1}{2}-\frac{c_1^2}{2l_1}+\frac{c_1^3}{5l_1^2}\right)\right\}\right.$$
$$\left.+\frac{2K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)\right]\cdots\cdots\cdots(33)$$

## 第九章　温度ノ變化ノ爲メニ受クル扶構ノ撓度

### (第一)中央扶構ノ撓度

今中央扶構ノ中點ニ $P$ ナル補助荷重ヲ掛ケタリトシ又温度カ正規温度ヨリ $t^0$ 丈ケ變化シタリト假定ス

温度ノ變化ヨリ起レル鏁條ノ水平分力ヲ $H_t$ トシ $P$ ヨリ起レル鏁條ノ水平分力ヲ $H_p$ トスレハ

$$H_p=\frac{\frac{5fl^2}{48I}P}{\text{Denominator eq. (12) or (13)}}=KP$$

又中央徑間及側徑間ニ於ケル水平每呎ノ吊材張力ヲ $p$ 及 $p_1$ トスレハ

$$p=\frac{8(H_t+KP)f}{l^2} \qquad p_1=\frac{8(H_t+KP)f_1}{l_1^2}$$

（一）扶構ノ內働ト其ノ微係數

兩側扶構ノ彎曲率

$$M=-\frac{4}{l_1}(H_t+KP)f_1x+\frac{4(H_t+KP)f_1}{l_1^2}x^2 \qquad \frac{dM}{dP}=\frac{4Kf_1}{l_1}\left(-x+\frac{x^2}{l_1}\right)$$

中央扶構ノ彎曲率

$$M=\frac{1}{l}\left\{\frac{1}{2}Pl-4(H_t+KP)f\right\}x+\frac{4(H_t+KP)f}{l^2}x^2$$

$$\frac{dM}{dP}=\frac{1}{l}\left\{\left(\frac{1}{2}l-4Kf\right)x+\frac{4Kf}{l}x^2\right\}$$

$$\omega=2\int_0^{l_1}\frac{M^2}{2EI_1}dx+2\int_0^{\frac{l}{2}}\frac{M^2}{2EI}dx$$

$$\frac{d\omega}{dP}=2\int_0^{l_1}\frac{M}{EI_1}\left(\frac{dM}{dP}\right)dx+2\int_0^{\frac{l}{2}}\frac{M}{EI}\left(\frac{dM}{dP}\right)dx=\frac{16H_tKf_1^2l_1}{15EI_1}+\frac{H_tfl}{3EI}\left(\frac{8}{5}Kf-\frac{5}{16}l\right)$$

（二）鏈條ノ內働ト其ノ微係數

$$T=(H_t+KP)\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}} \qquad \frac{dT}{dP}=K\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}$$

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$$\omega=2\int_0^{\frac{l}{2}}\frac{T^2ds}{2EA}+2\int_0^{l_1'}\frac{T_1^2ds}{2EA}+2\frac{T_2^2l_2}{2EA}-2\int_0^{\frac{l}{2}}T(\theta t\cdot ds)-2\int_0^{l_1'}T_1(\theta t\cdot ds)-2T_2\theta t l_2$$

$$\frac{d\omega}{dP}=2\int_0^{\frac{l}{2}}\frac{T}{EA}\left(\frac{dT}{dP}\right)ds+2\int_0^{l_1'}\frac{T_1}{EA}\left(\frac{dT_1}{dP}\right)ds+2\frac{T_2l_2}{EA}\left(\frac{dT_2}{dP}\right)$$

$$-2\int_0^{\frac{l}{2}}\left(\frac{dT}{dP}\right)(\theta t\,ds)-2\int_0^{l_1'}\left(\frac{dT_1}{dP}\right)(\theta t\,ds)-2\left(\frac{dT_2}{dP}\right)\theta t l_2$$

$$=\frac{2H_tK}{EA}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)$$

$$-2K\theta t\left(\frac{l}{2}+l_1+l_2+\frac{8f^2}{3l}+\frac{3h_1^2+16f_1^2}{3l_1}\right)$$

(三)吊材ノ內働ト其ノ徽係數

中央徑間及側徑間ニ於ケル吊材ノ張力ヲ$S_1$及$S_2$トスレハ

$$S_1=\frac{8(H_t+KP)f}{nl}\qquad\frac{dS_1}{dP}=\frac{8Kf}{nl}$$

$$S_2=\frac{8(H_t+KP)f_1}{n_1l_1}\qquad\frac{dS_2}{dP}=\frac{8Kf_1}{n_1l_1}$$

$$\omega=\frac{S_1^2L_1}{2EA_1}+2\frac{S_2^2L_2}{2EA_2}-S_1(\theta t\,L_1)-2S_2(\theta t\,L_2)$$

$$\frac{d\omega}{dP}=\frac{S_1L_1}{EA_1}\left(\frac{dS_1}{dP}\right)+2\frac{S_2L_2}{EA_2}\left(\frac{dS_2}{dP}\right)-\left(\frac{dS_1}{dP}\right)\theta t\,L_1-2\left(\frac{dS_2}{dP}\right)\theta t\,L_2$$

$$=\frac{64H_tKf^2}{EA_1nl^2}\left(h-\frac{2}{3}f\right)+\frac{128H_tKf_1^2}{EA_2n_1l_1^2}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)$$

$$-8K\theta t\left\{\frac{f}{l}\left(h-\frac{2}{3}f\right)+\frac{2f_1}{l_1}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)\right\}$$

(四) 塔ノ內働ト其ノ微係數

$$S=(H_t+KP)\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)\qquad\frac{dS}{dP}=K\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)$$

$$\omega=2\frac{S^2h_2}{2EA_3}+2S(\theta t\,h_2)$$

$$\frac{d\omega}{dP}=2\frac{Sh_2}{EA_3}\left(\frac{dS}{dP}\right)+2\left(\frac{dS}{dP}\right)\theta th_2$$

$$=\frac{2H_tK}{EA_3}\left(\frac{4f}{l}+\frac{h_1+4f_1}{h_1}\right)^2h_2+2K\theta t\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)h_2$$

(五) 中央扶構ノ撓度ノ決定

溫度ノ變化ノ爲メニ起ル中央扶構ノ中點ノ撓度$\delta_t$ハ次ノ如シ

$$\delta_t=\frac{2H_t}{E}\left[\frac{8Kf_1^2l_1}{15I_1}+\frac{fl}{6I}\left(\frac{8}{5}Kf-\frac{5}{16}l\right)+\frac{K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)\right.$$

$$\left.+32K\left\{\frac{f^2}{A_1nl^2}\left(h-\frac{2}{3}f\right)+\frac{2f_1^2}{A_2n_1l_1^2}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)\right\}+\frac{K}{A_3}\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)^2h_2\right]$$

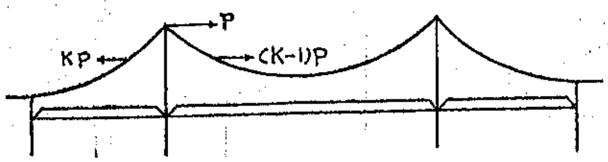
$$-2K\theta t\left[\left(\frac{l}{2}+l_1+l_2+\frac{8f^2}{3l}+\frac{3l_1^2+16f_1^2}{3l_1}\right)\right.$$

$$\left.+4\left\{\frac{f}{l}\left(h-\frac{2}{3}f\right)+\frac{2f_1}{l_1}\left(h-\frac{l_1}{2}-\frac{2}{3}f_1\right)\right\}-\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)h_1\right]\quad\cdots\cdots(34)$$

吊材及塔ノ影響ヲ省略スルトキハ

$$\delta_c=\frac{2H_t}{E}\left[\frac{8Kf_1^2l_1}{15I_1}+\frac{fl}{6I}\left(\frac{8}{5}Kf-\frac{5}{16}l\right)+\frac{K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3l_1^2+16f_1^2}{2l_1}\right)\right]$$

$$-2K\theta t\left(\frac{l}{2}+l_1+l_2+\frac{8f^2}{3l}+\frac{3l_1^2+16f_1^2}{3l_1}\right)\quad\cdots\cdots(35)$$




第二十一圖

(第三)側扶構ノ撓度

前同樣側扶構ノ中點ニ$P$ナル補助荷重ヲ掛ケタリトシ$P$ノ爲メニ起ル鏃條ノ水平分力ヲ$H_p$トスレバ

$$H_p=\frac{\frac{5f_1l_1^2}{48I_1}P}{\text{Denominator eq. (12) or (13)}}=KP$$

溫度ノ變化ノ爲メ側扶構ノ中點ニ起ル撓度ハ吊材及ヒ塔ノ影響ヲ略スル時ハ次ノ如シ

$$\delta_c=\frac{2H_t}{E}\left[\frac{4Kf^2l}{15I}+\frac{f_1l_1}{6I_1}\left(\frac{16}{5}Kf_1-\frac{5}{16}l_1\right)+\frac{K}{A}\left(\frac{l}{2}+l_1+l_2+\frac{4f^2}{l}+\frac{3l_1^2+16f_1^2}{2l_1}\right)\right]$$

$$-2K\theta t\left(\frac{l}{2}+l_1+l_2+\frac{8f^2}{3l}+\frac{3l_1^2+16f_1^2}{3l_1}\right)\quad\cdots\cdots(36)$$

## 第十章　活荷重ノ爲メニ受クル支鞍(Saddle)ノ塔上ニ於ケル水平移動

今左ノ塔ノ頂上ニ於テ水平ノ方向ニ$P$ナル補助荷重ヲ掛ケタリトシ$P$ノ爲メニ起リシ鍊條應力ノ水平分力ヲ左側鍊條ニテ$KP$トシ中央鍊條及右側鍊條ニテ$(K-1)P$トスレハCastiglianoノ第二定理ノ應用ニヨリ次ノ如キ$K$ノ値ヲ得ヘシ

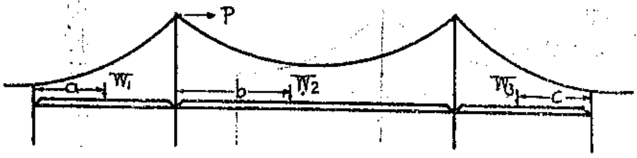
$$K=\frac{\left[\begin{array}{l}\frac{8}{15}\left(\frac{f^2l}{I}+\frac{f_1^2l_1}{I_1}\right)+\frac{1}{A}\left(l+l_1+l_2+\frac{8f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)\\ +64\left\{\frac{f^2}{A_1nl^2}\left(h-\frac{2}{3}f\right)+\frac{f_1^2}{A_2n_1l_1^2}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)\right\}\\ +\frac{h_2}{A_3}\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)\left(\frac{8f}{l}+\frac{h_1+4f_1}{l_1}\right)\end{array}\right]}{\text{Denominator eq. (12)}}$$

吊材及塔ノ影響ヲ省略スル時ハ

$$K=\frac{\frac{8}{15}\left(\frac{f^2l}{I}+\frac{f_1^2l_1}{I_1}\right)+\frac{1}{A}\left(l+l_1+l_2+\frac{8f^2}{l}+\frac{3h_1^2+16f_1^2}{2l_1}\right)}{\text{Denominator eq. (13)}}\quad\cdots\;\cdots\;\cdots(38)$$

算式(37)(38)ニ於テ$K$ノ値ハ殆ト一(one)ニ等シキヲ以テ$P$ナル補助荷重ノ爲メニ起ル鍊條ノ水平分力ヲ左側鍊條ニテ$P$トシ中央鍊條及右側鍊條ニテ0ト假定スルモ大差ナシ

次ニ$W_1$ノ爲メニ起ル支鞍ノ移動ヲ見出サントス




第二十二圖

今$W_1$ト共ニ補助荷重$P$ヲ塔ノ頂上ニ於テ水平ノ方向ニ掛クレハ水平毎呎ノ吊材張力ハ次ノ如シ

但シ$\overline{W}_1$ニヨリ起リシ鍊條ノ水平分力ヲ$H$トス

$$p_1=\frac{8(H+P)f_1}{l_1^2} \qquad \text{for left span}$$

$$p=\frac{8Hf}{l^2} \qquad \text{for center span}$$

$$p_2=\frac{8Hf_1}{l_1^2} \qquad \text{for right span}$$

## (一)扶構ノ内働ト其ノ微係數

左側扶構ノ彎曲率

$x<a$

$$M=\frac{1}{l_1}\Big\{W_1(l_1-a)-4(H+P)f_1\Big\}x+\frac{4(H+P)f_1}{l_1^2}x^2$$

$$\frac{dM}{dP}=\frac{4f_1}{l_1}\left(-x+\frac{x^2}{l_1}\right)$$

$x>a$

$$M=\frac{1}{l_1}\Big\{W_1(l_1-a)-4(H+P)f_1\Big\}x+\frac{4(H+P)f_1}{l_1^2}x^2-W_1(x-a)$$

$$\frac{dM}{dP}=\frac{4f_1}{l_1}\left(-x+\frac{x^2}{l_1}\right)$$

中央扶構ノ彎曲率

$$M=-\frac{4Hf}{l}x+\frac{4Hf}{l^2}x^2 \qquad \frac{dM}{dP}=0$$

右側扶構ノ彎曲率

$$M=-\frac{4Hf_1}{l_1}x+\frac{4Hf_1}{l_1^2}x^2 \qquad \frac{dM}{dP}=0$$

$$\omega=\int_0^a\frac{M^2}{2EI_1}dx+\int_a^{l_1}\frac{M^2}{2EI_1}dx+\int_0^{l}\frac{M^2}{2EI}dx+\int_0^{l_1}\frac{M^2}{2EI_1}dx$$

$$\frac{d\omega}{dP}=\int_0^a\frac{M}{EI_1}\left(\frac{dM}{dP}\right)dx+\int_a^{l_1}\frac{M}{EI_1}\left(\frac{dM}{dP}\right)dx+\int_0^{l}\frac{M}{EI}\left(\frac{dM}{dP}\right)dx+\int_0^{l_1}\frac{M}{EI}\left(\frac{dM}{dP}\right)dx$$

$$=\frac{Hf_1}{3EI_1}\left\{\frac{8}{5}f_1l_1-\frac{W_1a}{H}\left(l_1-\frac{2a^2}{l_1}+\frac{a^3}{l_1^2}\right)\right\}$$

(二) 鏁條ノ內働ト其ノ微係數

left span

$$T_1=(H+P)\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}$$

$$\frac{dT_1}{dP}=\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}$$

$$T_2=(H+P)\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}\qquad\frac{dT_2}{dP}=\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}$$

center span

$$T=H\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}\qquad\frac{dT}{dP}=0$$

right span

$$T_1=H\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}\qquad\frac{dT_1}{dP}=0$$

$$T_2=H\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}\qquad\frac{dT_2}{dP}=0$$

$$\omega=\int_0^{l_1'}\frac{T_1^2ds}{2EA}+\frac{T_2^2l_2}{2EA}+2\int_0^{\frac{l'}{2}}\frac{T^2ds}{2EA}+\int_0^{l_1'}\frac{T_1^2ds}{2EA}+\frac{T_2^2l_2}{2EA}$$

$$\frac{d\omega}{dP}=\int_0^{l_1'}\frac{T_1}{EA}\left(\frac{dT_1}{dP}\right)ds+\frac{T_2l_2}{EA}\left(\frac{dT_2}{dP}\right)+2\int_0^{\frac{l'}{2}}\frac{T}{EA}\left(\frac{dT}{dP}\right)ds$$

$$+\int_0^{l_1'}\frac{T_1}{EA}\left(\frac{dT_1}{dP}\right)ds+\frac{T_2l_2}{EA}\left(\frac{dT_2}{dP}\right)$$

$$=\frac{H}{EA}\left(l_1+l_2+\frac{3h_1^2+16f_1^2}{2l_1}\right)\qquad\text{nearly}$$

(三) 吊材ノ内働ト其ノ微係數

Left span $\quad S_2=\dfrac{8(H+P)f_1}{n_1l_1}\qquad\dfrac{dS_2}{dP}=\dfrac{8f_1}{n_1l_1}$

Center span $\quad S_1=\dfrac{8Hf}{nl}\qquad\dfrac{dS_1}{dP}=0$

Right span $\quad S_2=\dfrac{8Hf_1}{n_1l_1}\qquad\dfrac{dS_2}{dP}=0$

$$\frac{d\omega}{dP}=\frac{S_2L_2}{EA_2}\left(\frac{dS_2}{dP}\right)+\frac{S_1L_1}{EA_1}\left(\frac{dS_1}{dP}\right)+\frac{S_2L_2}{EA_2}\left(\frac{dS_2}{dP}\right)=\frac{64Hf_1^2}{EA_2n_1l_1^2}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)$$

(四) 塔ノ内働ト其ノ微係數

Left tower $\quad S=H\dfrac{4f}{l}+(H+P)\dfrac{h_1+4f_1}{l_1}\qquad\dfrac{dS}{dP}=\dfrac{h_1+4f_1}{l_1}$

Right tower $\quad S=H\left(\dfrac{4f}{l}+\dfrac{h_1+4f_1}{l_1}\right)\qquad\dfrac{dS}{dP}=0$

$$\frac{d\omega}{dP}=\frac{Sh_2}{EA_3}\left(\frac{dS}{dP}\right)+\frac{Sh_2}{EA_3}\left(\frac{dS}{dP}\right)$$

$$=\frac{H}{EA_3}\left(\frac{4f}{l}+\frac{h_1+4f_1}{l}\right)\left(\frac{h_1+4f_1}{l_1}\right)h_2$$

(五)支鞍ノ水平移動ノ決定

$W_1$ノタメニ起ル支鞍ノ水平移動$\Delta l$ハ次ノ如シ

$$\Delta l=\frac{H}{E}\left[\frac{f_1}{3I_1}\left\{\frac{8}{5}f_1l_1-\frac{W_1a}{H}\left(l_1-\frac{2a^2}{l_1}+\frac{a^3}{l_1^2}\right)\right\}+\frac{1}{A}\left(l_1+l_2+\frac{3h_1^2+16f_1^2}{2l_1}\right)\right.$$
$$\left.+\frac{64f_1^2}{A_2n_1l_1^2}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)+\frac{h_2}{A_3}\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)\left(\frac{h_1+4f_1}{l_1}\right)\right]\quad\cdots\cdots(39)$$

吊材及塔ノ影響ヲ省略スルトキハ

$$\Delta l=\frac{H}{E}\left[\frac{f_1}{3I_1}\left\{\frac{8}{5}f_1l_1-\frac{W_1a}{H}\left(l_1-\frac{2a^2}{l_1}+\frac{a^3}{l_1^2}\right)\right\}+\frac{1}{A}\left(l_1+l_2+\frac{3h_1^2+16f_1^2}{2l_1}\right)\right]\quad\cdots\cdots(40)$$

支鞍ノ移動ニ對スル吊材及塔ノ影響ハ頗ル小ナルヲ以テ今後ハ之ヲ略ス

$w$ナル等布荷重カ$a_1$マテ乘リシ時ハ

$$\Delta l=\frac{H}{E}\left[\frac{f_1}{3I_1}\left\{\frac{8}{5}f_1l_1-\frac{wa_1^2}{H}\left(\frac{l_1}{2}-\frac{a_1^2}{2l_1}+\frac{a_1^3}{5l_1^2}\right)\right\}+\frac{1}{A}\left(l_1+l_2+\frac{3h_1^2+16f_1^2}{2l_1}\right)\right]\quad\cdots\cdots(41)$$

同様ニシテ$W_2$ノ爲メニ起ル支鞍ノ移動ヲ求ムレハ次ノ如シ

$$\Delta l=\frac{H}{E}\left[\frac{8f_1^2l_1}{15I_1}+\frac{1}{A}\left(l_1+l_2+\frac{3h_1^2+16f_1^2}{2l_1}\right)\right]\quad\cdots\cdots(42)$$

等布荷重カ$b_1$マテ乘リシ時ニモ算式(42)ハ其ノ儘之ヲ適用スル事ヲ得ヘシ

又$W_3$ノ爲メニ起ル支鞍ノ移動ハ次ノ如シ

$$\Delta l=\frac{H}{E}\left[\frac{8f_1^2l_1}{15I_1}+\frac{1}{A}\left(l_1+l_2+\frac{3h_1^2+16f_1^2}{2l_1}\right)\right]\quad\cdots\cdots(43)$$

等布荷重カ$c_1$マテ乘リシ時ニモ算式(43)ハ其ノ儘之ヲ適用スル事ヲ得ヘシ.

## 第十一章　温度ノ變化ノ爲メニ受クル支鞍ノ水平移動

前同様左ノ塔ノ頂上ニ於テ水平ノ方向ニ$P$ナル補助荷重ヲ掛ケ$P$ノ爲メニ起リシ左側鏁條ノ水平分力ヲ$P$トシ温度ノ變化ノ爲メニ起リシ鏁條ノ水平分力ヲ$H_t$トス

### (一)扶構ノ内働ト其ノ微係數

左側扶構ノ彎曲率

$$M=-\frac{4(H_t+P)f_1}{l_1}x+\frac{4(H_t+P)f_1}{l_1^2}x^2 \qquad \frac{dM}{dP}=\frac{4f_1}{l_1}\left(-x+\frac{x^2}{l_1}\right)$$

$$\frac{d\omega}{dP}=\int_0^{l_1}\frac{M}{EI_1}\left(\frac{dM}{dP}\right)dx=\frac{8H_tf_1^2l_1}{15EI_1}$$

### (二)鏁條ノ内働ト其ノ微係數

左側徑間ニ於ケル鏁條ノ應力ハ次ノ如シ

$$T_1=(H_t+P)\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}} \qquad \frac{dT_1}{dP}=\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}$$

$$T_2=(H_t+P)\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}} \qquad \frac{dT_2}{dP}=\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}$$

$$\frac{d\omega}{dP}=\int_0^{l_1'}\frac{T_1}{EA}\left(\frac{dT_1}{dP}\right)ds+\frac{T_2l_2}{EA}\left(\frac{dT_2}{dP}\right)-\int_0^{l_1'}\left(\frac{dT_1}{dP}\right)\theta t ds-\left(\frac{dT_2}{dP}\right)\theta t l_2$$

$$=\frac{H_t}{EA}\left(l_1+l_2+\frac{3h_1^2+16f_1^2}{2l_1}\right)-\theta t\left(l_1+l_2+\frac{3h_1^2+16f_1^2}{3l_1}\right)\ \text{nearly}$$

### (三)吊材ノ内働ト其ノ微係數

左側徑間ニ於ケル吊材ノ應力ハ次ノ如シ

$$S_2=\frac{8(H_t+P)f_1}{n_1 l_1}\qquad\qquad \frac{dS_2}{dP}=\frac{8f_1}{n_1 l_1}$$

$$\frac{d\omega}{dP}=\frac{S_2L_2}{EA_2}\left(\frac{dS_2}{dP}\right)-\left(\frac{dS_2}{dP}\right)\theta t\,L_2$$

$$=\frac{64H_tf_1^2}{EA_2n_1l_1^2}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)-8\theta t\frac{f_1}{l_1}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)$$

(四)　塔ノ內側ト其ノ微係數

左ノ塔ニ於ケル應力ハ次ノ如シ

$$S=H_t\frac{4f}{l}+(H_t+P)\frac{h_1+4f_1}{l_1}\qquad\qquad \frac{dS}{dP}=\frac{h_1+4f_1}{l_1}$$

$$\frac{d\omega}{dP}=\frac{Sh_2}{EA_3}\left(\frac{dS}{dP}\right)+\frac{dS}{dP}\theta t h_2$$

$$=\frac{H_th_2}{EA_3}\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)\left(\frac{h_1+4f_1}{l_1}\right)+\theta t\frac{h_1+4f_1}{l_1}h_2$$

(五)　支鞍ノ水平移動ノ決定

今溫度ノ爲メニ起ル支鞍ノ水平移動ヲ$\Delta l_t$トスレハ次ノ如シ

$$\Delta l_t=\frac{H_t}{E}\left[\frac{8f_1^2l_1}{15I_1}+\frac{1}{A}\left(l_1+l_2+\frac{3h_1^2+16f_1^2}{2l_1}\right)\right.$$

$$\left.+\frac{64f_1^2}{A_2n_1l_1^2}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)+\frac{h_2}{A_3}\left(\frac{4f}{l}+\frac{h_1+4f_1}{l_1}\right)\left(\frac{h_1+4f_1}{l_1}\right)\right]$$

$$-\theta t\left\{\left(l_1+l_2+\frac{3h_1^2+16f_1^2}{3l_1}\right)+\frac{8f_1}{l_1}\left(h-\frac{h_1}{2}-\frac{2}{3}f_1\right)-\left(\frac{h_1+4f_1}{l}\right)h_2\right\}\quad\cdots\cdots\cdots\cdots(44)$$

吊材及塔ノ影響ヲ省略スルトキハ

$$\Delta l_t=\frac{H_t}{E}\left[\frac{8f_1^2l_1}{15I_1}+\frac{1}{A}\left(l_1+l_2+\frac{3l_1^2+16f_1^2}{2l_1}\right)\right]-\theta t\left(l_1+l_2+\frac{3l_1^2+16f_1^2}{3l_1}\right)\quad\cdots\cdots\cdots(45)$$

## 第十二章　死荷重ノ爲メニ受クル鏁條ノ撓度

鏁條ハ其ノ初メ架設中ニアッテハ只タ自己ノ重量ヲ受クルノミナレトモ築造完了ノ後ニハ床構並ニ扶構等ヨリ來タル死荷重ノ爲メ可ナリ大ナル張力ヲ受ケ從ッテ幾等カノ撓度ヲ起スモノナリ故ニ鏁條架設ノ初メニ當ッテハ鏁條ノ中央部ヲシテ其ノ最後ノ位置ヨリモ少シク高ク置カシムルコト必要ナリ

今床構並ニ扶構等(但シ鏁條ノ自重ハ含マス)ヨリ來ル水平毎呎ノ死荷重ヲ中央徑間及側徑間ニテ$w$及$w_1$トシ又是等ノ死荷重ヲ受ケシ後ノ最後ノ垂矢ヲ中央徑間及側徑間ニテ$f$及$f_1$トス

鏁條ハ自重ノ爲メ垂曲線(Catenary curve)ヲナスモノナレトモ床構及扶構等ヨリ來タル死荷重ヲ受クル時ハ拋線トナリ從ッテ自重ノ爲メニ生セル鏁條應力ニモ多少ノ變化ヲ生ス然レトモ其ノ差餘リ大ナラサルヲ以テ鏁條ノ自重ノ爲メニ生セル鏁條應力ニハ前後變化ナキモノト假定ス

### (第一)中央鏁條ノ撓度

(A)　第一解法

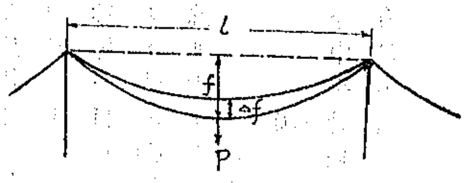
今中央鏁條ニ水平毎呎$w$ナル死荷重ヲ掛ケ又鏁條ノ中點ニ補助荷重$P$ヲ掛クレハ鏁條ノ水平分力$H$ハ次ノ如シ

$$H=\frac{wl^2}{8f}+\frac{Pl}{4f}\qquad\frac{dH}{dP}=\frac{l}{4f}$$

$$T=H\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}\qquad\frac{dT}{dH}=\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}$$

$w$ ナル死荷重ヲ受ケザル以前ニ於テハ垂曲線ヲナスモノナレトモ之ヲ拋線ナリト假定スレハ極小曲線長 $ds$ ハ次ノ如シ

$$ds=\left\{1+\left(\frac{8f-8\Delta f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}dx=\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}dx \quad \text{nearly}$$




第三十二圖

Castigliano ノ第一定理ニヨリ撓度 $\Delta f$ ハ次ノ如シ

$$\Delta f=\frac{d\omega}{dP}=2\int_0^{\frac{l}{2}}\frac{T}{EA}\left(\frac{dT}{dH}\right)\left(\frac{dH}{dP}\right)ds=\frac{2}{EA}\int_0^{\frac{l}{2}}H\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{3}{2}}\frac{l}{4f}dx$$

$$\therefore\ \Delta f=\frac{Hl}{2EAf}\left(\frac{l}{2}+\frac{4f^2}{l}\right) \quad \text{nearly}$$

$$\text{or} \quad =\frac{wl^3}{16EAf^2}\left(\frac{l}{2}+\frac{4f^2}{l}\right) \quad \text{nearly} \quad \cdots\cdots(46)$$

即鋼條ノ中央部ヲシテ $\Delta f$ 丈ケ上方ニ置カシムレハ可ナリ

(B)　第二解法

第一解法ニ於テハ原曲線 (Original curve) ヲ拋線ナリト假定シタレトモ實際ニ於テハ垂曲線ナルヲ以テ次ニ最モ正確ナル方法ヲ述ヘントス

先ツ初メニ垂曲線ノ方程式ヲ導ケハ次ノ如シ

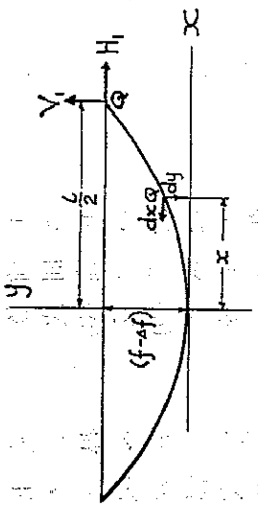
今 $w'$ ヲ鋼條ノ單位長カ有スル自重トシ鋼條ノ一部分 OQ ノ平衡ヲ考フルトキハ容易ニ次ノ關係式ヲ得ヘシ

$$\frac{dy}{dx}=\frac{V_1-w'\int_x^{\frac{l}{2}}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx}{H_1}$$

$$=\frac{V_1}{H_1}-\frac{w'}{H_1}\int_x^{\frac{l}{2}}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

上式ヲ微分スルトキハ($V_1$ $H_1$ $w'$ $l/2$ 等ハ常數ナルコトニ注意スルヲ要ス)

$$\frac{d^2y}{dx^2}=\frac{w'}{H_1}\sqrt{1+\left(\frac{dy}{dx}\right)^2},\qquad \frac{w'}{H_1}=c,\qquad \frac{dy}{dx}=Z$$ ト置ケハ $$\frac{d^2y}{dx^2}=\frac{dZ}{dx}$$ ナルヲ以テ




第二十四圖

$$\frac{dZ}{dx}=c\sqrt{1+Z^2}$$

$$\int\frac{dZ}{\sqrt{1+Z^2}}=c\int dx+C_1$$

$$log(Z+\sqrt{1+Z^2})=cx+C_1$$

$x=0$ ナルトキ $\frac{dy}{dx}=Z=0$ ナルヲ以テ $C_1=0$ ナリ

$$Z+\sqrt{1+Z^2}=e^{cx}\qquad\qquad \sqrt{1+\left(\frac{dy}{dx}\right)^2}=e^{cx}-\frac{dy}{dx}$$

兩邊ヲ自乘シ簡單ニスレハ

$$\frac{dy}{dx}=\frac{1}{2}\left(e^{cx}-e^{-cx}\right)\qquad\qquad y=\frac{1}{2}\left(\frac{1}{c}e^{cx}+\frac{1}{c}e^{-cx}\right)+C_2$$

$x=0$ ナルトキ $y=0$ ナルヲ以テ $C_2=-\frac{1}{c}$ ナリ

$$\therefore\quad y=\frac{1}{2c}\left(e^{cx}+e^{-cx}-2\right)\quad\cdots\cdots\cdots(47)$$

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又 $x=\frac{l}{2}$ ノトキ $y=(f-\Delta f)$ ナルヲ以テ

$$(f-\Delta f)=\frac{1}{2c}\left(e^{\frac{cl}{2}}+e^{-\frac{cl}{2}}-2\right)$$

$$=\frac{1}{2c}\left(\frac{c^2l^2}{4}+\frac{c^4l^4}{8\times24}+\cdots\cdots\right)$$

$$=\frac{cl^2}{8}+\frac{c^3l^4}{384}\quad\text{nearly}$$

$$\left(\frac{l^4}{384}\right)c^3+\left(\frac{l^2}{8}\right)c-(f-\Delta f)=0\quad\cdots\cdots(48)$$

上ノ三次方程式ヲ解キテ$c$ノ値ヲ見出シ得ヘク或ハ$c^3$ナル項ヲ省略スルトキハ次ノ如キ$c$ノ略値ヲ得ヘシ

$$c=\frac{8(f-\Delta f)}{l^2}\quad\cdots\cdots(49)$$

算式(48)及(49)ニ於テ$\Delta f$ノ値ハ初メニ之ヲ假定シ試算スルコトヲ要ス

前同様$w$ヲ水平毎呎ノ死荷重トシ$P$ヲ鏻條ノ中點ニ於ケル補助荷重トシ又$H$ヲ鏻條ノ水平分力トスレハ最後ノ曲線カ$f$ナル垂矢ヲ有スル拋線ナルヲ以テ

$$H=\frac{wl^2}{8f}+\frac{Pl}{4f}\qquad\frac{dH}{dP}=\frac{l}{4f}$$

$$T=H\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}\qquad\frac{dT}{dH}=\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}^{\frac{1}{2}}$$

又原曲線ナル垂曲線ノ極小曲線長$ds$ハ次ノ如シ

$$ds=\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\frac{1}{2}(e^{cx}+e^{-cx})dx$$

$$\Delta f=\frac{d\omega}{dP}=2\int_0^{\frac{l}{2}}\frac{T}{EA}\left(\frac{dT}{dH}\right)\left(\frac{dH}{dP}\right)ds$$

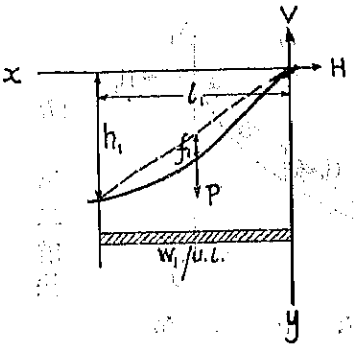
$$=\frac{2}{EA}\int_0^{\frac{l}{2}}H\left\{1+\left(\frac{8f}{l^2}\right)^2x^2\right\}\frac{l}{4f}\,\frac{l}{2}\,\frac{1}{2}\left(e^{cx}+e^{-cx}\right)dx$$

$$\therefore\ \Delta f=\frac{Hl}{4EAfc}\left[e^{\frac{cl}{2}}-e^{-\frac{cl}{2}}+\left(\frac{8f}{l^2}\right)^2\left\{e^{\frac{cl}{2}}\left(\frac{l^2}{4}-\frac{l}{c}+\frac{2}{c^2}\right)-e^{-\frac{cl}{2}}\left(\frac{l^2}{4}+\frac{l}{c}+\frac{2}{c^2}\right)\right\}\right]\cdots\cdots\cdots(50)$$

上ノ式ニ於テ$c$ノ値ハ算式(48)或ハ(49)ヲ以テ計算シ得ヘク從ッテ$\Delta f$ノ値ヲ求メ得ルナリ

算式(46)ト(50)ハ殆ト同値ヲ與フルカ故ニ原曲線ヲ拋線ト假定スルモ大差ナシ

(第二)側鉄條ノ撓度




第二十五圖

第一解法

(A) 原曲線ヲ拋線ト假定シ$w_1$ヲ水平毎呎ノ死荷重トシ$P$ヲ鉄條ノ中點ニ於ケル補助荷重トスレハ

$$H=\frac{w_1l_1^2}{8f_1}+\frac{Pl_1}{4f_1}\qquad\frac{dH}{dP}=\frac{l_1}{4f_1}$$

$$T_1=H\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}\qquad\frac{dT_1}{dH}=\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}$$

$$T_2=H\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}\qquad\frac{dT_2}{dH}=\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}$$

$$ds=\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}dx\ \cdots\cdots\ \text{nearly}$$

$$\Delta f_1=\frac{d\omega}{dP}=\int_0^{l_1'}\frac{T_1}{EA}\left(\frac{dT_1}{dH}\right)\left(\frac{dH}{dP}\right)ds+\frac{T_2}{EA}\left(\frac{dT_2}{dH}\right)\left(\frac{dH}{dP}\right)_{l_2}$$

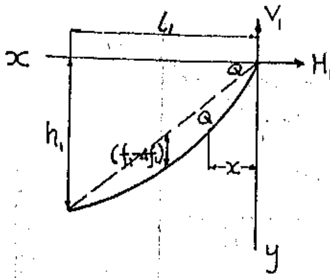
$$=\frac{Hl_1}{4EAf_1}\left(l_1+l_2+\frac{3h_1^2+16f_1^2}{2l_1}\right)\quad\text{nearly}\quad\cdots\cdots\cdots(51)$$

(*B*)　第二解法

原曲線ハ垂曲線ナルヲ以テ最モ正確ナル算式ヲ得ルニハ內働ノ計算ニ於テ $ds$ ハ垂曲線ノ方程式ニ依ラサルヘカラス然レトモ普通ノ設計ニ於テハ第一解法ヲ以テ充分ナリトスレトモ只タ參考ノ爲メ(實際ハ不必要ナレトモ)垂曲線ノ $ds$ ヲ以テ計算スレハ次ノ如シ

先ツ初メニ垂曲線ノ方程式ヲ導カンカ爲メ第二十六圖ノ左端ニテ彎曲率ヲ取レハ

$$V_1=\frac{H_1h_1+w'\int_0^{l_1}\sqrt{1+\left(\frac{dy}{dx}\right)^2}(l_1-x)dx}{l_1}$$




第二十六圖

今 QQ ナル鐵條ノ平衡ヲ考フレハ

$$\frac{dy}{dx}=\frac{V_1-w'\int_0^x\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx}{H_1}$$

$$=\frac{\frac{H_1h_1}{l_1}+\frac{w'}{l_1}\int_0^{l_1}\sqrt{1+\left(\frac{dy}{dx}\right)^2}(l_1-x)dx-w'\int_0^x\sqrt{1+\left(\frac{dy}{dx}\right)^2}dx}{H_1}$$

上式ヲ微分スレハ

$$\frac{d^2y}{dx^2}=-\frac{w'}{H_1}\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

今 $\frac{w'}{H_1}=c$, $\frac{dy}{dx}=Z$ ト置ケバ $\frac{d^2y}{dx^2}=\frac{dZ}{dx}$ ナルヲ以テ

$$\frac{dZ}{dx}=-c\sqrt{1+Z^2} \qquad log(Z+\sqrt{1+Z^2})=-cx+C_1$$

$C_1$ ハ或ル常數トス

$$Z+\sqrt{1+Z^2}=e^{-cx+C_1}=e^{C_1}e^{-cx}$$

$$\frac{dy}{dx}=\frac{1}{2}\left(e^{C_1}e^{-cx}-e^{-C_1}e^{cx}\right)$$

$$y=\frac{1}{2c}\left(-e^{C_1}e^{-cx}-e^{-C_1}e^{cx}\right)+C_2$$

$x=0$ ナルトキ $y=0$ ナルヲ以テ $C_2=\frac{1}{2c}\left(e^{C_1}+e^{-C_1}\right)$ ナリ

$$\therefore \quad y=\frac{1}{2c}\left(e^{C_1}+e^{-C_1}-e^{-C_1}e^{cx}-e^{C_1}e^{-cx}\right) \quad \cdots\cdots\cdots(52)$$

$x=l_1$ ナルトキ $y=h_1$ ニシテ $x=\frac{l_1}{2}$ ナルトキ $y=\frac{h_1}{2}+(f_1-\Delta f_1)$ ナルヲ以テ

$$h_1=\frac{1}{2c}\left(e^{C_1}+e^{-C_1}-e^{-(C_1-cl_1)}-e^{(C_1-cl_1)}\right)$$

$$\frac{h_1}{2}+(f_1-\Delta f_1)=\frac{1}{2c}\left\{e^{C_1}+e^{-C_1}-e^{-(C_1-\frac{cl_1}{2})}-e^{(C_1-\frac{cl_1}{2})}\right\}$$

上ノ二式ヨリ理論上$C_1$及$c$ノ値ヲ求メ得ルモノナレトモ$C_1$及$c$ノ略値ヲ求ムレハ次ノ如シ

$$h_1=\frac{1}{2}(2C_1l_1-cl_1^2)$$

$$\frac{h_1}{2}+(f_1-\varDelta f_1)=\frac{1}{2}\left(C_1l_1-\frac{1}{4}cl_1^2\right)$$

$$\therefore\quad c=\frac{8(f_1-\varDelta f_1)}{l_1^2}\qquad C_1=\frac{h_1+4(f_1-\varDelta f_1)}{l_1}\quad\cdots\cdots\cdots\cdots\cdots\cdots\cdots(53)$$

前同樣$w_1$ヲ水平每呎ノ死荷重トシ$P$ヲ鏈條ノ中點ニ於ケル補助荷重トシ又$H$ヲ是等ノ荷重ヨリ起レル鏈條ノ水平分力トスレハ最後ノ曲線カ$f_1$ナル垂矢ヲ有スル拋線ナルヲ以テ

$$H=\frac{w_1l_1^2}{8f_1}+\frac{Pl_1}{4f_1}\qquad\qquad \frac{dH}{dP}=\frac{l_1}{4f_1}$$

$$T_1=H\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}\qquad \frac{dT_1}{dH}=\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}^{\frac{1}{2}}$$

$$T_2=H\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}\qquad \frac{dT_2}{dH}=\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}^{\frac{1}{2}}$$

$$ds=\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx=\frac{1}{2}\left(e^{-C_1}e^{cx}+e^{C_1}e^{-cx}\right)dx$$

$$\varDelta f_1=\frac{d\omega}{dP}=\int_0^{l_1'}\frac{T_1}{EA}\left(\frac{dT_1}{dH}\right)\left(\frac{dH}{dP}\right)ds+\frac{T_2}{EA}\left(\frac{dT_2}{dH}\right)\left(\frac{dH}{dP}\right)l_2$$

$$=\frac{Hl_1}{8EAf_1}\int^{l_1}\left\{1+\left(\frac{h_1+4f_1}{l_1}-\frac{8f_1}{l_1^2}x\right)^2\right\}\left(e^{-C_1}e^{cx}+e^{C_1}e^{-cx}\right)dx+\frac{Hl_1}{\cdots}\left\{1+\left(\frac{h_1-4f_1}{\cdots}\right)^2\right\}l_2$$

$$\therefore\ \Delta f_1=\frac{H_1}{8EAf_1c}\Bigg[\left\{1+\left(\frac{h_1+4f_1}{l_1}\right)^2\right\}\left\{e^{-C_1}\left(e^{cl_1}-1\right)-e^{C_1}\left(e^{-cl_1}-1\right)\right\}$$
$$-\frac{16f_1(h_1+4f_1)}{l_1^3}\left\{l_1\left(e^{-C_1}e^{cl_1}-e^{C_1}e^{-cl_1}\right)-\frac{1}{c}\left(e^{-C_1}e^{cl_1}+e^{C_1}e^{-cl_1}-e^{-C_1}-e^{C_1}\right)\right\}$$
$$+\left(\frac{8f_1}{l_1^2}\right)^2\left\{e^{-C_1}e^{cl_1}\left(l_1^2-\frac{2l_1}{c}+\frac{2}{c^2}\right)-e^{C_1}e^{-cl_1}\left(l_1^2+\frac{2l_1}{c}+\frac{2}{c^2}\right)\right.$$
$$\left.+\frac{2}{c^2}\left(e^{C_1}-e^{-C_1}\right)\right\}+2l_1c\left\{1+\left(\frac{h_1-4f_1}{l_1}\right)^2\right\}\Bigg]\quad\cdots\cdots\cdots(54)$$

$C_1$及$c$ノ値ハ算式(53)ヲ以テ之ヲ求メ得可ク從ツテ$\Delta f_1$ノ値ヲ計算シ得ルナリ然レトモ此ノ第二解法ハ全ク徒勞ニシテ第一解法ヲ以テ充分ナリトス(完)