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COMPARISON BETWEEN CAMPBELL AND VAN GENUCHTEN MODELS BY INVERSE ESTIMATION OF UNSATURATED PROPERTIES

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Inverse technique to evaluate the unsaturated hydraulic properties of soil/rock was studied using Campbell and van Genuchten models, as the functions of the hydraulic properties. When the parameters of these models are given, the evaporation change under the constant climate and the simple boundary conditions can be calculated with considering the liquid and vapor flows in soil/rock. So that, the parameters can be inversely estimated by fitting the calculated evaporation change to the measured one. Genetic Algorithm (GA) was adapted to find the best parameters in these two models. It was found that the parameters in those models could be well estimated by this technique and the transient evaporation change and the moisture profile in homogeneous soil could be well simulated.

Key Words: *Evaporation, unsaturated hydraulic properties, genetic algorithm, inverse estimation*

1. INTRODUCTION

Evaporation and unsaturated hydraulic properties are important factors for evaluating the transportation of water in shallow underground and the movement of salinity and pollutant to soil surface. Evaporation is depending on soil hydraulic properties, climatic conditions on the surface and boundary conditions in soil, such as groundwater depth etc. Campbell[1)] proposed a numerical technique to calculate evaporation rate under the constant climate conditions and well-described boundary conditions with considering both liquid and vapor flow in soil. Also, he proposed a technique to inversely estimate the properties from the transient change of evaporation. This technique involves analytical or numerical solution of commonly used non-linear Richards' equation[2)]. Solution of Richards' equation needs to define the unsaturated hydraulic functions. He used the Campbell model as the function of unsaturated hydraulic properties.

In recent, van Genuchten model has been more commonly and widely adapted[3),4)] to formulate the unsaturated properties. For the reason, some researchers have proposed other inverse estimation techniques based on the van Genuchten formula to estimate the hydraulic properties using not only transient evaporation but also capillary suction change[5),6),7)]. However, most of them have evaluated the water flow in unsaturated soil and the vapor flow was not much taken into consideration. The aim of this study was to compare the applicability of Campbell model[8)] and van Genuchten model to predict unsaturated hydraulic properties of soil/rock from evaporation data. In this study both water and vapor flows were taken into consideration as Campbell has proposed. The Genetic Algorithm (GA)[9),10),11)] was used in the inverse estimation to optimize the parameters in both models.

2. ANALYSIS OF EVAPORATION

(1) Campbell model and van Genuchten model

Campbell and van Genuchten models have been used to formulate the unsaturated hydraulic properties. Campbell model[8)] can be written as:

$$k(\theta) = k_s \left(\frac{\theta}{\theta_s}\right)^m \tag{1}$$

$$\theta = \theta_s \left(\frac{\psi_e}{\psi}\right)^{1/b} \qquad (2)$$

$$m = 2b + 3 \qquad (3)$$

where θ and θ_s are volumetric and saturated water contents (m^3m^{-3}) of soil, respectively, ψ_e is air entry potential of soil (JKg^{-1} ~ 0.102 m of water), $\psi(\theta)$ is the matric potential of water (JKg^{-1} ~ 0.102 m of water), b and m are parameters related to pore size distribution. $k(\theta)$ is soil hydraulic conductivity (ms^{-1}) and k_S is saturated hydraulic conductivity (ms^{-1}). On the other hand, van Genuchten model[3),4)] can be written as:

$$k(\theta) = k_s \theta_e^{1/2} [1-(1-\theta_e^{1/l})^l]^2 \qquad (4)$$

$$\theta_e = \left(1 + |\alpha\psi|^n\right)^{-l} \qquad (\alpha > 0) \qquad (5)$$

$$\theta_e = (\theta - \theta_r)/(\theta_s - \theta_r) \qquad (0 \le \theta_e \le 1) \qquad (6)$$

$$n = 1/(1-l) \qquad (0 < l < 1, n > 1) \qquad (7)$$

where α (m^{-1}), l and n are parameters of soil. θ_r and θ_s are the residual and the saturated water contents (m^3m^{-3}), respectively. θ_e is the effective water content (m^3m^{-3}).

(2)Liquid and vapor flows in unsaturated soil/rock

When potential evaporation rate (E_v) in $mmday^{-1}$ is known, vapor flux (evaporation) from soil surface (q_{vs}) can be written as[1)]:

$$q_{vs} = E_v \frac{(h_s - h_a)}{(1 - h_a)} \qquad (8)$$

where h_s is the soil/rock surface humidity and h_a is the atmospheric humidity. h_s indicates only the saturation at the top surface. This equation implies that evaporation is a function of surface humidity (h_s) under the constant air humidity (h_a)[1)]. Therefore, it is estimated through the analysis of both liquid and the vapor flows in soil/rock.

a) Liquid-phase flow

Vertical liquid flow in unsaturated porous media is mostly described by Darcy-Buckingham formula:

$$q_L = -k[(\partial\psi / \partial z) + 1] \qquad (9)$$

where q_L is water flux (ms^{-1}), $k(\theta)$ is soil hydraulic conductivity (ms^{-1}), $\psi(\theta)$ is soil matric head (≤ 0, m) and z is vertical coordinate (m, positive upward).

b) Vapor-phase flow

Flux density of vapor (q_v) in isothermal soil is described by Fick's law[1)]:

$$q_v = -D_v \frac{dc_v}{dz} \qquad (10)$$

where q_v is vapor flux (ms^{-1}), c_v is soil vapor concentration (kgm^{-3}) and D_v is water vapor diffusivity in soil (m^2s^{-1}). With some arrangements Eq. (10) becomes[9)]:

$$q_v = -K_v \frac{d\psi}{dz} \qquad (11)$$

where K_v is vapor conductivity (ms^{-1}). Equations (9) and (11) are combined for estimating the total water flux. In this approximation, it is assumed that the vapor content is in equilibrium condition with suction pressure[1)].

3. INVERSE ESTIMATION USING GENETIC ALGORITHM

Fig. 1 schematically shows the procedures of the inverse estimation technique. Transient evaporation ($E_m(t)$) from soil surface, porosity (ϕ), θ_r, θ_s and K_s are measured by the experiments. By giving the boundary conditions, one-dimensional flow equation is numerically calculated for evaluating evaporation ($E_c(t)$) with assuming parameters in Campbell model or van Genuchten model. Then, the ($E_m(t)$) and ($E_c(t)$) are compared. When the summation of square differences (SSD) is less than the critical value (CrSSD), the iteration is stopped. Otherwise, new values of the parameters are assumed. The procedures[9)] are repeated till the best parameter sets are found, by which the calculated evaporation is well fitted to the measured one.

Many researchers have been using different optimization algorithms such as Newton's method, Gauss method, Levenberg-Marquard method etc. to solve inverse problems numerically[9)]. Recently Genetic Algorithm (GA) has been frequently

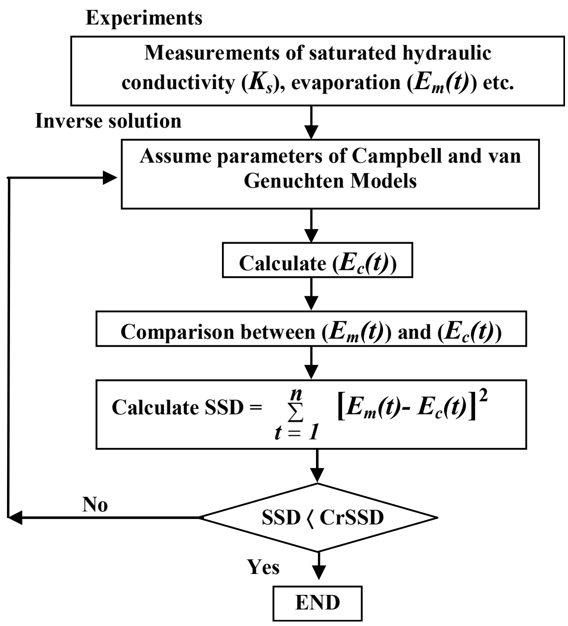

Fig. 1 Procedure of the inverse solution technique.

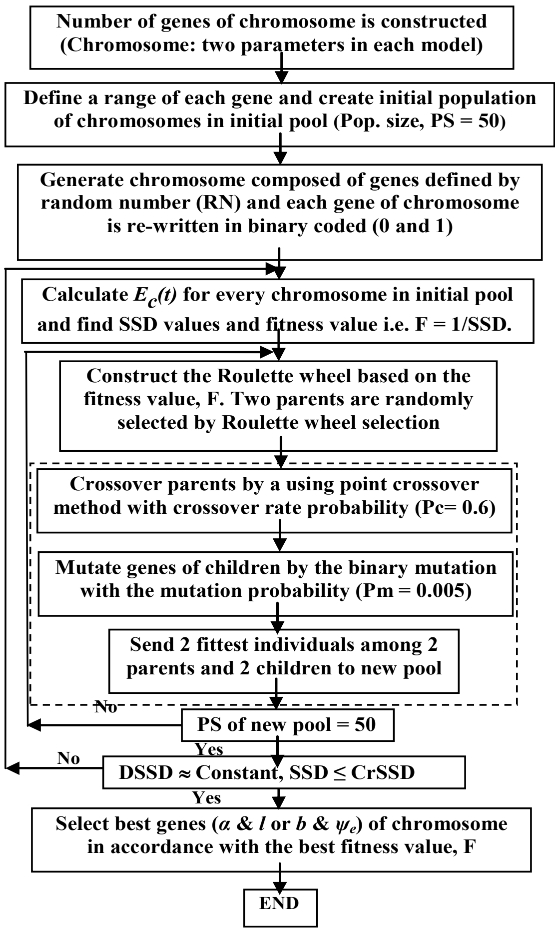

Fig. 2 Flow chart of GA application in the inverse solution.

applied to solve both science and engineering problems[9),10),11)]. It was reported that GA makes fast convergence and more efficient in finding the best solution as compared with other methods[9)]. Therefore, GA is adopted and is used for the parameter estimation in this study.

Fig. 2 illustrates the flow chart of GA application in the inverse solution technique[9)]. Each of the Campbell and the van Genuchten models has a combination of two unknown parameters (b and ψ_e) or (α and l) respectively and the combination is called as a chromosome. In this technique, each parameter (gene) is given as the random numbers (RN) within the range defined by the maximum and minimum values. Then, population size (PS) of combinations of two parameters generated by RNs are sent to the initial "pool". The transient evaporation rates ($E_c(t)$) are calculated for all combinations, and the summation of square difference (SSD) between ($E_c(t)$) and ($E_m(t)$) are calculated. Then the fitness values (F), which are the reverse of SSD, are calculated. For changing combination in pool, two arbitrary combinations (parents) are selected. A roulette wheel based on F values is formed and used for selecting parents. Then new generation (children) is generated through the crossover and the mutation operations. The two parent combinations and two children combinations are compared on their F values. Then the two combinations having the maximum F values are selected and sent to a new pool. This procedure of new generation creation is repeated till the population of new pool reaches to PS, which is 50. This process is named as the generation change. Whole procedure was repeated till SSD becomes lower than the critical value of CrSSD given and DSSD (difference between maximum and minimum SSD) becomes constant. In this study, the probabilities of crossover and mutation occurrence were assumed as 0.6 and 0.005, respectively. The convergence of GA was checked before the analysis and explained later by **Fig. 7**.

4. TESTS FOR TRANSIENT EVAPORATION CHANGE

The creation of unsaturated flow induced by evaporation under simple boundary condition is needed for the inverse technique. In this research, one dimensional unsaturated flow was created in a disk shaped sedimentary rock specimen and in a soil box.

(1) Sedimentary Rock

A disk shaped rock specimen of Tertiary fine sandstone was used. The specimen was initially saturated by submerging it in a container filled with distilled water and sucking air by a vacuum pump[9)]. Rock specimen was completely sealed from all surfaces except the upper surface to allow the evaporation (see **Fig. 3**). Then under the controlled humidity (40%) and temperature (25^0C) conditions, the transient evaporation change was calculated from the weight change of specimen.

(2) Sandy Soil

Fig. 4 shows a schematic view of the evaporation measurement equipment proposed by Ali *et al.*[4)]. Fine sand (Toyoura sand) and course sand (average diameter 1.34 mm) were used under the constant relative humidity (55%) and air temperature (≈ 28^0C) conditions. A soil box with interior dimensions of 10 cm width, 100 cm length and 50 cm height was covered by a wind tunnel (see **Fig 4**). Total depth of sand layer was set as 44 cm. Both sides of the soil box were connected to a constant head water tank to change the groundwater table in the soil box. As the initial condition, the water table was maintained at the soil surface. Then, the water table was rapidly dropped by 42 cm as schematically shown in **Fig. 5**. Air was injected into

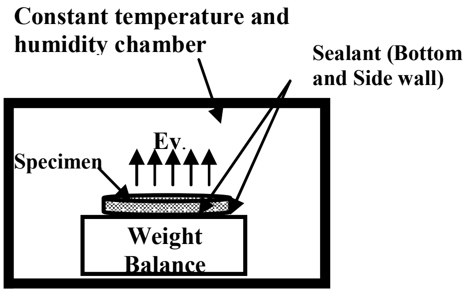

Fig. 3 Experimental condition for the measurements of evaporation from a rock specimen (Maung, *et al.*).

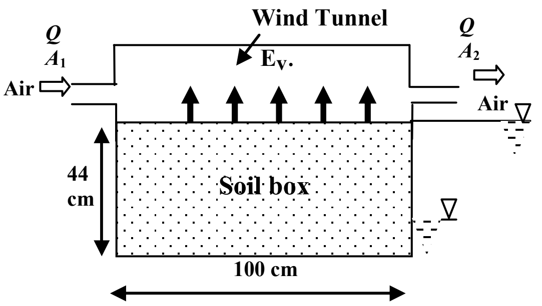

Fig. 4 Schematic view of evaporation measurement equipment.

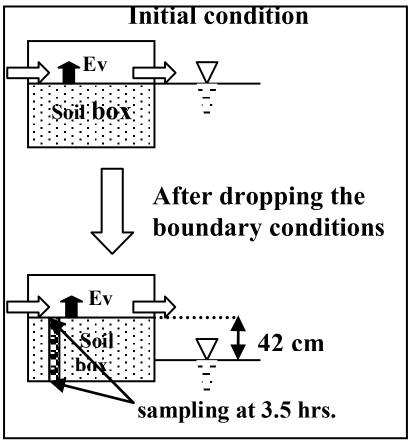

Fig. 5 Procedure of the experiment.

the wind tunnel, and exhausted from the other side for measuring transient evaporation after the groundwater table drop.

The evaporation rate (E_v) in mmday^{-1}, could be calculated using the following equations[5),12),13)].

$$E_v = Q(A_2 - A_1) \quad (12)$$

$$A = f(R,T) \quad (13)$$

where, Q, R, T and A are air flow rate (m^{3}s^{-1}), relative humidity, temperature (^{0}C) and absolute humidity (Mgm^{-3}), respectively. Subscripts 1 and 2 represent injected and exhausted air respectively. During the experiment, relative humidity, temperature of injected and exhausted air and the air flow (Q) are measured.

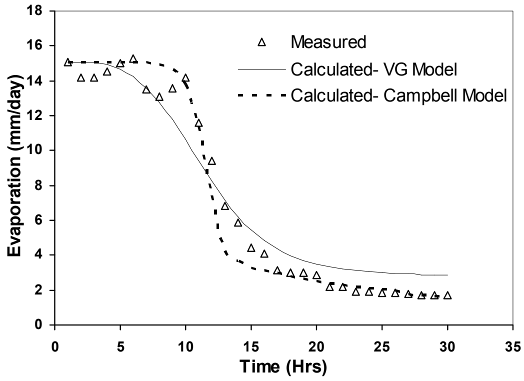

Fig. 6 Measured and calculated evaporation for sedimentary rock.

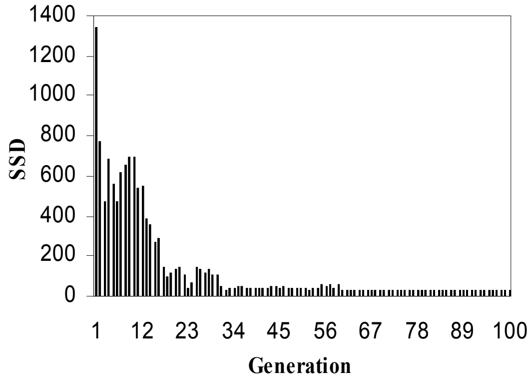

Fig. 7 Relation between generations and the SSD values in GA calculations for sedimentary rock specimen.

5. UNSATURATED PROPERTY ESTIMATION

(1)Unsaturated property estimation for sedimentary rock

Because this research deals with drying process due to evaporation, hysteresis in retention curve is not taken into account. **Fig. 6** shows the measured and the calculated evaporation for the fine sandstone using the Campbell and the van Genuchten models.

An example of the convergence of parameter estimation by GA is displayed in **Fig. 7**. It shows how the SSD values change during GA calculation for sedimentary rock using Campbell model. Although the SSD values started with large variation, after 30th generation the variation became small. As shown in this figure, the convergence of GA is usually good.

In general, it is found from **Fig. 6** that the evaporation change could be well analyzed by using both models. However, when comparing the calculated trends with measured evaporation, van Genuchten model showed a gentle change and after 15 hours gave a higher evaporation. Therefore, from this result, it can be said that Campbell model gives better estimation of evaporation for the sedimentary

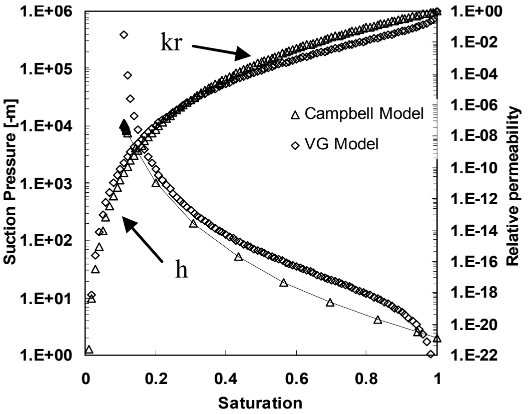

Fig. 8 Estimated hydraulic functions of sedimentary rock.

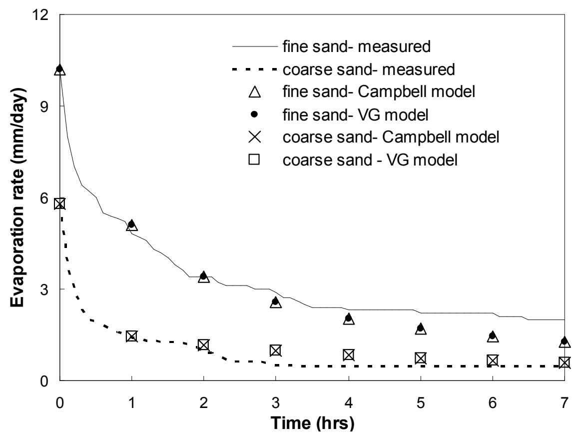

Fig. 9 Measured and simulated evaporation from soils.

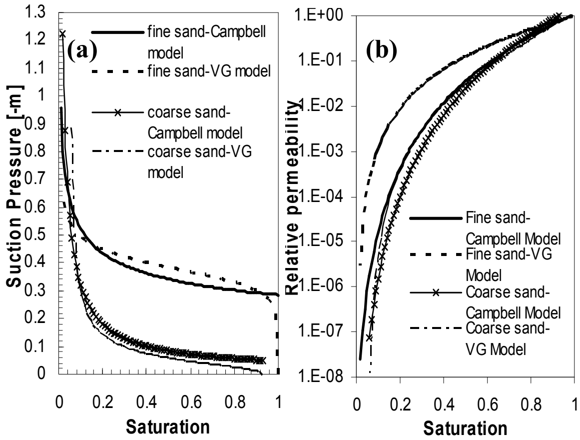

Fig. 10 Estimated hydraulic functions of soils.

rock. From evaporation itself we cannot conclude which model gives better estimation. Evaporation change is occurred in the top layer, therefore we have to study the saturation distribution of the sample. As the sedimentary rock is too small to measure the saturation distribution, a soil box was used to measure the saturation distribution as explained in **Fig 4** and **5**.

Fig. 8 shows the unsaturated hydraulic properties (the retention between capillary head [h] and relative hydraulic conductivity [kr]) of the sedimentary rock for both models. Even though the calculated evaporation changes were different to each other, the water retention curves by two models were not so different.

(2) Unsaturated property estimation for sand

Fig. 9 shows the comparison between the measured and the calculated evaporations for the experiment using two types of soils; fine sand and coarse sand (see **Table 1**). The evaporation from the coarse sand is lower in comparison with the fine sand. Lower evaporation implies that the saturation at the soil surface of coarse sand was low. It was observed that evaporation rate from these two types of soils was high at the beginning and decrease with the time. Higher evaporation implies the high water content at the soil surface. High water content condition was decreased by evaporation and by downward infiltration. As the general tendency, both calculated evaporations of fine sand were little bit lower than the measured one after 4 hours.

Fig. 10 (a) and **(b)** display the estimated hydraulic functions of soil for both Campbell and van Genuchten models. From **Fig. 10 (a)** it was found that the relation between saturation and capillary suction pressure are almost same for both models. However, the relation between saturation and relative permeability estimated for fine sand by using those models were different to each other. Relative permeability estimated by the Campbell model was 1~2 order lower than the van Genuchten model when the saturation is lower than 0.7. Although this difference was large, the evaporation could be well estimated using both models as explained in **Fig. 9**. The reason for such result is that the evaporation occurred from the soil surface. Therefore, to see the applicability of each model, a detailed study should be done using the saturation distribution. To investigate more in detail on the difference, the saturation distribution of fine sand was studied and described in section (4).

(3)Parameters estimated for sedimentary rock and sand samples

The measured and estimated parameters of two soils and the sedimentary rock are summarized in **Table 1** for both models.

Table 1 Measured and estimated parameters for soils and sedimentary rock.

Sample	$ø$ (%)	K_s (ms^{-1})	θ_r	θ_s	Campbell model		VG model	
					b -	Ψ_e (m)	α (m^{-1})	m -
F. sand	44.5	2.0E-4	0.01	0.99	3.8	-.28	2.64	.897
C. sand	39.6	4.7E-4	0.05	0.93	1.5	-.05	21.7	.601
S. rock	58.0	5.0E-6	0.10	1.00	4.6	-1.9	0.01	0.30

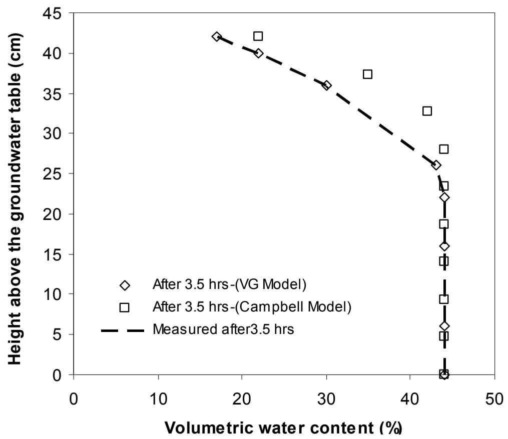

Fig. 11 Comparison between measured and calculated water contents for fine sand.

(4) Saturation distribution for fine sand

Vertical saturation profile of fine sand was measured at 3.5 hours after the sudden drop of groundwater table. Then, the profile was calculated by using both models. The results are displayed in **Fig. 11**. As shown in **Fig. 9**, the transient evaporation change can be well analyzed by using both models. However, from the comparison of saturation profiles van Genuchten model was in good agreement with measured one. From this fact it can be said that the van Genuchten model is better for sandy soil. As presented in **Fig. 9**, evaporation is approximated as the function of the saturation at the surface. It implies that the evaporation gives only the information of the saturation at the surface in the analysis. For the reason, it can be said that the parameters inversely estimated from evaporation must be also checked by the saturation profile.

6. CONCLUSION

In this study, inverse technique for estimating unsaturated hydraulic parameters of soil/rock from the transient evaporation change was studied. Both Campbell and van Genuchten models were used. The obtained results are as follows.

1. Transient evaporation change can be well estimated by both Campbell and van Genuchten models.
2. Parameters of both models can be well estimated from the transient evaporation change.
3. However, difference between both models was found in the relation between saturation and the relative permeability for fine sand.
4. From the analysis of vertical saturation profile, it can be said that the van Genuchten model is better than the Campbell model for fine sand.
5. The parameters which were inversely estimated from the evaporation should be further checked by the saturation distribution.

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