

A NUMERICAL MODEL TO STUDY BEDFORM DEVELOPMENT IN HYDRAULICALLY SMOOTH TURBULENT FLOWS

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A numerical model has been built to study the mechanism of sedimentary bedform development in hydraulically smooth turbulent flows. Sediment (bedload) flux is estimated by van Rijn (1984) formula corresponding to bed shear stress distribution obtained from flow solution by a Large-Eddy-Simulation (LES) method coupled with an Immersed-Boundary-Method (IBM). Evolution of bed surface is described by the Exner-Polya equation and simultaneously updated with the flow field.

Initiation and development of bedforms from an initially flat bed to fully developed forms has been successfully reproduced. Particularly, formation of newly and successively formed bedforms, growing and downstream propagating process of existing bedforms are very close to reported experimental observations. Bed shear stress and corresponding sediment flux around evolving bedforms, which are difficult to observe in experiments, have been well produced with this model.

Key Words: *sediment transport, bedforms, turbulent flows, Large-Eddy-Simulation*

1. Introduction

In hydraulic engineering, the problem of sedimentary bedform development has been studied for a long time. However, although abundant statistics on the bedforms in rivers have been reported and major consensus points have been reached¹⁾, there is still little understanding on the physics of the problem²⁾. Consequently, works to obtain more understanding on the mechanism of bedform development are still demanded.

Aiming to contribute more understanding of that mechanism, this work is conducted to study initiation and evolution of bedforms from erodible, initially flat sediment beds in turbulent flows. Flows in hydrodynamically smooth regions, i.e. with small particle Reynolds numbers are considered. Large-Eddy-Simulation (LES), a highly accurate method for solving the flow field, that has not yet been applied to this problem in the literature, is employed, and interactions of the evolving flow field and bed surface are described simultaneously.

2. Numerical model

We now summarize the computational model, and refer the reader to the previous works^{3, 4)} for more details.

In hydraulically smooth flows, by definition, roughness of the bed surface has negligible influence on the flow fields. Yalin¹⁾ argued that the maximum particle Reynolds

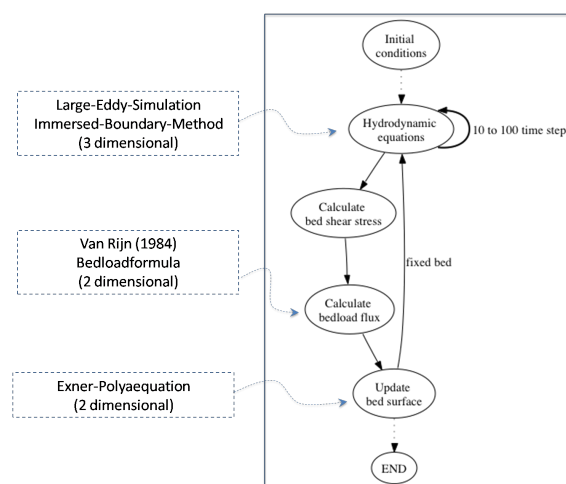


Figure-1 Computational procedure

number below which effects of individual grains on the flow field can be ignored is about 2.5. Adapting to that assumption, for the present study, the bed surface is treated as *smooth*, continuum, and the maximal particle Reynolds number is limited 2.5. At low particle Reynolds numbers, the only reported type of bedform is ripple, whose development and dimensions are assumed to be independent of the flow depth¹⁾. Accordingly, effects of the flow depth are also ignored in this model.

The computational procedure of the numerical model is shown in Figure 1. From desired initial conditions of flow fields and sediment properties, the three-dimensional flow field is solved by Large-Eddy-Simulation (LES) coupled

with the Immersed-Boundary-Method (IBM).

With IBM, the same governing equations are applied in the whole domain including both solid and fluid portions, on a *fixed* Cartesian grid, and an *artificial* body force $\mathbf{f}$ is added to the Navier-Stokes equation to account for the presence of the solid portions. IBM simplifies the grid-generating process and avoids re-generating the computational grid as the solid portions move.

The governing equations for LES coupled with IBM become⁵⁾:

$$\frac{\partial \bar{u}_j}{\partial x_j} = 0 \quad (1)$$

$$\frac{\partial \bar{u}_i}{\partial t} + \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} + f_i \quad (2)$$

where $\bar{\cdot}$ indicates the filtered quantities of the velocity u_i and the pressure p ; ρ and ν are the fluid density and viscosity, respectively; f_i is the artificial force in the i direction; τ_{ij} is the sub-grid stress (SGS), which is computed by the Shear-Improved Smagorinsky eddy viscosity model proposed by Leveque et al⁶⁾

The artificial body-force is evaluated by the direct forcing method⁵⁾. Eq.(2) is time-discretized as:

$$\bar{u}_i^{n+1} - \bar{u}_i^n = \Delta t (RHS + f_i)$$

where Δt is the time step and RHS contains the advection, pressure, subgrid stress, and the viscous terms. To impose the desired velocity v_{bi} within the solid body:

$$f_i = -RHS + (v_{bi} - \bar{u}_i^n) / \Delta t$$

inside the flow region occupied by the solid body and *zero* elsewhere, in the fluid portion(s). For grid cells completely inside the solid or fluid portion, implementation of f_i is straight-forward. For interfacial grid cells, f_i at the grid points closest to, but inside, the solid surface is linearly interpolated between the value that would yield v_{bi} inside the solid body and the value of zero inside the fluid portion. These implementations are evaluated at every time step.

The LES+IBM module was validated with test cases of flows over fixed sinusoidal bed surfaces and the computed results were compared to the conventional body-fitted Direct Numerical Simulation (DNS) methods. Results with mean flow quantities was reported in the previous publication³⁾; as an additional check, Figure 2 shows the results of bed shear stress distribution, which is the most important desired output from this module, obtained from two different grid resolutions, and validated with benchmark data of De Angelis et al⁷⁾ with the Reynolds number based on the mean friction velocity about 170.

Figure 4 showed that this LES+IBM module was able to produce reasonably instantaneous flow fields, and particularly bed shear stress adapting to evolving bed surfaces.

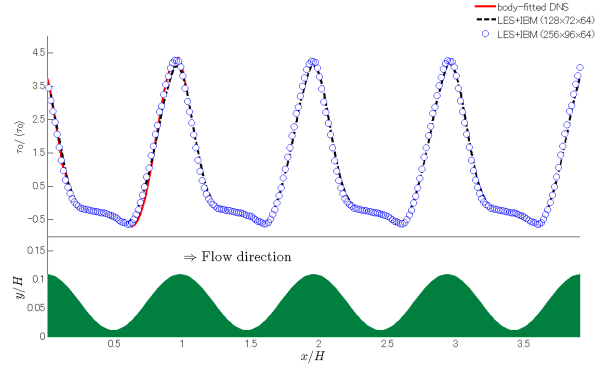


Figure-2 Bed shear stress distribution along a sinusoidal surface: LES+IBM vs.DNS results

More validation tests with mobile bed surface can be found in reference⁸⁾.

For sediment transport, only bedload, which dominates ripple formation, is considered in the model. For simplicity, the sediment transport and the bed surface are treated in two dimensions. Correspondingly, the bed shear stress, obtained from the flow solution with LES+IBM, is averaged in the span, z , direction. The equilibrium bedload flux is estimated by van Rijn formula⁹⁾ which is, to the authors' knowledge, the only formula valid at low particle Reynolds number. The bedload flux is then modified by effects of gravity or bedslope¹⁰⁾, which also affects critical condition for bedload motion¹¹⁾.

To model the well-known lag between local bedload flux and local bed shear stress¹²⁾, non-equilibrium bedload flux is then computed from the equilibrium one via an adaption-length¹⁰⁾ through a relaxation law. Finally, the local bed surface is evolved adapting to the local non-equilibrium bedload flux by the Exner-Polya equation.

To prevent the bedslope from being larger than the friction angle, which is non-physical, maximum bedslope is fixed to an angle ϕ_{fixed} , set slightly lower than the friction angle, to satisfy the following constraints:

$$\begin{cases} \partial h / \partial x = \phi_{fixed} \\ \int h \cdot dx = const \end{cases} \quad (3)$$

where h and x are the bed elevation and the stream direction, respectively. Here, the second constraint, together with employment of the Exner-Polya continuity equation of bedload flux, ensures conservation of the sediment mass. The particular choice of ϕ_{fixed} was observed to affect only downstream slopes of the bedform, and not to affect their growth rate nor downstream migration speeds.

It is assumed that time scale of flow development is much shorter than that of the bedform development¹²⁾. Accordingly, bed surface is treated as a fixed one while the flow field is solved in an interval of 10 to 100 timesteps to allow the flow field to adapt to the new bed profile. The process is repeated continuously (Figure 3).

To approximate a free surface, a stress-free boundary condition is applied on the upper surface. In the stream and span directions, periodic conditions are used. In the vertical direction, uniform grid spacing of about 0.9 wall unit is applied up to $y/H = 0.2$, where H is the total flow depth, and then a hyper-tangential grid distribution is used. In the stream and span directions, uniform grid spacings of 8.8 wall units are used. Compared to the considered range of the particle Reynolds number of less than 2.5, these grid resolutions offer fine and converged flow and sediment solutions.

Compared to relevant reported models on this problem in the literature which also treats the bed surface as a continuum^{10, 12, 15, 16, 21}, the present model has a similar treatment of the bedload flux and bed surface evolution, though the bedload flux was determined by a different formula, but has advanced treatment of the flow field. With the employment of LES+IBM, the present model offers highly accurate three-dimensional flow solutions compared to that of two-dimensional RANS (Reynolds-Averaged Navier-Stokes equations)^{10, 16}, the linearized equations^{12, 15}, or depth-average equations²¹, particularly when dealing with flow separations^{16, 18} which is clearly an dominant effect in bedform development.

In addition, using a three-dimensional flow solver for a two-dimensional bed profile may be thought to be excessive, but is required for solving three-dimensional turbulent structures¹⁶.

3. Result examples and discussions

Initial conditions (inputs) for the numerical model are:

- (a) initial bed level
- (b) initial flow conditions: domain size, computational grid, mean flow Reynolds number, $Re_{\tau_0} = H^+ = u_{\tau_0} H / \nu$, where u_{τ_0} is the mean initial bed friction velocity.
- (c) sediment conditions: mean initial particle Reynolds number, $Re_p = u_{\tau_0} d / \nu$; mean initial Shields number $\theta_0 = u_{\tau_0}^2 / (s - 1)gd$, where d is the particle diameter, s is the relative density, ratio between the sediment density and the fluid density, and g is the gravitational acceleration

Figure 3 shows an example of bedform development from an initially flat bed surface with $H^+ = 300$, $Re_p = 0.125$, and $\theta_0 = 1.7$. Bed profile is plotted at different nondimensionalized time, $\tilde{t} = tu_{\tau_0}/H$, up to $\tilde{t} = 360$. The initial flat bed surface was introduced with a small disturbance of a sinusoidal wave with the amplitude less

than *one* wall unit. From this disturbance, a small bedform appeared³) and triggered successive visible bedforms as observed in this figure.

The successive bedforms were not initiated simultaneously but appeared one by one. An existing beform triggered initiation of a new one downstream. The newly formed bedform grew in time, and once gaining a critical height, it again triggered another new one downstream. The process is repeated and after a certain interval, a chain of bedforms, which is named *ripple train* as observed in experiments¹⁷), is seen on the bed surface, at $\tilde{t} = [40 - 80]$ in Figure 3.

Once a bedform appeared, it already had a certain height and length which continued increasing in time until fully developed dimensions. At this time, the bedform simply migrated downstream. Examples of developed bedforms are the first and the second ones from the right at $\tilde{t} = 360$ in Figure 3.

The process of initiation and migration of the *ripple train* is now examined in two catagories:

- (a) How a new bedform was initiated, and
- (b) How an existing bedform migrated downstream.

Figure 4 shows an example of instantaneous flow field and bed shear stress, which were averaged in the span direction, around a newly formed bedform.

A new bedform is identified when a new front is formed downstream of an existing beform. First, the existing upstream bedform caused a flow separation behind it. Associated with this separation was a strong positive gradient of bed shear stress and correspondingly bedload flux, by which the bed was eroded. Farther downstream, the bed shear stress reached a peak and then experienced a negative gradient before returning to the base value. With the negative gradient of the bed shear stress, the sediment was deposited and a bump was formed. The more the sediment was eroded, the deeper that area was, and hence the stronger the gradient of the bed shear stress was. Therefore, the bump was continuously fed and grew gradually.

When the bump gained a certain threshold height, its downstream part turned into a clear front whose slope was still smaller than that of the fixed value, ϕ_{fixed} . Examination of the bed shear stress showed that this happened after the bed shear stress, or the Shields number, θ , behind the bump was smaller than the critical one, θ_c , for the bedload transport. Once $\theta < \theta_c$, the bedload flux was zero, hence there was discontinuity of the sediment transport on that area and that the scoured sediment was deposited right before that discontinuity point helped the bump to become a visible front. At this instant, the bed shear stress downstream of the front also dropped to zero, meaning that a new separation was going to be created and the associ-

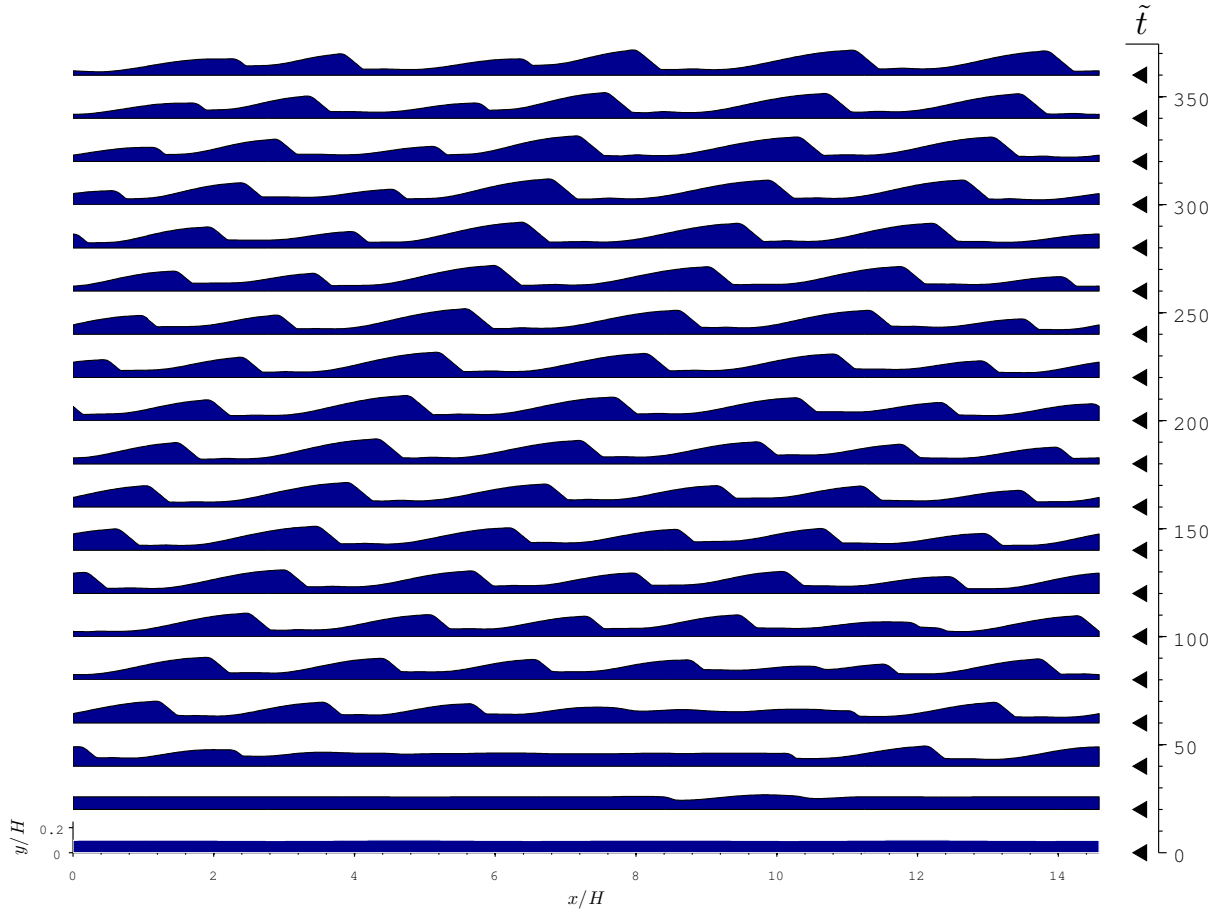


Figure-3 An example of bedform initiation and evolution from an initially flat bed surface. Test cases of $Re_p = 0.125$, $\theta_0 = 1.7$, and $H^+ = 300$.

ated strong positive gradient of the bed shear stress behind this point scoured the bed, and helped the front to be more visualized.

Tests with different Re_p and θ_0 showed that the height of such a newly formed bedform was $\eta^+ = \eta u_{\tau_0} / \nu \approx [12.0 - 13.0]$, thus approximately equal to the thickness of the viscous sublayer, $\delta_v \approx 11.6^{19)}$. Therefore, a newly formed bedform was just extruded above this layer, and its continuity was disrupted. The bedform now started directly affecting the turbulent core flow and a separation was expected to appear¹⁾. This point is confirmed with the zero bed shear stress mentioned above.

Figure 5 helps to clarify the question of how an existing beform grows and propagates downstream. Similar to the observations for the newly formed bedforms, separations before and after the bedform produced strong variation of the bed shear stress. Adapting to the bed shear stress, the bedload flux at the two ends of the bed form, where $\theta < \theta_c$ was zero, hence the sediment transport is limited within the surface of the bedform. Adapting to the gradient of the bed shear stress, the sediment was scoured over the upstream lee, particularly just behind the reattachment point, where the gradient was positive, and deposited over the down-

stream lee where the gradient was negative. This cyclic process led the bedform to grow and to propagate downstream.

Phase lag between the bed shear stress and the bed surface also helped the bedform gain height. The peak of the bed shear stress distribution was at a position upstream of the peak of the bed profile. Downstream of the peak of the bed shear stress, the bed shear stress gradient turned negative, and led to the sediment deposition from top of the bedform afterward, thus allowed the bedform to increase its height.

It is observed that the height of a fully developed bedform was about $[45 - 50]$ wall units, and the length was about $[950 - 1000]$ wall units, also from tests with other Re_p 's and θ_0 's, as shown in Figure 6. These dimensions are comparable to the reported ones in the literature. Mantz¹⁴⁾ reported that the minimum 'modal height' is about 38.0 wall units and the 'modal' length is about 700 wall units. Raudkivi²⁾ claimed that the experimental ripple length for $Re_p \leq 2.5$ in Yalin's works¹⁾ can be approximated in $[1000 - 2500]$ wall units.

Figure 7 shows variation of the bulk velocity normalized by the initial mean friction velocity, $\langle U \rangle / u_{\tau_0}$, and

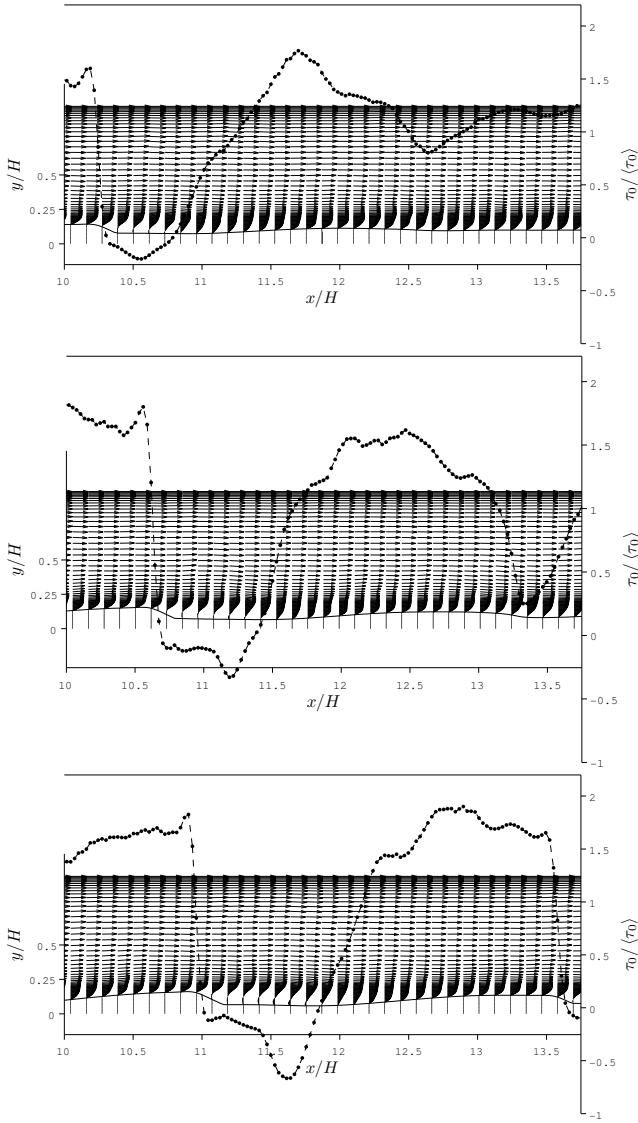


Figure-4 Successive instantaneous flow field and bed shear stress distributions, with a time interval of $\Delta \tilde{t} = 5$: *top*: behind an existing bedform; *middle*: around a growing bump; *bottom*: around a newly formed bedform

the Reynolds number $Re = \langle U \rangle H / \nu$, as the bedform developed, for the test case presented in Figure 3. Due to increasing form drag on the bedforms while the driving pressure gradient was kept constant, up to $\tilde{t} = 100$, these values decreased rapidly as during this time, the bedforms developed rapidly. Later, the dropping rate was steady and very low as the bedforms already filled up the domain, and simply continued growing and migrating downstream. At $\tilde{t} = 350$, these values dropped almost 40% from the initial ones. Such drops have also observed in the experiments by Coleman et al¹³⁾.

It is still remaining a big question that whether the assumption of ignoring effects of the flow depth is correct or not, i.e. whether the obtained bedforms are ripples so that their development process is independent of the flow power and the flow depth or not. To check this, tests with

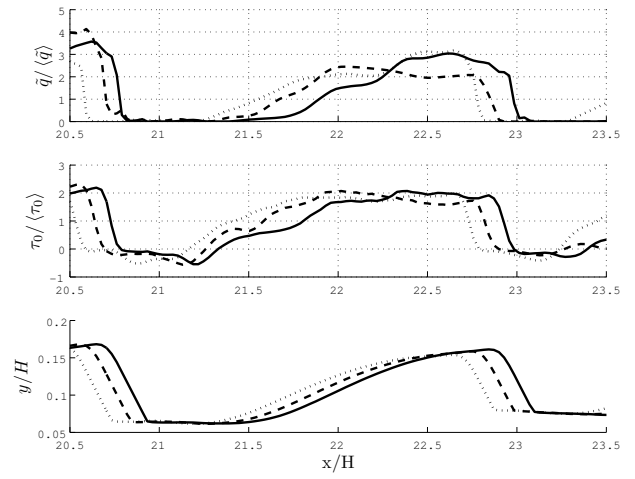


Figure-5 Instantaneous bedload flux, bed shear stress, and bed profile around an evolving bedform at successive time steps with a time interval of $\Delta \tilde{t} = 10$. *Top*: nondimensional bedload flux, $\tilde{q} = q / [(s-1)g]^{0.5} d^{1.5}$; *middle*: bed shear stress; *bottom*: bed profile

higher H^+ and/or with rigid upper surfaces will be conducted, and the development process of the bedforms as well as their mature dimensions in either cases will be compared to each other and to reported experimental results.

It is useful to contrast the present results with those by Sekine²²⁾, who coupled a 2D RANS flow model with a "saltation model" for an erodible sediment bed composed of spheres with a Shields parameter of 0.08 and Re_p of 36.6, which was much higher than the considered range in this study. After simulating the flight of 25000 particles in a streamwise-periodic 2D domain that was 2000 particle diameters long, random ripple-like features appeared on the bed. After moving 10^6 bed particles, two sand waves came to dominate which were clearly related to corresponding depressions in the free-surface elevation. Although the final waves had something of the fore-aft asymmetry of typical bedforms, they do not possess a clear downstream slip face. Furthermore the downstream-propagating cycle of ripple birth is not evident.

4. Summaries

To the authors' knowledge, there are two obvious advancements of the present model compared to the past ones. Firstly, the employment of three-dimensional Large-Eddy-Simulation (LES) allows the flow field, particularly bed shear stress and the flow separations downstream of the bedforms, to be solved at accuracy higher than the other two-dimensional models^{10, 16, 21, 22)}. Secondly, the

¹due to extremely high computational cost, only tests with $H^+ = 300$ has been conducted so far.

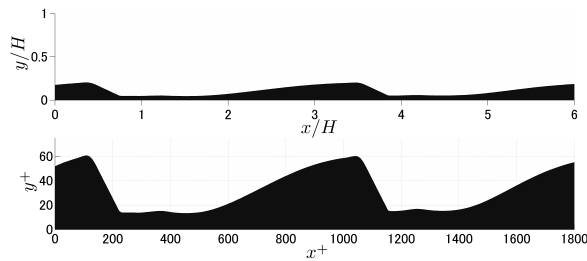


Figure-6 Developed dimensions of a bedform, nondimensionalized by: the total flow depth, *above*, in non-distorted scales; viscous lengthscale, *below*, with the vertical direction stretched for legibility.

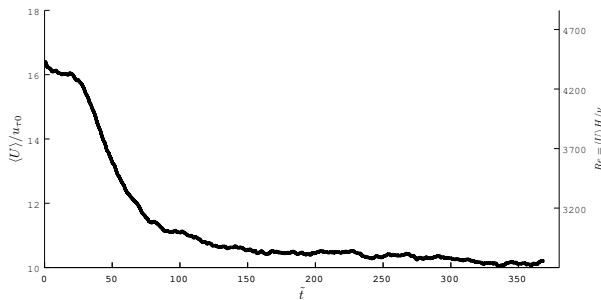


Figure-7 Variation of the bulk velocity due to development of bedforms, for the test case in Figure 3.

employment of Immersed-Boundary-Method (IBM) simplifies the process of grid generation and avoids re-gridding when the bed-surface evolves, hence avoids possible numerical errors associated with interpolations employed in re-gridding in traditional body-fitted methods.

To increase its accuracy as well as to widen its applicability, the model is going to be checked about sensitivity of each of its component to the computed bedform development process, such as estimation of bedload flux, effects of the adaption length (see references^{3,4}), effects of the flow power and flow depth (as discussed above)...

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