

CONDENSATION MASS TRANSFER IN UNSATURATED HUMID AIR INSIDE TUBULAR SOLAR STILL

Amimul AHSAN¹ and Teruyuki FUKUHARA²

¹Member of ASCE, M.Sc. Eng., D Phil Scholar, Dept. of Architecture and Civil Engineering, University of Fukui (3-9-1 Bunkyo, Fukui 910-8507, Japan)

²Member of JSCE, Dr. of Eng., Professor, Dept. of Architecture and Civil Engineering, University of Fukui (3-9-1 Bunkyo, Fukui 910-8507, Japan)

A production model of a Tubular Solar Still based on a film-wise condensation theory that takes into account of thermal resistance in unsaturated humid air inside the still is developed in this study. The condensation coefficient due to thermal resistance is inversely proportional to the dry air pressure fraction. In the present model, the overall heat transfer coefficient between the humid air inside the still and the ambient air outside the still is used for the first time because the measurement of the ambient air temperature is easier to obtain than the inner surface temperature of the tubular cover. The analytical solution of this condensation theory could provide a good agreement with the observed production flux obtained from this laboratory experiment. Furthermore, in order to assess the model accuracy, the model prediction is compared with field experimental data and proves that the model can be used to precisely predict the production flux.

Key Words : Condensation, heat and mass transfer, Tubular Solar Still (TSS), unsaturated humid air

1. INTRODUCTION

Solar distillation may be one of the viable options for providing drinking water for a single house or a small community in an arid or coastal region. A solar still can be easily built on-site without using special tools. This easy assembly helps to shorten the transportation distance of the water from the still to the point of use. A basin type still is the most popular among solar stills, but is difficult to quickly repair and also to construct. A new type of solar distillation called a Tubular Solar Still (TSS) was designed by the authors' group to overcome the disadvantages in the maintenance and management of the still. The TSS consists of a transparent tubular cover made of vinyl chloride and a trough inside the cover. Consequently, the weight of the TSS becomes much lighter than that of the basin type with a typical glass cover.

Many researchers¹⁾⁻⁴⁾ have proposed production models of the basin type still using evaporative heat and mass transfer correlations to predict the distilled water output, i.e. production. The distilled water is, however, produced after the condensation which

takes place on the inner surface of the tubular cover, following the evaporation from the water surface in a trough as shown in **Fig. 1**. Therefore, there exists a time lag between the beginning of the production of

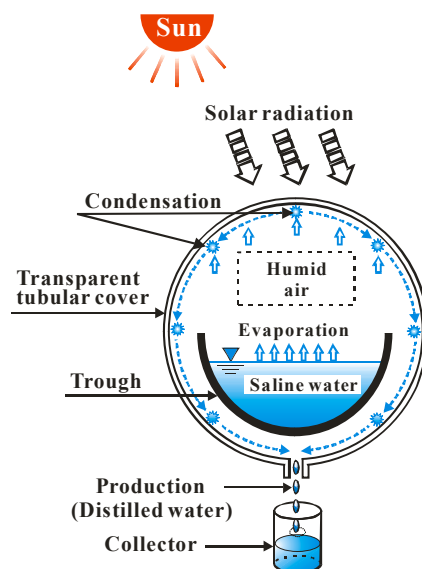


Fig.1 Production mechanism of TSS.

distilled water and the occurrence of the evaporation⁵). By observing the time lag in the evaporation → condensation → production process in the still, it can be deduced that the theory of condensation may give a better result predicting the hourly production than the past evaporation theories.

Many condensation theories⁶⁻⁸) have been applied to a film flow achieved on a plate or a tube that derive the condensate mass flow and the heat transfer rate. However, the condensation theory for solar stills has not been fully examined compared with the other evaporation theories, because the data of the temperature and relative humidity of humid air is inadequate. We recently found from our field experiments^{9,10}) that the relative humidity of the humid air in the daytime is by no means saturated (50-95%). From these experiments, the condensate mass transfer coefficient was obtained empirically, but the theoretical expression was not fully developed.

This paper aims to propose a production model based on a film-wise condensation theory that takes into account the thermal resistance of unsaturated humid air. Furthermore, this paper will also show the validity of the proposed condensation theory by comparing the field experimental results of hourly production with the theoretical predictions.

2. CONDENSATION THEORY OF TSS

(1) Momentum and energy equations

The following assumptions are made to simplify the analysis:

1. The TSS is cylindrical and set horizontally.
2. The condensate liquid film (liquid film) is a laminar flow and flows only along the inner circumference of the tubular cover, i.e. in the angular (θ) direction shown in **Fig. 2**.
3. The liquid film is a single homogeneous flow in the axial (x) direction shown in **Fig. 3(a)**.
4. The temperatures of the humid air, T_{ha} , and the tubular cover, T_{ci} , are uniform.
5. The heat transfer at the liquid film-humid air interface (interface) is caused only by condensation. The convection in the liquid film associated with the angular velocity, v_θ does not contribute to the heat transfer in the liquid film.
6. The boundary layer of T_{ha} exists on the interface and the boundary layer thickness, δ_{ha} , or the surface temperature of the liquid film, T_{ha}' , might be affected by the thermal resistance associated with the presence of a non-condensable gas (dry air) in the humid air (see **Fig. 3(b)**).
7. The interfacial shear stress at the interface is negligible, i.e. $\tau_{i\theta}=0$.

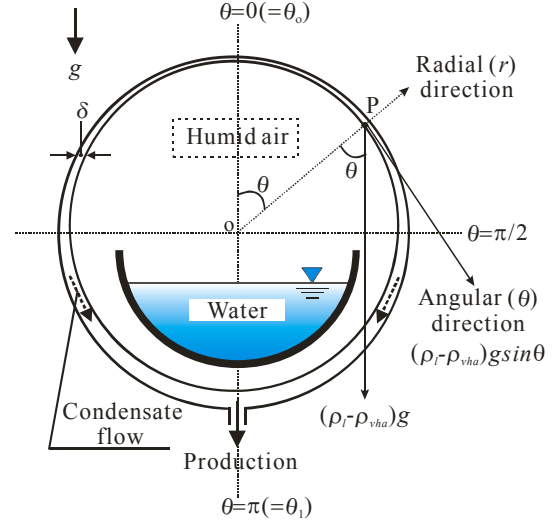


Fig.2 Condensate liquid film flowing on inner surface of tubular cover and component of gravity force at point P.

8. The velocity (v_θ) gradient in the θ direction is negligible compared to the radial (r) direction.

Then the governing equations of momentum and energy for a liquid film under a steady state condition can be simplified as follows:

Momentum equation:

$$\left(\frac{d^2 v_\theta}{dr^2} + \frac{1}{r} \frac{dv_\theta}{dr} - \frac{v_\theta}{r^2} \right) + \frac{g}{\mu_l} (\rho_l - \rho_{vha}) \sin \theta = 0 \quad (1)$$

Energy equation:

$$\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = 0 \quad (2)$$

The governing equations are subjected to the following boundary conditions:

$$v_\theta = 0; T = T_{ci} \text{ at } r = R, 0 \leq \theta \leq \pi \quad (3)$$

$$\tau_{i\theta} = \mu_l \left[r \frac{d}{dr} \left(\frac{v_\theta}{r} \right) \right] = 0; T = T_{ha}' \quad (4)$$

$$\text{at } r = R - \delta(\theta), 0 \leq \theta \leq \pi$$

When integrating Eq.(1) is subjected to the boundary conditions of Eqs.(3) and (4), the profile of v_θ in the r direction becomes

$$v_\theta = \left[\frac{XR}{3} + \frac{X(R-\delta)^3}{6R^2} \right] r - \frac{X(R-\delta)^3}{6r} - \frac{Xr^2}{3} \quad (5)$$

where $X = \frac{g}{\mu_l} (\rho_l - \rho_{vha}) \sin \theta$.

Using Fourier's law, the substantial heat flux for the neat area of the liquid film, q_s , at the interface ($r=R-\delta$) is given by

$$q_s = -\lambda_l \frac{dT}{dr} \Big|_{r=R-\delta} \quad (6)$$

where λ_l is the thermal conductivity of the condensate liquid.

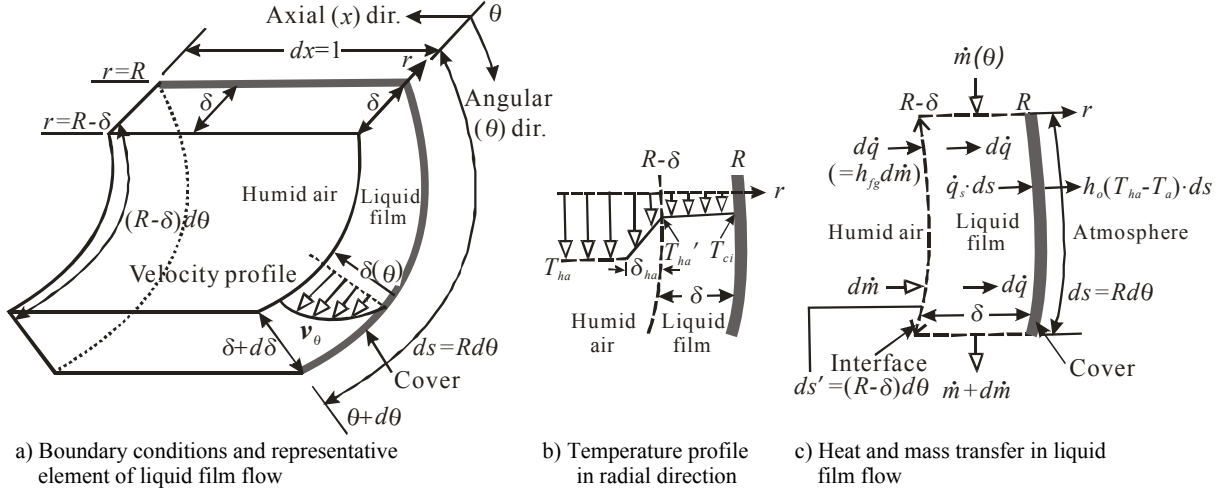


Fig.3 Boundary conditions and heat and mass transfer in liquid film flow.

Integrating Eq.(2) subjected to the boundary conditions, $T = T_{ci}|_{r=R}$ and $T = T_{ha}|_{r=R-\delta}$, yields

$$\left. \frac{dT}{dr} \right|_{r=R-\delta} = \frac{T_{ha} - T_{ci}}{(R-\delta) \ln\left(\frac{R-\delta}{R}\right)} \quad (7)$$

Substituting Eq.(7) into Eq.(6), q_s is given by

$$q_s = \frac{\gamma_1 \lambda_l (T_{ha} - T_{ci})}{(R-\delta) \ln\left(\frac{R}{R-\delta}\right)} \quad (8)$$

where $\gamma_1 = \frac{T_{ha} - T_{ci}}{T_{ha} - T_{ci}} (<1)$. γ_1 is called the temperature correction coefficient, which might be affected by the thermal resistance at the interface.

Since the liquid film thickness is fairly small compared to the radius of the TSS (i.e. $\delta \ll R$), Eq.(8) is approximated as

$$q_s = \frac{\gamma_1 \lambda_l (T_{ha} - T_{ci})}{\delta(\theta)} \quad (9)$$

The apparent heat flux for the whole area of the tubular cover, \dot{q}_s , can be expressed using Eq.(9) as

$$\dot{q}_s = q_s \gamma_2 = \frac{\gamma_c \lambda_l (T_{ha} - T_{ci})}{\delta(\theta)} \quad (10)$$

where $\gamma_2 (<1)$ is the area fraction (=net area of the liquid film/whole area of the tubular cover) and might be influenced by the wettability of the cover material. $\gamma_c (= \gamma_1 \gamma_2)$ is called the condensation coefficient.

According to the assumption 5. in Section 2.(1), the heat flow across the interface per unit length ($dx=1$) due to condensation is equal to that across the inner surface of the tubular cover shown in Fig. 3(c). That is,

$$d\dot{q} = h_{fg} d\dot{m} = \dot{q}_s \cdot ds \quad (11)$$

Using $ds=Rd\theta$ and Eq.(11), \dot{q}_s is written as

$$\dot{q}_s = h_{fg} \frac{d\dot{m}}{Rd\theta} = h_{fg} \frac{d\dot{m}}{d\delta} \frac{d\delta}{Rd\theta} \quad (12)$$

(2) Condensate liquid film thickness

The condensate mass flow for $dx=1$, $\dot{m}(\theta)$, can be obtained by the following integration:

$$\dot{m}(\theta) = \int_{R-\delta}^R \rho_l v_\theta dr \quad (13)$$

Substituting Eq.(5) into Eq.(13) yields

$$\dot{m}(\theta) = \frac{\rho_l X \delta^3}{3} = \frac{g \rho_l (\rho_l - \rho_{vha}) \delta^3 \sin \theta}{3 \mu_l} \quad (14)$$

Differentiating Eq.(14) in terms of δ yields

$$\frac{d\dot{m}}{d\delta} = \frac{g \rho_l (\rho_l - \rho_{vha}) \delta^2 \sin \theta}{\mu_l} \quad (15)$$

Substituting Eqs.(10) and (15) into Eq.(12), the following relation is derived.

$$\delta^3 d\delta = \frac{\gamma_c \lambda_l \mu_l D (T_{ha} - T_{ci})}{2g \rho_l (\rho_l - \rho_{vha}) h_{fg}} \frac{d\theta}{\sin \theta} = A \frac{d\theta}{\sin \theta} \quad (16)$$

where $A = \frac{\gamma_c \lambda_l \mu_l D (T_{ha} - T_{ci})}{2g \rho_l (\rho_l - \rho_{vha}) h_{fg}}$ and $D=2R$.

Integrating Eq.(16) subjected to the boundary condition, $\delta=0$ at top of the tubular cover, i.e. $\theta=\theta_o (\approx 0)$, yields the variation of δ in the θ direction,

$$\delta(\theta) = \left[4A \ln \left| \frac{\tan \theta / 2}{\tan \theta_o / 2} \right| \right]^{1/4} \quad (17)$$

(3) Heat transfer coefficient of unsaturated humid air

\dot{q}_s can be also expressed using the local heat transfer coefficient of a liquid film, h_l , in the form

$$\dot{q}_s = h_l (T_{ha} - T_{ci}) \quad (18)$$

Substituting Eq.(10) into Eq.(18), h_l is given as

$$h_l = \frac{\gamma_c \lambda_l}{\delta(\theta)} \quad (19)$$

Substituting Eq.(17) into Eq.(19) yields

$$h_l = \gamma_c \lambda_l \left[4A \ln \left| \frac{\tan \theta / 2}{\tan \theta_o / 2} \right| \right]^{-1/4} \quad (20)$$

The average heat transfer coefficient, h_c , is defined as (see Fig. 2)

$$h_c = \frac{1}{(\theta_1 - \theta_o)} \int_{\theta_o}^{\theta_1} h_l d\theta \quad (21)$$

Since $\theta_o=0^\circ$ cannot be applied to the integration of Eq.(21) because 0° cannot be calculated, $\theta_o=1^\circ$ and $\theta_1=180^\circ$ are chosen in this study. Thus, h_c is,

$$h_c = 0.996 \gamma_c \frac{3}{4} \left[\frac{g \rho_l (\rho_l - \rho_{vha}) h_{fg} \lambda_l^3}{\mu_l D (T_{ha} - T_{ci})} \right]^{1/4} \quad (22)$$

However, Eq.(22) is not practical because the measurement of T_{ci} is not as easy to obtain, compared to the measurement of T_w , T_{ha} and T_a . We, therefore, propose a new overall heat transfer coefficient, h_o , defined by the following equation:

$$h_o (T_{ha} - T_a) = h_c (T_{ha} - T_{ci}) \quad (23)$$

Eq.(23) means that the heat flux across the tubular cover is given by the product of h_o and the difference in temperature between the inside and outside of the TSS, i.e. $T_{ha}-T_a$.

Supposing that the temperature difference fraction, $a=(T_{ha}-T_{ci})/(T_{ha}-T_a)$, is constant, h_o is expressed as

$$h_o = 0.996 \gamma_c \frac{3}{4} \left[\frac{g \rho_l (\rho_l - \rho_{vha}) h_{fg} a^3 \lambda_l^3}{\mu_l D (T_{ha} - T_a)} \right]^{1/4} \quad (24)$$

Thus, the condensate mass flux is defined as

$$m = h_o (T_{ha} - T_a) / h_{fg} \quad (25)$$

Substituting Eq.(24) into Eq.(25), γ_c is given by

$$\gamma_c = \left(\frac{m}{0.996} \right)^{4/3} \left[\frac{\mu_l D h_{fg}^3}{g \rho_l (\rho_l - \rho_{vha}) a^3 \lambda_l^3 (T_{ha} - T_a)^3} \right]^{1/3} \quad (26)$$

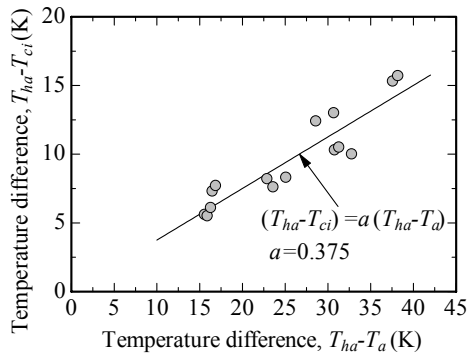


Fig.4 Relation between temperature difference, $T_{ha}-T_{ci}$, and temperature difference, $T_{ha}-T_a$.

Table 1 Laboratory experimental conditions and observed steady state values.

Case No.	Experimental conditions			Observed values	
	R_s (W/m ²)	T_a (°C)	RH_a (%)	T_{ha} (°C)	RH_{ha} (%)
1		35.3		66.1	78
2		32.1		63.4	78
3	1200	26.6	35	59.4	77
4		22.2		59.8	73
5		17.0		55.2	72
6		35.3		58.2	81
7		32.0		55.6	81
8	800	26.5	35	51.6	81
9		22.4		51.0	76
10		16.2		46.9	75
11		35.1		50.7	85
12		31.7		47.6	86
13	500	26.5	35	42.8	86
14		22.7		39.2	90
15		17.0		33.9	90

3. EXPERIMENTAL METHOD AND CONDITIONS

(1) Laboratory experiments⁵⁾

To find the properties of γ_c and a , our laboratory experimental results are cited in this paper. Laboratory experiments on the productivity of a TSS were conducted in a thermostatic room to keep a constant ambient air temperature and a relative humidity. The equipment consisted of a TSS, a solar simulator, two electric balances, a data logger and two computers. The TSS was comprised with a tubular cover and with a semicircular black trough in it. The tubular cover used in the experiment was made of a curled transparent vinyl chloride sheet of 0.5mm in thickness. The length and outside diameter of the TSS were 0.52m and 0.13m, respectively. The trough was made of vinyl chloride with 1.0mm in thickness, 0.1m in outside diameter and 0.49m in length. Production was enhanced by using 12 infrared lamps (125W). The radiant heat flux, R_s , was controlled by changing the height of the lamps. The temperatures (T_{ha} , T_{ci} and T_a), RH_{ha} and R_s were measured by thermo-couples, a

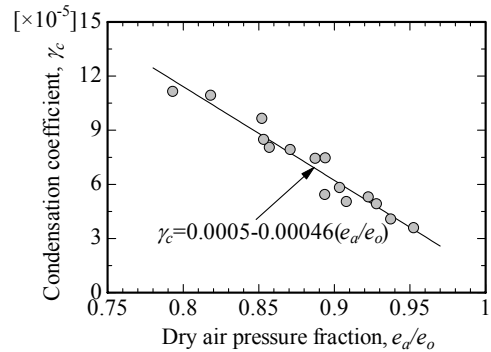


Fig.5 Relation between condensation coefficient, γ_c , and dry air pressure fraction, e_a/e_o .

thermo-hygrometer and a pyranometer, respectively. All the data were automatically recorded into a data logger and computers at one-minute intervals.

Table 1 shows the conditions of the laboratory experiments. The production was measured under three different levels of R_s and five different T_a , respectively.

(2) Field experiments⁹⁾

Our field experimental results are cited to support the validity of the condensation theory. The same specifications mentioned in the section above for the TSS were fabricated for the field experiments.

4. RESULTS AND DISCUSSIONS

(1) Laboratory experiments

Fig. 4 shows the relation between $T_{ha}-T_{ci}$ and $T_{ha}-T_a$. As can be seen from the straight regression line, the value of a in Eq.(26) is 0.375. Since the condensation rate was not measured, γ_c was calculated by Eq.(26) under the assumption that the condensation flux is equal to the production flux.

Since the humid air is definitely not saturated, it is inferred that γ_c would be affected by the thermal resistance due to the presence of a dry air. Based on this concept, **Fig. 5** shows the relation between γ_c and the dry air pressure fraction, e_a/e_o (=partial dry air pressure/total atmospheric pressure). The value of γ_c is inversely proportional to e_a/e_o and the regression can be given as

$$\gamma_c = 0.0005 - 0.00046 \frac{e_a}{e_o} \quad (27)$$

where $\frac{e_a}{e_o} = 1 - \frac{e_{vha}}{e_o}$; $e_{vha} = f(T_{ha}, RH_{ha})$.

Substituting Eq.(27) and $a=0.375$ into Eq.(24), h_o is expressed by

$$h_o = \left(1.86 - 1.72 \frac{e_a}{e_o} \right)^{\frac{3}{4}} \left[\frac{g \rho_l (\rho_l - \rho_{vha}) h_{fg} \lambda_l^3}{\mu_l D (T_{ha} - T_a)} \right]^{\frac{1}{4}} \times 10^{-3} \quad (28)$$

The hourly condensation mass flux is defined as

$$m_h = \frac{3600 h_o (T_{ha} - T_a)}{h_{fg}} \quad (29)$$

Fig. 6 shows the comparison of m_h calculated by Eq.(29) with the observed hourly production flux, p_h . Eq.(29) corresponds nicely with the observed p_h .

(2) Field experiments

Fig. 7 shows the observed diurnal variations of T_{ha} , T_a and RH_{ha} obtained in Fukui, Japan on September 29 and October 6, 2005. The temperature, T_{ha} rose rapidly after sunrise (approximately 6:00) and peaked between 12:00 and 13:00 on both days.

The value of RH_{ha} was remarkably below 100% in the daytime (minimum 50%) but about 100% during the night.

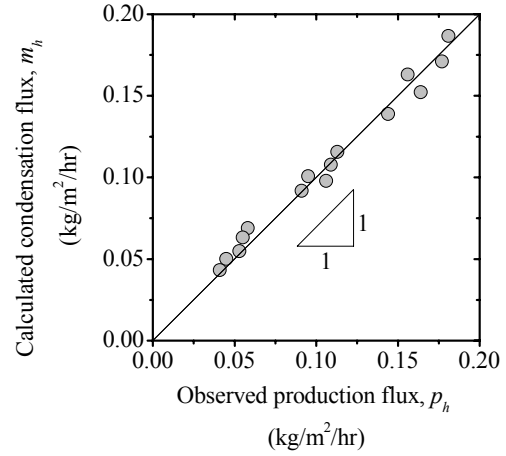


Fig.6 Comparison of calculated condensation flux, m_h , with observed production flux, p_h .

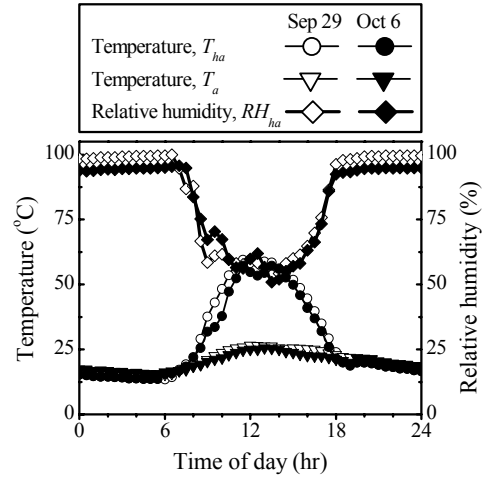


Fig.7 Diurnal variations of temperatures and relative humidity obtained on September 29 and October 6, 2005 in Fukui, Japan.

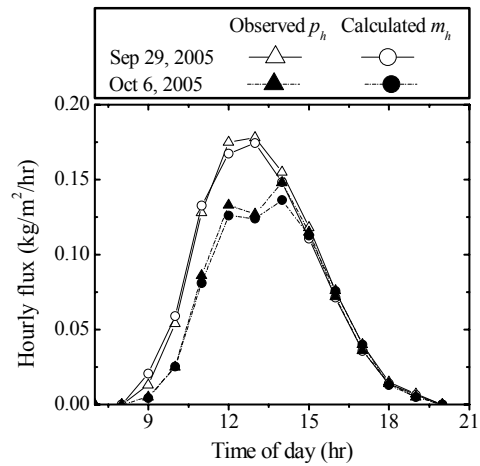


Fig.8 Diurnal variations of observed production flux, p_h , and calculated condensation flux, m_h .

Fig. 8 shows the diurnal variations of p_h obtained from the field experiment and of m_h calculated by Eq.(29). The profile of the p_h is almost the same as that of the m_h . The time lag between the condensation and the production is negligible as far as the TSS used in the experiment is concerned.

Fig. 9 shows the comparison between the p_h and the m_h . Both correspond well with each other. From **Figs. 8** and **9**, it is revealed that the p_h can be substituted for m_h .

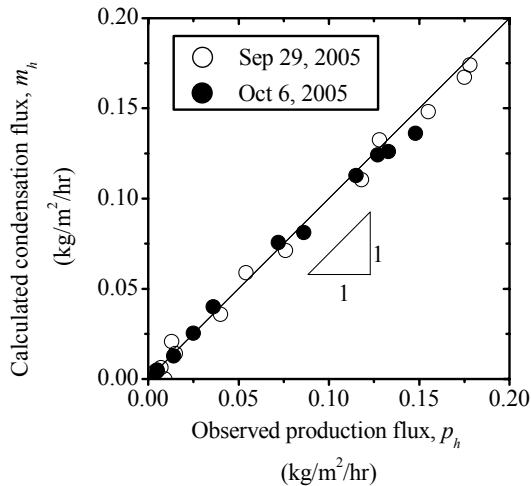


Fig.9 Comparison of calculated condensation flux, m_h , with observed production flux, p_h .

5. CONCLUSIONS

A production model of a Tubular Solar Still based on a film-wise condensation theory that takes into account of thermal resistance in unsaturated humid air inside the still is developed in this study. It was revealed that the condensation coefficient was in reverse proportion to the dry air pressure fraction. The present condensation theory using an overall heat transfer coefficient between humid and ambient air could provide a good agreement with the hourly production flux obtained from laboratory experiments. Moreover, the validity of the condensation theory was evaluated with a comparison between the field experimental data and the theoretical results. Due to the correspondence of these results it can be concluded that the present condensation theory can be used to precisely predict the production flux of a TSS.

NOMENCLATURE

D	: diameter of TSS	(m)
e_a	: partial dry air pressure	(Pa)
e_o	: total atmospheric pressure	(101325 Pa)
e_{vha}	: partial water vapor pressure in humid air	(Pa)
g	: gravitational acceleration	(9.807 m/s ²)
h	: heat transfer coefficient	(W/m ² ·K)
h_{fg}	: latent heat of vaporization	(J/kg)
\dot{m}	: condensate mass flow per unit length	(kg/m·s)

m	: condensation mass flux	(kg/m ² ·s)
p_h	: hourly production mass flux	(kg/m ² ·hr)
\dot{q}	: heat flow per unit length	(W/m)
q_s	: substantial heat flux	(W/m ²)
\dot{q}_s	: apparent heat flux	(W/m ²)
R	: radius of TSS	(m)
R_s	: radiative heat flux	(W/m ²)
RH	: relative humidity	(%)
s	: wetted perimeter	(m)
T	: temperature	(K)
T'	: temperature of liquid film	(K)
v_θ	: angular velocity	(m/s)

Greek letters:

γ_c	: condensation coefficient	(-)
γ_1	: temperature correction coefficient	(-)
γ_2	: area fraction	(-)
δ	: condensate film thickness	(m)
θ	: azimuthal angle	(radians)
λ	: thermal conductivity	(W/m·K)
μ	: dynamic viscosity	(kg/m·s)
ρ	: density	(kg/m ³)
τ	: shear stress	(Pa)

Subscripts:

a	: ambient air
c	: condensation; condensate
ci	: inner surface of tubular cover
h	: hourly
ha	: humid air
$i\theta$: interfacial shear stress in θ direction
l	: saturated liquid; local
o	: overall between humid and ambient air
vha	: water vapor in humid air

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