REFINED SIMULATION OF SOLITARY PLUNGING BREAKER BY CMPS METHOD

Abbas KHAYYER[1] and Hitoshi GOTOH[2]

[1] Member of JSCE, Doctoral Student, Dept. of Urban and Environmental Engineering, Kyoto University (Katsura Campus, Nishikyo-ku, Kyoto, 615-8540, Japan)

[2] Member of JSCE , Dr. of Eng., Associate Professor, Dept. of Urban and Environmental Engineering, Kyoto University (Katsura Campus, Nishikyo-ku, Kyoto, 615-8540, Japan)

As a meshfree particle method, the Moving Particle Semi-implicit (MPS) method is ideally suited for simulating the complicated behaviour of water surface with fragmentation. In this paper, the original formulations of MPS method are revisited from the view point of momentum conservation. Modifications and corrections are made to ensure the momentum conservation in a particle-based calculation of viscous incompressible free-surface flows. The excellent performance of Corrected MPS (CMPS) method in the exact (and nearly exact) conservation of linear (and angular) momentum is shown by a simple numerical test. The CMPS method is then applied to the simulation of wave breaking and post-breaking. Refined simulation of a solitary plunging breaker and resultant splash-up is demonstrated through comparisons with experiment. A tensor-type strain-based viscosity is also proposed for further enhanced CMPS reproduction of splash-up.

Key Words **:** *MPS method, CMPS method, momentum conservation, plunging breaker, splash-up, strain-based viscosity.*

1. INTRODUCTION

Accurate simulation of a free surface flow is a challenging hydraulic problem due to the presence of an arbitrary moving interface. Numerous grid-based interface capturing techniques such as the VOF method[1)] were developed to tackle the difficulty in free surface modeling. Nevertheless, the VOF-type models suffer from the problem of numerical diffusion arising from the grid-based discretization of advection terms in the Navier-Stokes equation. The numerical diffusion becomes more significant when the free surface undergoes large deformations accompanied by fluid fragmentations (as in the case of a plunging wave breaking and resultant splash-up). A few algorithms such as the CIP method[2)] have been proposed to attenuate the numerical diffusion; yet, the implementation of such sophisticated algorithms would further complicate the computational procedure for free surface modeling.

Recently, the meshfree particle methods have been applied in many engineering applications including the simulation of free-surface flows. Thanks to the fully Lagrangian treatment of discrete particles, the particle methods can easily handle the difficulty in free surface modeling without the numerical diffusion. Particle methods can be classified into those based on field estimations, as the Element Free Galerkin method, or those based on kernel approximations, as the Smoothed Particle Hydrodynamics (SPH) and the Moving Particle Semi-implicit (MPS) methods.

Originally developed by Koshizuka and Oka[3)], the MPS method has proven useful in a variety of problems. The MPS method has been improved and extended into Coastal Engineering to study wave breaking[4)] and overtopping[5)]. In spite of being a capable method for calculation of hydraulic phenomena, the MPS method suffers from some inherent difficulties, one of which is the non-conservation of momentum. This has been a critical theme in the SPH research[6)]. In case of the MPS method, however, there have been much less studies regarding to the mentioned difficulty.

This paper is focused on the momentum

conservation properties of the original MPS formulations. The aim is to improve the performance of the MPS method by modifying and correcting the formulations while maintaining their robustness and simplicity. The target phenomenon is a strong plunging breaker and the resultant splash-up. Moreover, as the splash-up is a highly deformed flow characterized by anisotropic strain rates, it would be preferable to calculate the viscous forces by applying a tensor-type strain-based viscosity. For this reason, we propose a strain-based viscosity when the CMPS method is supposed to simulate a highly anisotropically deformed flow as the splash-up.

2. MPS METHOD

In this section, the MPS method is briefly explained. Detailed description is provided by Gotoh *et al.*[5]. The fluid is modeled as an assembly of interacting particles, the motion of which is calculated through the interactions with neighboring particles. The governing equation is the Navier-Stokes equation:

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla p + \boldsymbol{g} + \nu \nabla^2 \boldsymbol{u} \tag{2}$$

where $\boldsymbol{u}$ = particle velocity; t = time; ρ = fluid density; p = particle pressure; $\boldsymbol{g}$ = gravitational acceleration and ν = laminar kinematic viscosity. The above equations are discretized by use of particle interaction models defined as[3]:

$$\langle \nabla \phi \rangle_i = \frac{D_s}{n_0} \sum_{j \neq i} \frac{\phi_j - \phi_i}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2} (\boldsymbol{r}_j - \boldsymbol{r}_i)\, w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{3}$$

$$\langle \nabla^2 \phi \rangle_i = \frac{2D_s}{\lambda n_0} \sum_{j \neq i} (\phi_j - \phi_i) w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{4}$$

where D_s = number of space dimensions, $\boldsymbol{r}$ = coordinate vector of fluid particle, $w(r)$ = the kernel function, n_0 = the constant particle number density and λ is a parameter introduced as follows:

$$\lambda = \frac{\sum_{j \neq i} w(|\boldsymbol{r}_j - \boldsymbol{r}_i|)\,|\boldsymbol{r}_j - \boldsymbol{r}_i|^2}{\sum_{j \neq i} w(|\boldsymbol{r}_j - \boldsymbol{r}_i|)} \tag{5}$$

Following Koshizuka *et al.*[7], the pressure gradient is defined by replacing ϕ_i in Eq. 3 by the minimum value of ϕ among the neighboring particles, such as:

$$\langle \nabla p \rangle_i = \frac{D_s}{n_0} \sum_{j \neq i} \frac{p_j - \hat{p}_i}{|\boldsymbol{r}_j - \boldsymbol{r}_i|^2} (\boldsymbol{r}_j - \boldsymbol{r}_i)\, w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \tag{6}$$

$$\hat{p}_i = \min_{j \in J} (p_i, p_j) \quad , \; J = \{ j : w(|\boldsymbol{r}_j - \boldsymbol{r}_i|) \neq 0 \} \tag{7}$$

This replacement improves the stability of the code by ensuring the interparticle repulsive force[7].

3. MOMENTUM CONSERVATION PROPERTIES OF MPS FORMULATIONS

(1) Conservation of linear momentum

The total linear momentum of a system of particles is given by:

$$\boldsymbol{G} = \sum_{i=1}^{N} m_i \boldsymbol{u}_i \tag{8}$$

where N = total number of fluid particles; m_i and $\boldsymbol{u}_i$ represent the mass and velocity of particle i, respectively. The motion of each particle is governed by the Newton's second law:

$$\boldsymbol{F}_i - \boldsymbol{A}_i = m_i \boldsymbol{a}_i \tag{9}$$

where $\boldsymbol{F}_i$ and $\boldsymbol{A}_i$ denote the external and internal forces acting on particle i and $\boldsymbol{a}_i$ is the instantaneous particle acceleration. In the absence of external forces, the rate of change of total linear momentum is:

$$\dot{\boldsymbol{G}} = \sum_{i=1}^{N} m_i \boldsymbol{a}_i = -\sum_{i=1}^{N} \boldsymbol{A}_i \tag{10}$$

Hence, the condition for preservation of linear momentum can simply be written as:

$$\sum_{i=1}^{N} \boldsymbol{A}_i = \sum_{i=1}^{N} \sum_{j \neq i}^{N_i} \boldsymbol{A}_{ij} = 0 \tag{11}$$

In Eq. 11, N_i = total number of neighboring particles of particle i; $\boldsymbol{A}_{ij}$ = the internal interacting force between particle i and its neighboring particle j. It can be shown that the linear momentum is exactly conserved for the viscous forces as the same magnitude of forces act in the opposite direction. From Eq. 4, the viscous force on particle i owing to j is:

$$\boldsymbol{A}^{\nu}_{j \to i} = m_i \nu \left(\nabla^2 \boldsymbol{u}\right)_{j \to i} \quad ; \; \left(\nabla^2 \boldsymbol{u}\right)_{j \to i} = \frac{2}{\lambda} (\boldsymbol{u}_j - \boldsymbol{u}_i)\, w(|r_j - r_i|) \tag{12}$$

which is exactly equal and opposite to the force on particle j owing to i. For the pressure interacting forces, however, the same is not true. From Eq. 6, the force due to pressure on particle i owing to j is:

$$\boldsymbol{A}_{j\to i}^{p} = \frac{-m_i}{\rho}\frac{p_j - \hat{p}_i}{\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|^2}(\boldsymbol{r}_j - \boldsymbol{r}_i)\, w\left(\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|\right) \tag{13}$$

while the pressure force on particle j owing to i would be:

$$\boldsymbol{A}_{i\to j}^{p} = \frac{-m_j}{\rho}\frac{p_i - \hat{p}_j}{\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|^2}(\boldsymbol{r}_i - \boldsymbol{r}_j)\, w\left(\left|\boldsymbol{r}_i - \boldsymbol{r}_j\right|\right) \tag{14}$$

Therefore, $\boldsymbol{A}_{j\to i}^{p} \neq -\boldsymbol{A}_{i\to j}^{p}$. Consequently, conservation of linear momentum is not guaranteed for the pressure forces. Even if p_i had not been replaced with the minimum pressure at neighboring particles, the pressure interacting forces were equal (if m_i=m_j) in magnitude but not opposite in direction.

(2) Conservation of angular momentum

The total angular momentum of a system of particles with respect to origin is given as:

$$\boldsymbol{H} = \sum_{i=1}^{N} \boldsymbol{r}_i \times m_i \boldsymbol{u}_i \tag{15}$$

By time differentiating and considering the law of motion in the absence of external forces, the rate of change of angular momentum of the system is:

$$\dot{\boldsymbol{H}} = \sum_{i=1}^{N} \boldsymbol{r}_i \times m_i \boldsymbol{a}_i = -\sum_{i=1}^{N} \boldsymbol{r}_i \times \boldsymbol{A}_i \tag{16}$$

Thus, conservation of angular momentum will be guaranteed if:

$$\sum_{i=1}^{N} \boldsymbol{r}_i \times \boldsymbol{A}_i = 0 \tag{17}$$

when $\boldsymbol{A}_{ij}$ = -$\boldsymbol{A}_{ji}$, the angular moment of the interacting forces between particles i and j will be:

$$\boldsymbol{r}_i \times \boldsymbol{A}_{ij} + \boldsymbol{r}_j \times \boldsymbol{A}_{ji} = -\boldsymbol{r}_{ij} \times \boldsymbol{A}_{ij} \tag{18}$$

The above term will vanish whenever the interaction force $\boldsymbol{A}_{ij}$ is co-linear with the position vector $\boldsymbol{r}_{ij}$. The interacting pressure forces between particles i and j are collinear with the vector $\boldsymbol{r}_{ij}$ as the pressure term (Eq. 6) is a product of a scalar and the vector $\boldsymbol{r}_{ij}$. However, since the interacting pressure forces are not opposite, similar to the linear momentum, the conservation of angular momentum is not ensured. In case of the viscous forces, the interactions do not necessarily lie on the same line with vector $\boldsymbol{r}_{ij}$; hence, conservation of angular momentum is not guaranteed either. Briefly speaking, in the MPS method, angular momentum is not conserved while linear momentum is conserved only in case of viscous forces.

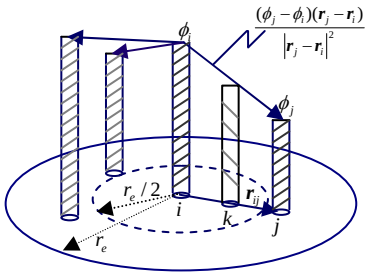

Fig. 1 Concept of gradient operator in MPS and CMPS methods

4. DERIVATION OF CMPS FORMULATIONS

(1) CMPS: conservation of linear momentum

As previously discussed in section 3.1, the pressure gradient term in the MPS method does not guarantee the conservation of linear momentum. For this reason, we propose another formulation for pressure gradient term. Eq. 6 is rewritten here, splitting the nominator of the fraction containing the pressure terms.

$$\left\langle \nabla p \right\rangle_i = \frac{D_s}{n_0}\sum_{j\neq i}\left\{\frac{p_j}{\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|^2}(\boldsymbol{r}_j - \boldsymbol{r}_i)\, w\left(\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|\right) - \frac{\hat{p}_i}{\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|^2}(\boldsymbol{r}_j - \boldsymbol{r}_i) w\left(\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|\right)\right\} \tag{19}$$

The concept of the gradient model in the MPS method is depicted in **Fig. 1**. In order to derive the new formulation, an imaginary point k is considered on the midpoint of the position vector $\boldsymbol{r}_{ij}$. The gradient term is now modified considering point k and the imaginary position vector $\boldsymbol{r}_{ik}$ (= $\boldsymbol{r}_k$ - $\boldsymbol{r}_i$).

$$\left\langle \nabla p \right\rangle_i = \frac{D_s}{n_{0-ik}}\sum_{j\neq i}\left\{\frac{p_k}{\left|\boldsymbol{r}_k - \boldsymbol{r}_i\right|^2}(\boldsymbol{r}_k - \boldsymbol{r}_i)\, w\left(\left|\boldsymbol{r}_k - \boldsymbol{r}_i\right|\right) - \frac{\hat{p}_i}{\left|\boldsymbol{r}_k - \boldsymbol{r}_i\right|^2}(\boldsymbol{r}_k - \boldsymbol{r}_i) w\left(\left|\boldsymbol{r}_k - \boldsymbol{r}_i\right|\right)\right\} \tag{20}$$

In Eq. 20, $n_{0\text{-}ik}$ refers to the particle number density in the new imaginary influence circle of particle i which contains the neighboring particles k. In the MPS method, originally a linear variation of pressure is assumed in the short distance between particles i and j. Hence, p_k can be substituted by $(p_j+p_i)/2$ while $\boldsymbol{r}_{ik}$ is also $\boldsymbol{r}_{ij}/2$. Therefore:

$$\left\langle \nabla p \right\rangle_i = \frac{D_s}{n_{0-ik}}\sum_{j\neq i}\left\{\frac{p_i + p_j}{\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|^2}(\boldsymbol{r}_j - \boldsymbol{r}_i)\, w\left(\left|\boldsymbol{r}_k - \boldsymbol{r}_i\right|\right) - \frac{2\hat{p}_i}{\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|^2}(\boldsymbol{r}_j - \boldsymbol{r}_i) w\left(\left|\boldsymbol{r}_k - \boldsymbol{r}_i\right|\right)\right\} \tag{21}$$

On the other hand, it can be shown that the weight function applied in the new imaginary influence circle is equal to the one in the initial influence circle:

$$w\left(\left|\boldsymbol{r}_k - \boldsymbol{r}_i\right|\right) = \left(\frac{r_{e-ik}}{r_{ik}} - 1\right) = \left(\frac{r_{e-ij}/2}{r_{ij}/2} - 1\right) = w\left(\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|\right) \quad (22)$$

Therefore, the summation of weight functions in the imaginary influence circle of particle *i* would be equal to that in the initial influence circle:

$$n_{0-ik} = \sum_{i \neq k} w\left(\left|r_k - r_i\right|\right) = \sum_{i \neq j} w\left(\left|r_j - r_i\right|\right) = n_{0-ij} = n_0 \quad (23)$$

Thus, the new pressure gradient term is written as:

$$\left\langle \nabla p \right\rangle_i = \frac{D_s}{n_0} \sum_{j \neq i} \left\{ \frac{p_i + p_j}{\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|^2} (\boldsymbol{r}_j - \boldsymbol{r}_i)\, w\left(\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|\right) - \frac{2\hat{p}_i}{\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|^2} (\boldsymbol{r}_j - \boldsymbol{r}_i) w\left(\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|\right) \right\} \quad (24)$$

Since the minimum pressure in the influence circle of particle *i* is not necessarily equal to that in the influence circle of particle *j*, Eq. 24 is not yet symmetric. In order to make it a full symmetric equation, $\hat{p}_i$ is replaced by $(\hat{p}_i + \hat{p}_j)/2$. Hence, the new pressure gradient term in the CMPS method is derived as:

$$\left\langle \nabla p \right\rangle_i = \frac{D_s}{n_0} \sum_{j \neq i} \frac{(p_i + p_j) - (\hat{p}_i + \hat{p}_j)}{\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|^2} (\boldsymbol{r}_j - \boldsymbol{r}_i) w\left(\left|\boldsymbol{r}_j - \boldsymbol{r}_i\right|\right) \quad (25)$$

The linear momentum is exactly conserved when the above symmetric equation is applied. Since the conservation of linear momentum is also guaranteed for the viscous forces (Eq. 12), in the CMPS method the total linear momentum of the system would be exactly conserved. To avoid the overestimation of pressure gradient calculated by Eq. 25, especially close to the free-surface, some modification is made in the averaging process.

(2) CMPS: conservation of angular momentum

The exact conservation of angular momentum is not ensured in the MPS method as viscous forces are not co-linear with the position vector of two neighboring particles and the pressure interacting forces are not opposite. In the CMPS method the new pressure gradient term is symmetric in addition to being radial (co-linear with the position vector $\boldsymbol{r}_{ij}$); thus, angular momentum is exactly conserved in case of the pressure interacting forces. For the viscous interacting forces, however, conservation of angular momentum is not strictly ensured. Nevertheless, in next chapter, we will show the significantly improved preservation of total angular momentum in the CMPS method.

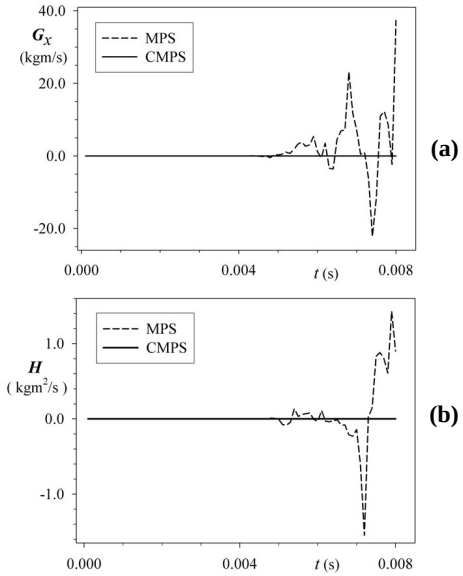

Fig. 2 Variation of total *x*-direction linear (a) and angular (b) momentum during the evolution of an elliptical drop

5. EVOLUTION OF A WATER DROP

In this section, a simple test is carried out to show the enhanced performance of CMPS in momentum conservation. The test is the simulation of an elliptical water drop[6]. The initial fluid configuration is a circle of radius 1 m subjected to no external forces but an initial velocity field as (-100*x*,100*y*) m/s. During the calculation due to the absence of external forces total linear and angular momentum should be preserved. **Fig. 2**(**a**-**b**) shows the time variation of total linear momentum in *x* direction and total angular momentum for both MPS and CMPS methods. The figure confirms that the conservation of linear and angular momentum is not guaranteed in the MPS method. In contrast, total linear momentum is exactly conserved in the CMPS method. Moreover, the new formulation of pressure in CMPS method has significantly improved the conservation of angular momentum. In the CMPS calculation performed here, the amplitude of fluctuations in total linear and angular momentum do not exceed $1O^{-12}$ and $1O^{-03}$, respectively.

6. SIMULATION OF A PLUNGING BREAKER AND RESULTANT SPLASH-UP

(1) Qualitative comparison

Breaking and post-breaking of a solitary wave with the incident relative wave height or the ratio of

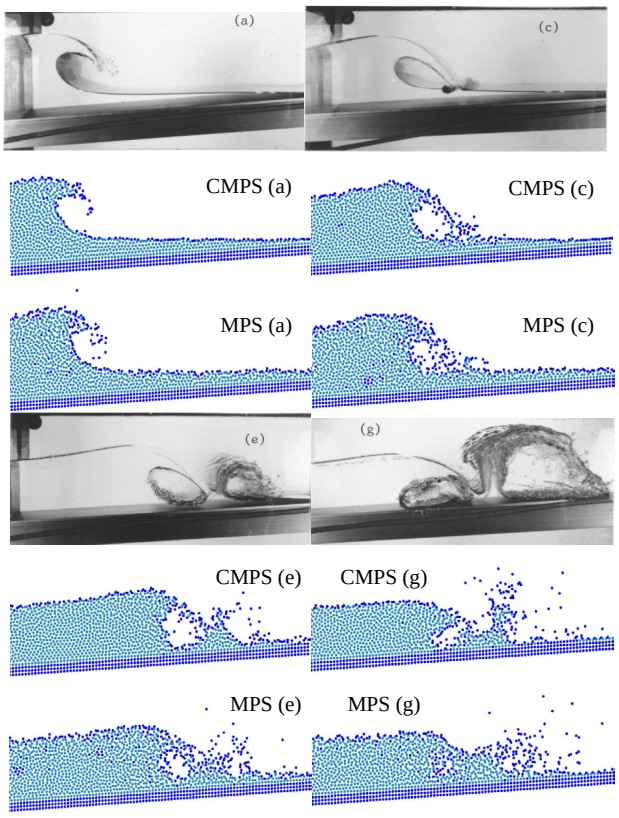

Fig. 3 A plunging breaker and resultant splash-up - qualitative comparison of CMPS and MPS snapshots with laboratory photographs[9)]

offshore wave height (=H_0) to offshore water depth (=h_0) of H_0/h_0=0.40 is simulated over a slope (=s) of 1:15. The prescribed conditions would lead to a strong plunging breaking in which the plunging jet hits the still water ahead of the wave, accordingly a secondary shoreward directed jet is generated from the impact point. The splash of water in form of a secondary jet, often known as splash-up, is a complex yet important process as it plays an essential role in the dissipation of wave energy and momentum transfer. The applicability of the MPS method in the simulation of splash-up is already shown by Khayyer and Gotoh[8)]. Here we show the refined simulation of splash-up by CMPS method.

Fig. 3 illustrates the CMPS and MPS snapshots together with the laboratory photographs[9)]. From the figure, it is evident that the simulation-experiment qualitative agreement is better in case of CMPS method. The CMPS results portray a clearer image of the plunging jet (**Fig. 3(a)**) and its impingement (**Fig. 3(c)**) with less particle scattering as seen in MPS snapshots. In addition, from **Fig. 3(e-g)**, the splash-up is more precisely simulated by the CMPS method as the reflected jet angel and the air chamber beneath the plunging jet are in better agreement with the experiment. Yet, the entire curl of the splash-up has not been well reproduced. One of the main reasons behind this disagreement is expected to be the employment of a simplified Laplacian model (Eq. 4) which treats the viscosity as a scalar quantity. Here, we propose a tensor-type strain-based viscosity which helps the viscous accelerations to be calculated from a strain rate tensor.

In a kernel-based particle method such as the MPS method, the divergence of a function $f(x)$ can be calculated from the following equation:

$$\langle \nabla \cdot f(x_i) \rangle = \sum_{j \neq i}^{N_i} V_j f(x_j) \cdot \nabla_i w_{ij} \tag{26}$$

where V_j=tributary volume of particle j being equal to the inverse of n_0. Therefore:

$$\begin{aligned} \left(\nu \nabla^2 \boldsymbol{u}\right)_i &= \left(\frac{1}{\rho} \nabla \cdot \boldsymbol{T}\right)_i = \frac{1}{\rho} \sum_{j \neq i}^{N_i} V_j \boldsymbol{T}_{ij} \cdot \nabla_i w_{ij} \\ &= \frac{1}{\rho n_0} \sum_{j \neq i}^{N_i} \boldsymbol{T}_{ij} \cdot \nabla_i w_{ij} \end{aligned} \tag{27}$$

In Eq. 27, $\boldsymbol{T}$ = the viscous stress tensor which can be related to the strain rate of flow by the following equation:

$$\boldsymbol{T}_{ij} = 2\mu \boldsymbol{S}_{ij}$$
$$; \ \boldsymbol{S}_{ij} = \begin{bmatrix} \left(\frac{\partial u}{\partial x}\right)_{ij} & \left(\frac{1}{2}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]\right)_{ij} \\ \left(\frac{1}{2}\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right]\right)_{ij} & \left(\frac{\partial v}{\partial y}\right)_{ij} \end{bmatrix} \tag{28}$$

where, μ = dynamic viscosity; u and v = the components of the particle velocity in x and y directions, respectively. The velocity and kernel gradients are introduced for each particle as:

$$\left(\frac{\partial u}{\partial x}\right)_{ij} = \left(\frac{\partial u}{\partial r}\frac{\partial r}{\partial x}\right)_{ij} = \frac{u_{ij}}{r_{ij}}\frac{x_{ij}}{r_{ij}} = \frac{u_j - u_i}{r_{ij}}\frac{x_j - x_i}{r_{ij}} \tag{29}$$

$$\nabla_i w_{ij} = \left(\frac{\partial w_{ij}}{\partial x}\right)_i \boldsymbol{i} + \left(\frac{\partial w_{ij}}{\partial y}\right)_i \boldsymbol{j} = \frac{-r_e x_{ij}}{r_{ij}^3} \boldsymbol{i} + \frac{-r_e y_{ij}}{r_{ij}^3} \boldsymbol{j} \tag{30}$$

The strain-based viscosity introduced above (Eq. 27) exactly preserves linear momentum; yet, similar to the original MPS formulation of viscosity (Eq. 4) it does not exactly conserve angular momentum.

Fig. 4 shows the snapshots of standard MPS, CMPS, and CMPS with Strain-Based Viscosity (CMPS-SBV) and the laboratory photographs[9)]. The employment of a strain-based viscosity has resulted in a further enhanced reproduction of the splash-up development (**Fig. 4(f)**) and its curling back (**Fig. 4(g)**) by the CMPS-SBV method.

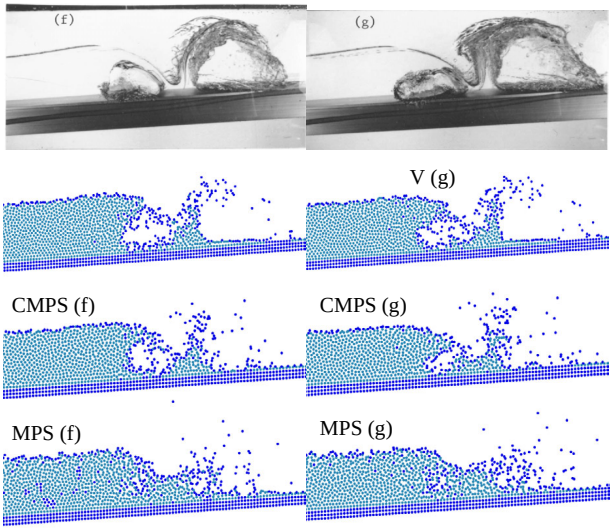

Fig. 4 Enhanced reproduction of splash-up - qualitative comparison between laboratory photographs[9] and CMPS-SBV, CMPS and MPS snapshots

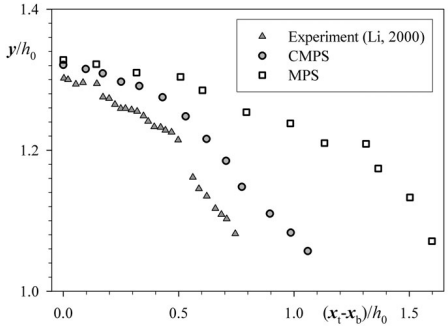

Fig. 5 CMPS and MPS predictions of the trajectory of the plunging jet tip

(2) Quantitative comparison

In order to further evaluate the accuracy of CMPS method, another case of solitary plunging breaker (H_0/h_0=0.30; s=1:15) is simulated and the results are quantitatively compared to the experimental data by Li[10]. Comparisons are made in terms of the trajectory of the plunging jet tip and plunging jet length (= the horizontal distance from the tip of the jet to the nearest location of the wave surface which is vertical). From **Fig. 5**, the CMPS method has given a more accurate prediction of the motion and location of the plunging jet tip. In this figure, x_t and x_b indicate the x-coordinate of plunging jet tip and the breaking point, respectively. The variation of the plunging jet length L_1 is also better predicted by the CMPS method (**Fig. 6**).

7. CONCLUSIVE REMARKS

The paper highlights the importance of momentum conservation in a particle-based calculation of free-surface flows. Refined simulations of a plunging breaker and resultant splash-up are presented through the application of

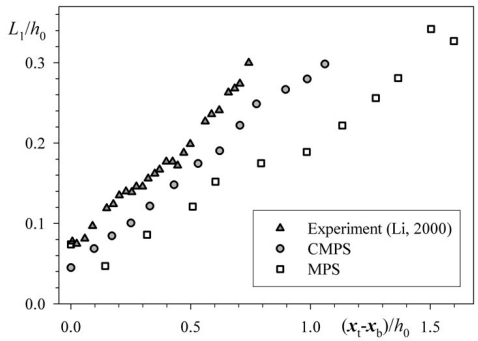

Fig. 6 CMPS and MPS predictions of the plunging jet length

two corrected versions of the MPS method, namely, the Corrected MPS (CMPS) and the Corrected MPS with Strain-Based Viscosity (CMPS-SBV). The step-by-step extension of CMPS method to a 3D multi-phase code with the SPS (Sub-Particle-Scale) turbulence modeling[11] is among the future works.

REFERENCES

1) Hirt, C. and Nichols, B. D.: Volume of fluid (VOF) method for the dynamics of free boundaries, *J. Comput. Phys*. 39, 201-225, 1981.
2) Yabe, T., Xiao, F., Utsumi, T.: The constrained interpolation profile method for multiphase analysis, *Journal of Computational Physics*, 169, 556-593, 2001.
3) Koshizuka, S. and Oka, Y.: Moving particle semi-implicit method for fragmentation of incompressible fluid, *Nuclear Science and Engineering*, 123, 421-434, 1996.
4) Gotoh, H. and Sakai, T.: Lagrangian simulation of breaking waves using particle method, *Coastal Eng. Jour.*, 41 (3 & 4), 303-326, 1999.
5) Gotoh, H., Ikari, H., Memita, T. and Sakai, T.: Lagrangian particle method for simulation of wave overtopping on a vertical seawall, *Coastal Eng. Jour.*, 47, 157-181, 2005.
6) Bonet, J. and Lok, T. S.: Variational and momentum preservation aspects of smooth particle hydrodynamic formulation, *Comput. Methods Appl. Mech. Eng*. 180, 97-115, 1999.
7) Koshizuka, S., Nobe, A. and Oka, Y.: Numerical analysis of breaking waves using the moving particle semi-implicit method, *Int. J. Numer. Meth. Fluid*, 26, 751-769, 1998.
8) Khayyer, A. and Gotoh, H.: Applicability of MPS method to breaking and post-breaking of solitary waves, *Annual Journal of Hydraulic Eng.*, JSCE, 51, 175-180, 2007.
9) Li, Y. and Raichlen, F.: Energy balance model for breaking solitary wave runup, *Journal of Waterway, Port, Coastal, and Ocean Engineering*, 129(2), 47-59, 2003.
10) Li, Y.: Tsunamis, Non-breaking and breaking solitary wave run-up, Rep. KH-R-60, W. M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology, Pasadena, CA, 2000.
11) Gotoh, H., Shibahara, T. and Sakai, T.: Sub-Particle-Scale Turbulence Model for the MPS Method - Lagrangian Flow Model for Hydraulic Engineering, *Comput. Fluid Dynamics Jour*., 9(4), 339-347, 2001.

(Received September 30, 2007)