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DEVELOPMENT OF REGIONAL RAINFALL INTENSITY-DURATION-FREQUENCY CURVES BASED ON SCALING PROPERTIES

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This study developed a regional Intensity–Duration–Frequency (IDF) relationship for ungauged locations based on the scaling theory. The scaling properties of extreme rainfall are examined to establish scaling relationship behaviors of statistical moments over different durations. The results show that a rainfall property in time does follow a simple scaling process. A scale invariance concept is explored for disaggregation (or downscaling) of rainfall intensity from low to high resolution and is applied to the derivation of scaling IDF curves. These curves are developed for ungauged sites based on scaling of the Extreme Value type 1 (EV1) or Gumbel probability distributions. The spatial distribution maps of three parameters: the scaling exponent and two statistical parameters are constructed. By using these maps, the IDF relationships are deduced from only daily rainfall with the scaling approach, which shows good agreements in comparison with the IDF curves obtained from traditional techniques.

Key Words: *regional analysis, scale invariance, IDF curves, design rainfall, ungauged.*

1. INTRODUCTION

The knowledge of rainfall Intensity Duration Frequency (IDF) relationships is of fundamental importance in hydrology. The IDF curves are used in developing design storms and are also needed in many hydrologic models and procedures in computation of water quantity and quality characteristics. For the site for which sufficient rainfall data available, frequency analyses are commonly used for design of various hydraulic structures. In particular, precipitation frequency analysis studies are necessary for development of a design storm. There have been considerable attentions and researches on the IDF relationship. Hershfield[1)] developed various rainfall contour maps to provide the design rain depths for various return periods and durations. Koutsoyiannis *et al.*[2)] cited that the IDF relationship is a mathematical relationship between the rainfall intensity, the duration and the return period. Sivapalan and Blöschl[3)] proposed an approach for constructing catchment IDF curves based on the spatial correlation structure of rainfall. Nhat *et al.*[4)] estimated IDF curves, constructed the parameter contour maps of IDF functions and generalized IDF formulas for the monsoon area of Vietnam.

However, these techniques have limitations to estimate the IDF characteristics for ungauge sites. For the site where rainfall record is unavailable or the data sample at the site is limited, “Regionalization” and “Scaling” methods should be studied more to transfer the rainfall information from one location to others.

Over the last two decades, concepts of scale invariance have come to the fore in both modeling and data analysis in hydrological precipitation research. Gupta and Waymire[5)] studied rainfall spatial variability by introducing the concepts of simple and multiple scaling to characterize the probabilistic structure of the precipitation processes. Burlando and Rosso[6)] showed that both the simple scaling and multiscaling lognormal models can be used to derive Depth Duration Frequency (DDF) curves of point precipitation. Nguyen *et al.*[7)] proposed a Generalized Extreme Value distribution model for regional estimation of short duration rainfall extremes based on the scaling theory. Menabde *et al.*[8)] developed a simple scaling methodology to use daily rainfall statistics to infer the IDF curves for rainfall duration less than one day. Pao-shan, *et al.* [9)] is an example of methodology in which the theories of scaling properties are employed to infer the IDF characteristics of short-duration rainfall from daily data. Kuzuha *et al.* [10)] showed the scaling framework and regional flood frequency analysis. Nhat *et al.*[11] showed the existence of the simple scaling in time and space within some ranges.

The purpose of this study is to investigate 'scale invariance' or 'scaling' properties of rainfall for derivation of IDF relationships at ungauged site. At first, the scaling exponent for each gauge and the space variation of the scale exponents were investigated by using the annual maximum rainfall intensity series for various duration and return periods at the selected twenty stations in the Yodo River catchment. The IDF formulas can be derived based on three parameters: the scale exponent and two statistical parameters of 24 hours rainfall data only. Secondly, three parameters are interpolated from the contour maps of these parameters and the IDF at any sites are generated. Finally, the IDF relationships at the ungauged sites with the regional scaling model are examined, which shows good agrrements with the IDF curves obtained from conventional techniques.

2. THEORETICAL FRAMEWORK OF THE SIMPLE SCALING

The scaling or scale-invariant models enable us to transform hydrologic information from one temporal or spatial scale to another one, and thus, help overcome the difficulty of inadequate hydrologic data. A natural process fulfills the simple scaling property if the underlying probability distribution of some physical measurements at one scale is identical to the distribution at another scale. The basic theoretical development of scaling has been investigated by many authors, including Gupta and Waymire[5] and Kuzuha *et al*.[10]

Rainfall intensity $I(d)$ with duration d, exhibits a simple scale invariance behavior if

$$I(\lambda d) \overset{dist}{=} \lambda^H I(d) \tag{1}$$

holds. The equality " $\overset{dist}{=}$ " refers to identical probability distributions in both sides of the equations; λ denotes a scale factor and H is a scaling exponent. From equation (1), it leads to a simple scaling law in a wide sense

$$E[\{I(\lambda d)\}^q] = \lambda^{qH} E[\{I(d)\}^q] \tag{2}$$

where E[] is the expected value operator and q is the moment order. The random variable $I(d)$ exhibits a simple scale invariance in a wide sense if Equation (2) holds. If H is a non-linear function of q, the $I(d)$ is a general case of multi-scaling. The moments E[] are plotted on the logarithmic chart versus the scale λ for different moment order q. The slope function of the order moment $K(q)$ is plotted on the linear chart versus the moment order q. If the plotted results are on a straight line and through the origin, the random variable shows simple scaling, while in other cases, the multi-scaling approach has to be considered[5].

The IDF curves are often fitted to the extreme value type I (EVI) distribution developed by Gumbel and it is still the most often used distribution by many national meteorological services in the world to describe rainfall extremes. It will also be used in this study along with the method of moments. The annual maximum rainfall intensity $I(d)$ has a cumulative probability distribution CDF, which is given by

$$\begin{aligned} F\left[I(d)\right] &= 1-\frac{1}{T} = \exp[-\exp\{-\left[I(d)-\mu\right]/\sigma\}] \\ I(d) &= \mu - \sigma\ln(-\ln T) \end{aligned} \tag{3}$$

where the location parameter μ and scale parameter σ to be calculated from data series based on L moment method [12].

According to the scaling theory, the IDF formula can be derived [8), 11)] with

$$\begin{cases} I_{d,T} = \dfrac{\mu + \sigma[-\ln(-\ln(1-1/T)]}{d^{-H}} \\ \mu = \mu_{24}.(\lambda d)^{-H}; \sigma = \sigma_{24}.(\lambda d)^{-H} \end{cases} \tag{4}$$

where the μ_{24} and σ_{24} are parameters of 24 hour data series. It is worthwhile to note that the simple scaling hypothesis leads to the equality between the scale factor and the exponent in the expression relating rainfall intensity and duration. The IDF relationship can be derived from 24 hours data series based on three parameters: scale exponent, the location and scale parameters of EVI distribution.

3. REGIONAL SCALING MODEL TO DERIVE RAINFALL IDF FORMULAS

(1) Study area and rainfall data

The Yodo River catchment (approximately 7281 km^2) was selected as the study area. The rainfall data for analysis herein were collected from twenty hourly rain gauges of which locations are indicated in **Fig. 1**. The name and record length for recording rain gauges are listed in **Table 1**. The Annual Maximum Rainfall Intensity (AMRI) series for various durations, including 1, 2, 3...24 hours were taken from Automated Metrological Data Acquisition System (AMeDAS).

(2) Spatial distribution of scale exponent

The formula of IDF curves (Equation 4) can be derived from scaling invariance of rainfall. It is a function of scaling exponent H, and the two statistical parameters: Location parameter μ_{24} of 24 hour rainfall and scale parameter σ_{24} of 24 hour rainfall

$$I(d,T) = f\ (H,\ \mu_{24}, \sigma_{24}) \tag{5}$$

where T is return period and d is duration of rainfall intensity.

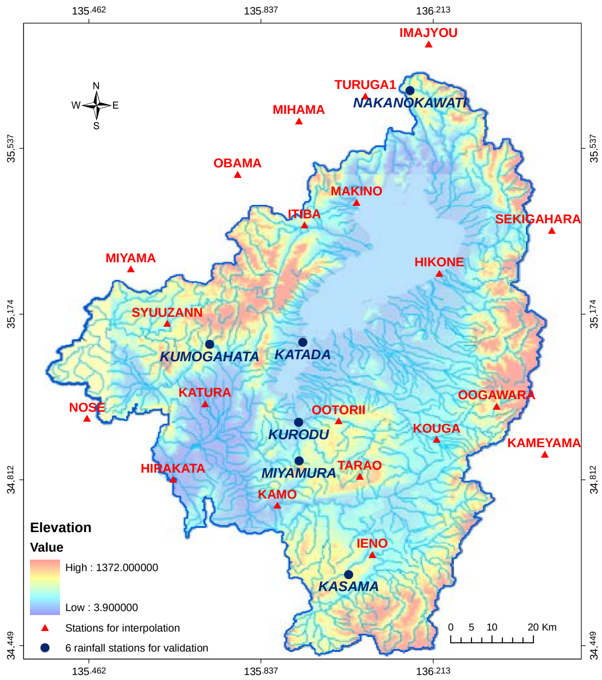

Fig. 1 Study area and rain gauge locations for analysis.

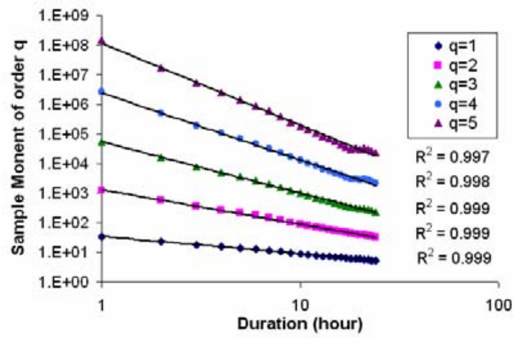

Fig. 2 Relationship between moment of order q and duration (hour) at Ootorii station.

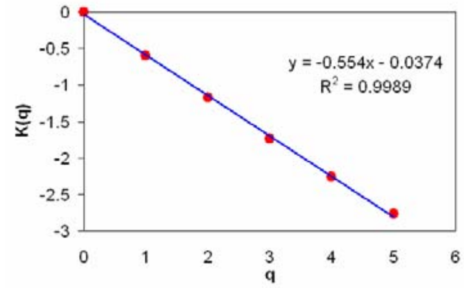

Fig. 3 Slope of the line provides an estimate of scaling exponent for the Ootorii station in the Yodo area.

Table 1 The stations and length of data.

No	Name of stations	Length of data
1	Ieno	1982-1996
2	Itiba	1982-2002
3	Oogawara	1982-2002
4	Ootorii	1982-2002
5	Katura	1982-2002
6	Kamo	1982-2001
7	Kouga	1982-2002
8	Syuuzann	1982-2002
9	Tarao	1982-1997
10	Hikone	1982-2002
11	Hirakata	1982-2002
12	Makino	1982-2002
13	Imajou	1976-2002
14	Turuga	1976-2002
15	Mihama	1976-2002
16	Obama	1976-2002
17	Miyama	1976-2002
18	Nose	1976-2002
19	Kameyama	1976-2002
20	Sekigahara	1976-2002

The regional IDF formula can be developed based on the three parameters above. First, the scaling exponent H was to be examined for the twenty stations in the Yodo River catchment. Based on the testing results, it is likely to conclude the existence of the simple scaling for area or region of the whole. Second, the two parameters of statistical analysis: μ_{24} and σ_{24} were derived from distribution of 24h ARMI data series.

With the values of the three parameters of all stations, spatial distributions of three parameters are constructed with the GIS interpolations technique. The IDF relationship for any point (ungauged) can be derived based on these maps and Equation (4).

To this end, after verification of scaling of the IDF estimates at Hikone stations in the previous studies[11], the assumption of scaling was examined at a representative sample of stations throughout the Yodo River catchment. The available hourly data at these stations were recorded by AMeDAS. An AMRI data set was derived from the data, and the first five moments were calculated for durations of 1 hour up to 24 hours. These moments were plotted against duration on a log–log scale to determine whether scaling could be assumed.

In **Figure 2** the qth moment of the intensity is plotted against the duration for the Ootorii station in central of the Yodo River catchment. It is clear that the data is linearly related, and there is not a break between 1 hour and 24 hour duration data[11]. The linearity of the moments seen here is similar to the other nineteen stations that were examined, indicating that for durations greater than 1 hour, scaling appears to be applicable throughout the Yodo River catchment. For each station, the scaling exponent, Hq, was plotted versus the moment, q, to determine if the simple scaling is applicable and to estimate the scale exponent for stations throughout the catchment. As shown in **Fig. 3**, the scaling exponent coefficient for the Ootorii station is found to be 0.55, with an R^2 value of 0.9989. This indicates that the simple scaling could be assumed at this station instead of the more complex multi-scaling.

Data from other gauges showed a similar scaling relationship, indicating that simple scaling may be applicable in the Yodo River area for the durations considered. In the **Table 2**, the results of the scaling exponent factor H of the twenty stations in the Yodo River catchment are shown with the high coefficients of determination (R^2) for each station ranging from 0.98 to 1.0. The result indicates a strong validity of the simple scaling property of the extreme rainfall in time series. The scaling coefficient for the Ootorii station of 0.55 was typical for the twenty stations examined, with a mean value of 0.61 and a standard deviation of 0.06. However, a few stations showed significantly higher scale exponents, with the highest being 0.69 (Kamo station). These stations were generally located on the southern part of the Yodo River catchment, as seen in **Fig. 4**, indicating that the effect of topography should be studied further. It can also be seen in **Fig. 4** that the scaling coefficients were generally lower in the northern part of the Yodo River catchment.

(3) Estimation of the parameters at ungauged sites

The three maps can be constructed by using Inverse-Distance Weighted interpolation method in ArcMap. The being estimated values were obtained:

$$Z^* = \sum_{i=1}^{n} Z_i d_i^{-p} \Big/ \sum_{i=1}^{n} d_i^{-p} \qquad (6)$$

where *di* is the distances from each of the *n* observed locations to the location being estimated, *Zi* are the observed values at those locations. The parameter is the power value (p), determined by minimizing the RMSE, using the cross validation technique. **Figure 4** displayed maps of spatial distributions of the scaling exponent parameters generated by the twenty stations (p=1). The location parameter μ_{24} and the scale parameter σ_{24} of the EV1 (Gumbel) distribution of 24 hours of AMRI can be derived by statistical analysis based on L-moment method as the results are shown at **Table 2**. With the same technique, two maps of statistical parameters can be constructed as shown in **Fig. 5** and **Fig. 6** (p=1.7 and 1.6). It is expected that those three maps can be applicable for any ungauged locations within the catchment. It is required to verify these maps for regional IDF relationships.

Table 2 The parameters at recording rain gauged.

	Name of stations	Scale exponent (-H)	Location Parameter (μ_{24})	Scale Parameter (σ_{24})
1	Kamo	0.69	3.80	0.82
2	Hirakata	0.68	4.04	1.26
3	Ieno	0.67	3.88	1.02
4	Katura	0.66	4.58	1.50
5	Hikone	0.61	3.83	1.36
6	Makino	0.60	4.34	1.38
7	Ootorii	0.55	4.43	1.44
8	Kouga	0.55	4.05	1.59
9	Oogawara	0.55	6.44	3.06
10	Tarao	0.53	4.93	1.76
11	Syuuzynn	0.53	4.41	1.52
12	Itiba	0.46	4.75	2.20
13	Imajou	0.67	4.06	0.92
14	Turuga	0.62	3.46	1.22
15	Mihama	0.62	3.82	1.73
16	Obama	0.60	3.70	1.64
17	Miyama	0.70	3.91	1.28
18	Nose	0.61	4.15	1.50
19	Kameyama	0.58	5.55	1.84
20	Sekigahara	0.60	5.53	2.04
	Mean	0.61	4.38	1.55
	Std.	0.06	0.74	0.49

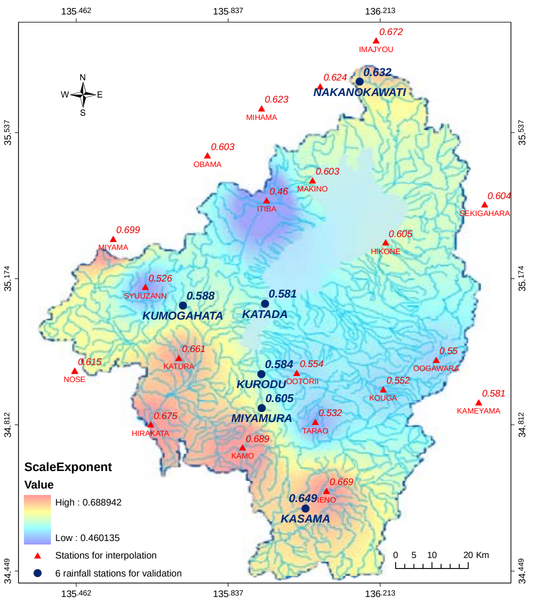

Fig. 4 Spatial distribution maps of the scale exponent (H).

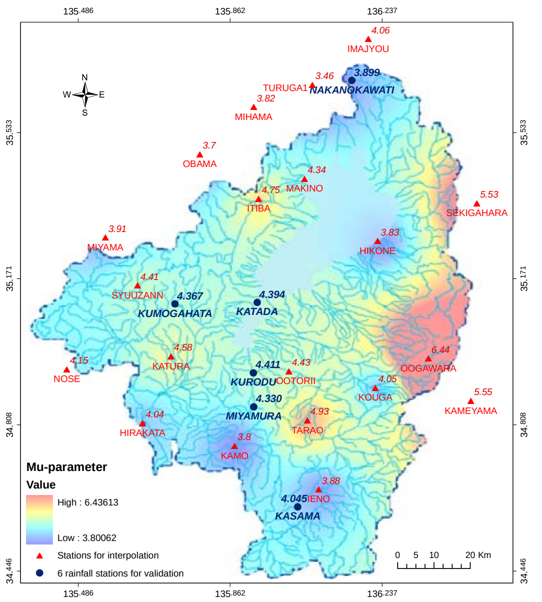

Fig. 5 Spatial distribution maps of the statistical parameters: the location (μ) of 24h rainfall AMRI.

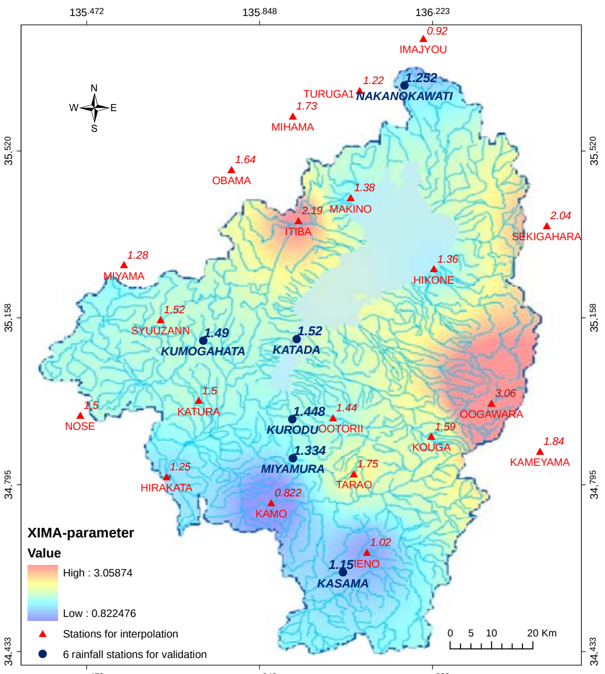

Fig. 6 Spatial distribution maps of the statistical parameters: the scale parameter (σ) of 24h rainfall AMRI.

4. APPLICATION AND VALIDATION OF THE REGIONAL SCALING MODEL

The model can be applied at any site in the study region for storm duration *d* equal to 1, 2..., 24 hours. For each storm duration *d* and return period *T*, the design storm *I*(*d*,*T*) can be evaluated as follows:

1) Estimate the local scale exponent (*H*) value for the site of interest. If this site is ungauged a spatial interpolation procedure can be used (see e.g., **Fig. 4**).
2) Estimate the statistical parameters: the location and the scale parameters of 24 hours of EV1(Gumbel) distribution by L-moment method. If the site is ungauged a spatial interpolation procedure can be used (see e.g., **Figs. 5–6**).
3) Compute an estimate of the design storm *I*(*d*,*T*) for ungauged site as a function of the scale exponent, the location parameter (μ_{24}) and the scale parameter (σ_{24}) of 24hour data only, where *T* is the return period in the years; *d* is duration in the hours.

Figure 7 shows the IDF curves derived from the regional scaling model in comparison with IDF curves derived from conventional method by EV1 distribution.

For demonstrating the performances of regional IDF model, which is developed in this study, the calculated rainfall intensities of the 7 return periods (2 to 200 years) and 24 durations (1-24hours) by using the regional IDF model were compared with

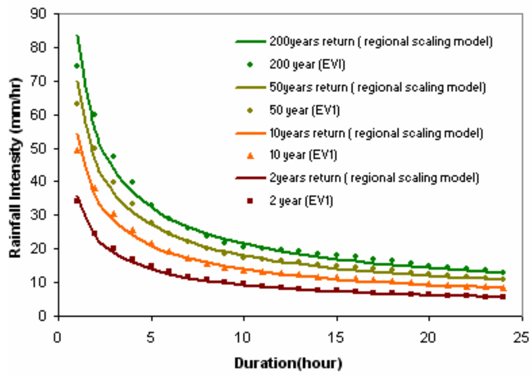

Fig. 7 Comparison the IDF cuvers derived from regional scaling model and IDF cuver with conventional method (EV1) at Kumogahata station.

Table 3 RMSE obtained for the estimation of rainfall intensity of 7 return periods (2-200yrs) and 24 durations (1-24h) for 6 stations with 21 years of observation data.

Years return	Kuro-du	Miya-mura	Kasa-ma	Nakano-kawati	Kumo-gahata	Katada
200	4.8	2.3	3.8	6.8	2.4	3.0
100	4.2	2.0	3.5	6.0	2.1	2.6
50	3.6	1.7	3.1	5.3	1.9	2.2
25	3.0	1.4	2.8	4.6	1.6	1.8
10	2.2	1.0	2.3	3.6	1.2	1.3
3	1.5	0.7	1.9	2.8	1.0	1.0
2	1.0	0.5	1.7	2.3	0.7	0.7

** Unit of RMSE value is mm/hr.*

the *I*(*d*,*T*) values estimated by conventional method, a frequency analysis based on the EV1 distribution with the observed data. Six stations, namely Kurodu, Miyamura, Kasama, Nakanokawati, Kumogahata and Katada, were chosen from the study area for validation. These validation gauges were considered as the ungauged sites (Regional scaling model did not use dada from these stations).

The values of the local scale exponent (*H*) and the two parameters μ_{24} and σ_{24} were estimated from isoline maps of the study area. Then the Root Mean Square Errors (RMSE) and Mean absolute Relative Percent Errors(MRPE) estimation values

$$RMSE(d,T)=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(I_i(d,T)^{*}-I_i(d,T))^2} \quad (7)$$

$$MRPE(d,T)=\frac{1}{n}\sum_{i=1}^{n}\left|\frac{I_i(d,T)^{*}-I_i(d,T)}{I_i(d,T)}\right|.100\% \quad (8)$$

were obtained, where the *I*(*d*,*T*)* indicates the rainfall intensity of duration *d* and return period *T* estimated by the regional scaling model and *I*(*d*,*T*) indicates the rainfall intensity from EV1 distribution. **Table 3** shows the RMSE values that were obtained for 7 return periods (2-200yrs) and 24 durations (1-24h) for six stations.

Figure 8 displays the comparison results of the two stations in the Yodo River catchment. The same calibration results were found in other four stations.

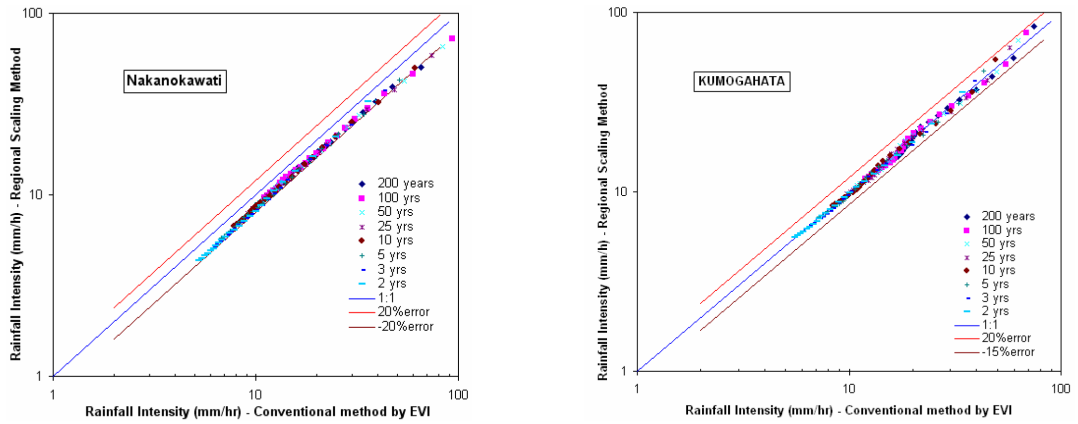

Fig. 8 The validation results for a) Nakanokawati station b) Kumogahata station.

The calibration results for all six stations reveal that the regional scaling models have good performances for Yodo River catchment and the mean relative percentage errors are less than 20 percent for the six ungauged stations.

5. CONCLUSION

The major findings of the present study can be summarized as follows: The properties of the time scale invariance of rainfall quantiles are examined in the Yodo River basin: for time scaling, rainfall properties follows the simple scaling. The hypothesis of piecewise simple scaling combined with the Gumbel distribution was used to develop the IDF scaling formulas depending on the three parameters: the scaling exponent, and the two statistical parameters, location and scale parameters of 24 hour rainfall. The spatial distribution maps of these parameters are used to derive the rainfall intensity duration frequency at ungauged points which were interested for designing analysis.

This study established the regional rainfall intensity duration frequency relationship for ungauged catchments based on three spatial distribution maps from scale invariance of rainfall in time. The paper presents a regional frequency analysis of annual maximum rainfall intensity for storm duration ranging from 1 hour to 24 hours, observed for a dense network of raingauges located in Yodo River cathment of Japan. The study investigates the scaling properties of rainfall extremes using the EV1 distribution (Gumbel) based on L-moments.

We developed a regional scaling model that enables one to estimate design storm at any ungauged sites of the study area. The uncertainty of the estimates for ungauged sites is quantified by using the RMSE. We obtained for the study area resampled estimates of the design storm characterised by mean relative errors, generally lower than 10% and never higher than 20%. The methodology should be applied in the different areas and climatology in order to generalize the results.

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