

LUMPING OF A PHYSICALLY-BASED DISTRIBUTED MODEL FOR SEDIMENT RUNOFF PREDICTION IN A CATCHMENT SCALE

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This paper proposes a lumping method of a distributed rainfall-sediment-runoff model. The lumping method is divided into two parts: lumping of the kinematic wave distributed rainfall-runoff model, and lumping of the distributed sediment transportation model that takes into account for erosion and deposition by raindrop and overland flow. A kinematic wave distributed rainfall-runoff model is lumped based on the assumption of steady state conditions. Soil detachment and deposition from an entire catchment are simulated by the balance between the current sediment storage and the maximum sediment storage, which is estimated from a distributed rainfall-sediment-runoff model as a function of total surface water storage of the lumped rainfall-runoff model. The proposed lumping method is examined by comparing water and sediment discharges simulated by the distributed and the lumped rainfall-sediment-runoff models in the Lesti River Basin, Indonesia.

Key Words: *Lumping, distributed rainfall-sediment-runoff model, erosion, Lesti River basin.*

1. INTRODUCTION

To predict sediment transport and runoff are very important to solve the problem of water resources. Design of reservoirs and dams, design of soil conservation, land-use planning, water quality and aquatic habitat management are some of the examples. Sediment transport is highly dependent on topography, land use, and soil type. Therefore, it is essential to incorporate physically reasonable functions in a rainfall-sediment-runoff model.

Complex distributed rainfall-sediment-runoff models have been developed. KINEROS, LISEM, WEPP, and EUROSEM¹⁾ are some of the examples. These models are normally designed to estimate sediment runoff during a single rainfall event.

The problem of those fully distributed models have limitation in application because those models generally require much computation time to conduct a simulation especially for large catchments and long-term simulations. In the past, lumped and semi-distributed models have been widely preferred for representing the rainfall-sediment-runoff process at scales ranging from few to several hundreds square kilometers. Unfortunately, given lack of

direct physical interpretation of their model parameters, long term observed data records are needed for their calibration, which may not always be available.

From the above considerations, the need is obvious for creating a new generation of a lumped model with parameters directly related to physically meaningful quantities derived from appropriate distributed scales. A concept for lumping a distributed rainfall-runoff model using digital topographic information to reduce computational burden required in runoff simulation was proposed²⁾. The governing equations and the lumped model parameters are derived from a distributed rainfall-runoff model by assuming that a rainfall-runoff process of the system reaches a steady state condition. The advantage of the lumping is that it can derive the lumped parameters keeping the physical meanings of the original distributed model without any additional calibration.

As an extension of the lumping method for a physically-based distributed rainfall-runoff model²⁾, the authors extend the method with adding a new lumping method for sediment transport processes incorporating surface and subsurface flow in

unsaturated and saturated zones. This research presents a method to lump a physically-based distributed rainfall-sediment-runoff model as a tool for estimating erosion, runoff, and sediment transport processes in a catchment scale. The lumped model is applied in the Lesti River located in the upper Brantas River basin, Indonesia.

2. THE DISTRIBUTED RAINFALL-SEDIMENT-RUNOFF MODEL

(1) Physically-based Distributed Rainfall-Runoff Model, CDRMV3

Cell Distributed Rainfall-Runoff Model Version 3 (CDRMV3) as a distributed hydrological model³⁾ is used as a base of a distributed sediment transport model. The model includes a stage-discharge, q - h , relationship for both surface and subsurface flows⁴⁾:

$$q = \begin{cases} v_m d_m (h/d_m)^\beta, & 0 \leq h \leq d_m \\ v_m d_m + v_a (h - d_m), & d_m \leq h \leq d_a \\ v_m d_m + v_a (h - d_m) + \alpha (h - d_a)^m, & d_a \leq h \end{cases} \quad (1)$$

$$v_m = k_m i, \quad v_a = k_a i, \quad k_m = k_a / \beta, \quad \alpha = \sqrt{i} / n$$

where q is discharge per unit width; h is water depth; i is the slope gradient; k_m is the saturated hydraulic conductivity of the capillary soil layer; k_a is the hydraulic conductivity of the non-capillary soil layer; d_m is the depth of the capillary soil layer; d_a is the depths of capillary and non-capillary soil layer; and n is the roughness coefficient based on the land cover classes. The continuity equation takes into account flow rate of each grid-cell or slope as:

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = r(t) \quad (2)$$

where t and x are time and distance along water flow, respectively; and r is the rainfall intensity.

The Lax-Wendroff finite difference scheme is used to solves the one-dimensional kinematic wave equation with the stage-discharge equation to simulate runoff generation and routing. The simulation area is divided into an orthogonal matrix of square grid-cells (250 m x 250 m).

(2) Coupling of CDRMV3 and Sediment Transport Model

The concept of spatially distributed sediment runoff modeling is shown in **Fig. 1**. A sediment transport algorithm is newly added to the CDRMV3. Runoff generation, soil erosion and deposition are computed for each grid-cell and are routed between grid-cells following water flow direction. The sediment transport algorithm includes multiple sources of sediment transport, which are soil detachment by raindrop (DR) and hydraulic detachment or deposition driven by overland flow

(DF). The basic assumption of this model is that the sediment is yielded when overland flow occurs. The eroded sediment is transported by overland flow to river channels.

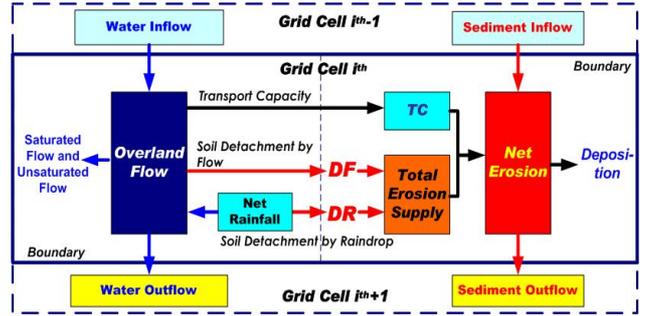


Fig.1 Schematic diagram of the physically based rainfall-sediment-runoff model in the i^{th} grid-cell scale.

Soil detachment and transport is handled with the continuity equation representing DR and DF as:

$$\frac{\partial (h_s c)}{\partial t} + \frac{\partial (q_s c)}{\partial x} = e(x, t) \quad (3)$$

$$e(x, t) = DR + DF$$

where C is the sediment concentration in the overland flow (kg/m^3); h_s is the water depth of overland flow (m); q_s is the discharge of overland flow (m^3/s); and e is the net erosion ($\text{kg}/\text{m}^2/\text{hr}$).

Soil detachment by raindrop is given by an empirical equation in which the rate is proportional to the kinetic energy of rainfall and decreases with increasing h_s . From the observation of rainfall characteristic in the study area⁵⁾ and dampening soil detachment rate by h_s ⁶⁾, the empirical equation for DR for the i^{th} grid-cell is expressed as:

$$DR_i = k KE e^{-b h_{si}} = k 56.48 r_i e^{-b h_{si}} \quad (4)$$

where k is the soil detachability (kg/J); KE is the total kinetic energy of the net rainfall (J/m^2); and b is an exponent to be tuned.

Following the theoretical work of EUROSEM⁶⁾, the concept of transport capacity is used to determine sediment transport rates in overland flow. Sediment transport capacity of overland flow (TC) is defined as the maximum value of sediment concentration to transport, which is estimated for each grid-cell. Then for the i^{th} grid-cell, DF is simulated as a result of overland flow and function of TC as follows:

$$DF_i = \alpha (TC_i / 1000 - C_i) h_{si} \quad (5)$$

where α is the detachment/deposition efficiency factor. Detachment or deposition by flow is assumed to be proportional to the TC deficit. Following the TC approach; if actual suspended sediment from upper grid-cells is lower than this capacity, detachment or erosion occurs, otherwise soil deposition excess.

The transportation capacity is calculated based

on the Unit Stream Power (USP) theory. The USP theory contributing to TC is defined as a product of the overland flow velocity, v , and slope, i , in the i^{th} grid-cell. A relationship between USP and the upper limit to the sediment concentration in the overland flow, C_t (ppm), can be derived⁷⁾ (see Eq. (6)). TC is the product of C_t as follows:

$$TC = \log C_t = I + J \log((vi - v_{critical}i) / \omega) \quad (6)$$

in which:

$$I = 5.435 - 0.386 \log(\omega D_{50} / NU) - 0.457 \log(U^* / \omega)$$

$$J = 1.799 - 0.409 \log(\omega D_{50} / NU) - 0.314 \log(U^* / \omega)$$

$$\omega = \sqrt{\frac{2}{3} + \frac{36}{\left(\frac{\rho_s}{\rho_w} - 1\right) g \left(\frac{D_{50}}{1000}\right)^{2/NU}}} - \sqrt{\frac{36}{\left(\frac{\rho_s}{\rho_w} - 1\right) g \left(\frac{D_{50}}{1000}\right)^{2/NU}}}$$

where vi is the unit stream power, m/s (v is flow velocity in m/s and i is the slope gradient m/m); $v_{critical}i$ is the critical unit stream power ($v_{critical}$ is the critical flow velocity); ω is the sediment fall velocity (m/s) calculated by Rubey's equation; ρ_s is the sediment particle density (kg/m^3); ρ_w is the water density (kg/m^3); g is the specific gravity (m/s^2); D_{50} is the median of grain size (mm); and NU is the kinematic viscosity of the water (m/s^2). $U^*(=\sqrt{g i h_s})$ is the average shear velocity (m/s).

This model does not explicitly separate rill and interrill erosion. Gullyng, river bed erosion, river bank erosion and lateral inflow of sediment to river channel are not considered.

3. THE LUMPED RAINFALL-SEDIMENT RUNOFF MODEL

The basic motivation in lumping lies in the development of a sediment runoff model applicable for a larger catchment scale by considering all the smaller scale processes and information, in which the derivation of lumped governing equations are associated to the physical meaning of model parameters, and it realizes to reduce computational time required in a simulation.

(1) Lumping of Physically-Based Distributed Rainfall-Runoff Model

The lumping method of physically-based distributed rainfall-runoff model²⁾ was used and extended. To obtain the lumped sediment runoff model, the process equation, namely the kinematic wave equation, is integrated over the entire system of grid-cells describing the catchment. This is done first by computing the total water volume stored in the soil, on the surface or in the channel network by adding up the single grid-cell volumes as a function of the geomorphology and topology of

the catchment under the assumption of steady state conditions of rainfall-runoff by spatially uniform rainfall input.

The relationship between total storage of water in the i^{th} grid-cell (s_{wi}) and the outflow discharge in the i^{th} grid-cell (Q_i) is theoretically derived. Flux Q_i is expressed as the product of rainfall intensity (\bar{r}) and the upslope contributing areas (U_i):

$$Q_i(x) = Q_i(0) + \bar{r} \int_0^x w_i(x) dx = \bar{r} U_i + \bar{r} x_i w(x)$$

$$q_i(x) = Q_i(x) / w_i(x) = \bar{r} U_i / w_i(x) + \bar{r} x_i \quad (7)$$

where x is the horizontal distance from the upstream end of a grid-cell and w is the width of grid-cell. The upslope contributing area can be generated from water flow accumulation data in each grid-cell.

The storage volume of overland flow for the i^{th} grid-cell (s_{wi}) as a function of h , w , and x is given as:

$$s_{wi} = \int_0^x h_i(x) w_i(x) dx \quad (8)$$

By substituting the variable of integration from x to q using the relationship given by (8):

$$s_{wi} = \frac{w}{\bar{r}} \int_{q(0)}^{q(x)} f(q) dq = \frac{w}{\bar{r}} [F(q(x)) - F(q(0))] \quad (9)$$

Figure 2 schematically shows a relationship between q , h , and F . $F(q(x))$ can be further estimated depending on the surface soil condition and q - h relationship for three layers. It is assumed that q is a function of h and can be analytically integrated with h . If the values of $q_i(x)$ using Eq.(7) and $h_i(x)$ numerically is obtained using Eq.(1), then $F(q(x))$ can be calculated. The storage of water at the catchment scale, S_w , can be calculated by adding up s_{wi} from each grid-cell. Finally, Q of the outlet is linked to S_w as a function of the topographic and physical characteristics of each grid-cell, as well as rainfall intensity. To relate Q of the catchment outlet and S_w , the Q - S_w relationship was established. Through variable rainfall intensities, the nonlinear Q - S_w discrete relationship at the study catchment was obtained (**Fig. 3a**). In order to obtain the dynamic distribution of overland flow in the catchment, a relationship between the surface water storage amount, S_{ws} , and S_w was developed (**Fig. 3a**).

(2) Lumping of Physically-Based Distributed Rainfall-Sediment-Runoff Model

The same concept is explored to the distributed rainfall-sediment-runoff model to derive how sediment load is related to catchment hydrological response, detachment source type and intensity, and depositional processes. The sediment-runoff processes in this study are affected by dynamic spatial distribution of overland flow.

In the case unsaturated flow ($q(x)$ less than or equal q_m) :

$$F(q(x)) = q(x)h(x) - \int_0^{h(x)} g(h) dh = q(x)h(x) - \frac{V_m h^{\beta+1}}{\beta+1 d_m^{\beta-1}}$$

In the case saturated flow ($q(x)$ less than or equal q_a) :

$$\begin{aligned} F(q(x)) &= q(x)h(x) - \int_0^{d_m} V_m d_m \left(\frac{h}{d_m}\right)^\beta dh - \int_{d_m}^{h(x)} (V_m d_m + V_a (h - d_m)) dh \\ &= q(x)h(x) - \frac{V_m d_m^2}{\beta+1} - 0.5V_a (h^2 - d_m^2) - d_m (h - d_m) (V_m - V_a) \end{aligned}$$

In the case surface flow ($q(x)$ great than q_a) :

$$\begin{aligned} F(q(x)) &= q(x)h(x) - \int_0^{d_m} V_m d_m \left(\frac{h}{d_m}\right)^\beta dh - \int_{d_m}^{d_a} (V_m d_m + V_a (h - d_m)) dh - \int_{d_a}^{h(x)} (V_m d_m + V_a (h - d_m) + \alpha (h - d_m)^m) dh \\ &= q(x)h(x) - \frac{V_m d_m^2}{\beta+1} - 0.5V_a (h^2 - d_m^2) - \frac{\alpha}{m+1} (h - d_a)^{m+1} - d_m (h - d_m) (V_m - V_a) \end{aligned}$$

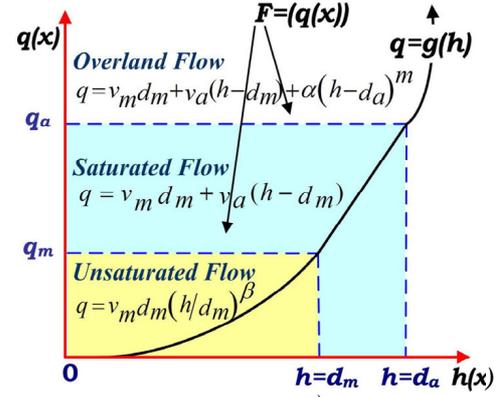


Fig.2 Schematic drawing of q , h , and F relationship.

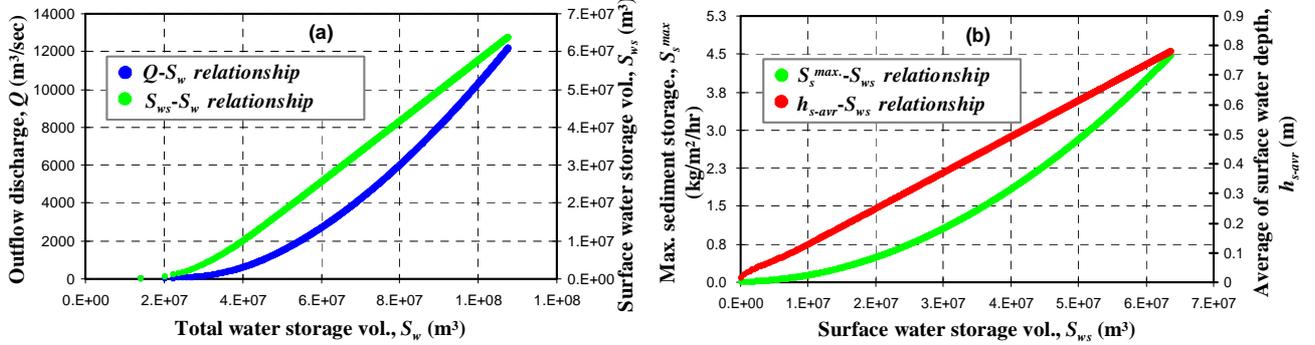


Fig.3 (a) Plots of discrete relationships between Q - S_w and S_{ws} - S_w ; and (b) discrete relationships between S_s^{\max} - S_{ws} and h_{s-avr} - S_{ws} .

a) The Maximum Sediment Storage (S_s^{\max})

The maximum sediment storage is defined as the total sediment transport capacity of overland flow in a whole of the catchment for each time step calculation. Therefore, we expressed the maximum sediment storage as the function of TC from the i^{th} grid-cell, surface water storage amount in the i^{th} grid-cell (S_{wsi}), and S_{ws} . The maximum sediment storage at catchment scale is calculated by adding up TC_i multiplied to S_{wsi} for all grid-cells as:

$$S_s^{\max} = \sum TC_i S_{wsi} / (S_{ws} 1000) \quad (10)$$

TC (ppm) for the i^{th} grid-cell has been estimated by:

$$TC_i = C_{ti} = 10^{5.0105 + 1.363 \log((USP_i - USP_{critical}) / \omega)} \quad (11)$$

where v in the i^{th} grid-cell is calculated if $h > d_a$ based on the q - h relationship as:

$$v_i = k_a i_i + \frac{\sqrt{i_i}}{n_i} (h_i - d_a)^{m-1} \quad (12)$$

b) Sediment Concentration (C)

Based on the relationship between the current sediment storage (S_s) ($\text{kg/m}^3/\text{hr}$) and S_w for each time step calculation, the value of C from watershed outlet can be solved as:

$$C = \frac{S_s}{S_{ws}} \quad (13)$$

For each time-step calculation C is assumed to be uniform over the catchment and this is the variable of sediment continuity (see Eq.(15)).

To relate the sediment transport variables and the rainfall-runoff variables at the study catchment scale, a discrete relationship between S_{ws} , S_s^{\max} , and the average of overland flow water depth at the catchment, h_{s-avr} , was resulted as shown in Fig. 3b using the above procedures.

Finally, the continuity equations of runoff (Eq. (14)) and sediment (Eq.(15)) on a catchment scale:

$$\frac{dS_w(t)}{dt} = rA - Q \quad (14)$$

$$\begin{aligned} \frac{dS_s(t)}{dt} &= DR + DF - QC \\ &= kKE e^{-b h_{s-avr}} + \alpha (S_s^{\max} - S_s) h_{s-avr} - QC 3600 / A \quad (15) \end{aligned}$$

where A is the total area of the catchment (m^2).

The relationship between detachment and redeposition represented by Eq.(15) depend on the balance between S_s and S_s^{\max} .

The most important feature of this method is that the relationship between Q , S_w , S_{ws} , h_{s-avr} , and S_s^{\max} at the catchment scale are deductively derived as a discrete relationship, which is estimated on the

basis of the grid-cell from the distributed model and serves basis to solve the continuity equations of the lumped model. Thus transforming the distributed model into a lumped model. The lumped parameters maintain the physical meaning of the original distributed model without any additional calibration. The continuity equations were solved numerically using a fourth-order Runge-Kutta algorithm.

4. EVALUATION OF THE LUMPED SEDIMENT RUNOFF MODEL

The lumped model was applied to the Lesti River catchment (351.3 km²), a tributary catchment in the Upper Brantas River basin (11,800 km²), in East Java, Indonesia. This area represents a tropical volcanic area, where land use types are largely dominated by agriculture lands. Most of urban lands, dense forests, and paddy fields are relatively small and concentrated in several spot areas (see Fig. 4). At the confluence point of the Lesti River and the Brantas main reach, the Sengguruh dam was constructed in 1998. Unexpectedly, most of the gross storage (21.5 million m³) has been already filled with the large amount of sedimentation from the Lesti River.

The parameter values of the lumped sediment runoff model were: $n=0.001-0.3 \text{ m}^{-1/3}\text{s}$, $d_a=0.08-1.2 \text{ m}$, $d_m=0.04-0.8 \text{ m}$, $K_a=0.015 \text{ m/sec}$, $m=1.667$, $\beta=8$, $D_{50}=0.062 \text{ mm}$, $k=0.004 \text{ kg/J}$, $\alpha=0.98$, $\rho_s=2650 \text{ kg/m}^3$, $\rho_w=10^3 \text{ kg/m}^3$, $g=9.8 \text{ m/s}^2$, and $NU=10^{-6} \text{ m}^2/\text{s}$.

Figures 5 and 6 show comparisons of simulated water and sediment concentration at the outlet for two cases of soil thickness which calculated by the distributed and the lumped models. The blue and black lines are simulated water and sediment concentration by the distributed model and red lines are simulated water and sediment concentration by the lumped model. In the case when soil thickness is shallow (Case 1), discrepancy between the results for both the water discharge and sediment concentration by the lumped and distributed is generally less than in the case surface soil thickness is more thick (Case 2). The difference increases in the case when soil thickness is very thick, where the lumped model generally tends to under estimate water discharge and sediment concentration at the rising limb, and over estimate at the falling limb of hydrograph and sedimentgraph. This difference is due to the assumed steady state condition in deriving the lumped model. In Case 2, the catchment system did not reach a steady state due to slow movement of water for deep soil layer. Thus the $Q-S_w$ relationship for the lumped model shows some difference from distributed one. In the distributed model, some parts of the catchment are saturated and overland flow happens thus the rising limb of the hydrograph is faster than the lumped one.

Within the range of possible soil depths, the difference is small and it does not cause severe problems for applications. Table 1 shows the differences value in cumulative total runoff and

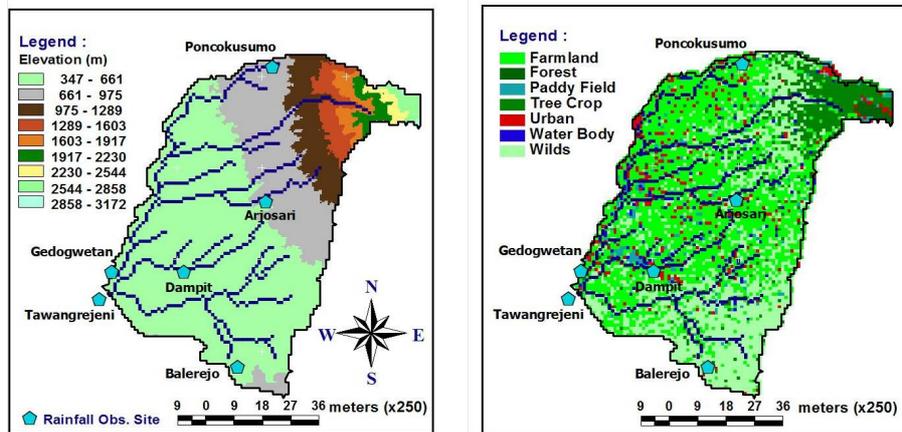


Fig.4 Lesti river catchment and land use in 2003.

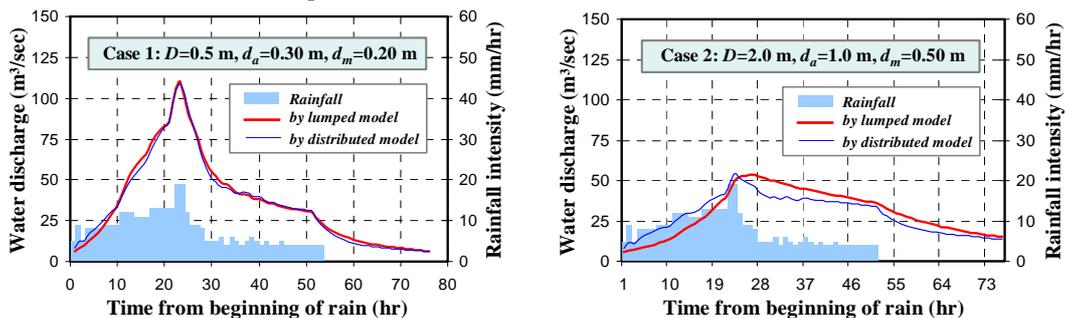


Fig.5 Computed water discharges with different values of soil thickness $D=0.5 \text{ m}$ (Case 1) and $D=2.0 \text{ m}$ (Case 2).

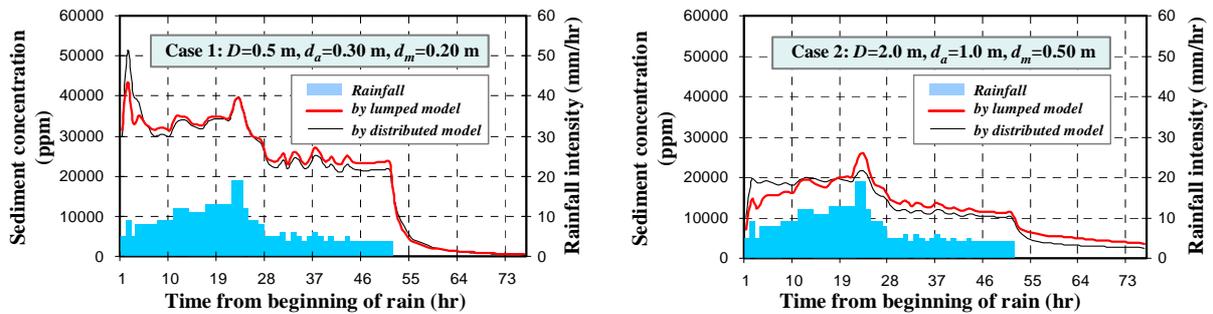


Fig.6 Computed sediment discharges with different combinations soil thickness $D=0.5$ m (Case 1) and $D=2.0$ m (Case 2).

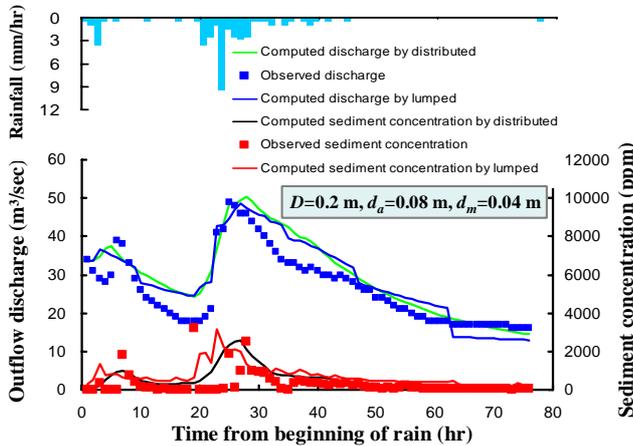


Fig.7 Event sediment-runoff observation and simulation at Tawangrejani station in 2003.

sediment yield calculated by lumped and distributed models, which are less than 7%.

Table 1 Simulated cumulative runoff and sediment yield for the two simulation cases

Variables	Cases	Distributed Mod.	Lumped Mod.	Difference (%)
Runoff (m ³)	Case 1	2724.20	2776.63	1.56
	Case 2	2192.48	2293.90	4.63
Sediment yield (kg)	Case 1	1517.05	1548.86	2.10
	Case 2	856.45	909.13	6.15

Figure 7 plots computed discharges and sediment concentrations by the distributed and lumped models compared to the observed data. The simulation results of the lumped model shows good agreement with the observed one. The lumped model successfully reduced the computational burden. Simulation time of the lumped model was about 1/37 of that of the original distributed model.

5. CONCLUSIONS

Simulation results computed by the lumped model agree well with the simulation results computed by the original distributed model. The lumped model runs efficiently in terms of calibrating and running time. The discrepancy of the lumped model and distributed model increases when the soil thicknesses increase, low accumulative rainfall amount, and/or spatial

temporal variation of rainfall is large.

The main advantage of lumping distributed model lies in the capability of being applied at large scale catchments without losing model and parameter physical interpretation. By incorporating hydrological and sediment transport processes derived from the distributed model, it is possible to reproduce the sediment runoff mechanism at catchment scale especially for large catchments.

River bed erosion or deposition, river bank erosion and lateral inflow of sediment to river channel were ignored. Its process will be considered in further developments of the lumped model.

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