A SIMPLE SCALING CHARATERISTICS OF RAINFALL IN TIME AND SPACE TO DERIVE INTENSITY DURATION FREQUENCY RELATIONSHIPS

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The properties of time and space scale invariance of rainfall are investigated and applied to Intensity-Duration-Frequency (IDF) relationships. The hypothesis of the simple scaling is examined in time and space in Yodo River basin and the simple scaling properties in time are confirmed, which is used to derive IDF relationships for short-duration rainfall of several hours from the statistical characteristics of daily data only. The derived IDF matches well with the one derived from historical observed data. Also, the simple scaling properties in space are examined. It is found that two ranges less than 1000 km^2 and more than 1000 km^2 show different scale properties.

Key Words: rainfall intensity, scaling invariance, IDF curves, design rainfall.

1. INTRODUCTION

The Intensity Duration Frequency (IDF) relationship of heavy storms is one of the most important hydrologic tools utilized by engineers for designing flood alleviation and drainage structures in urban and urban areas. Local IDF equations are often estimated on the basis of records of intensities abstracted from rainfall depths of different durations, observed at a given recording rainfall gauging station. In some regions, there may exist a number of recording rainfall gauging stations operating for a time period sufficiently long to yield a reliable estimation of the IDF relationships; in many other regions, especially in developing countries, however, these stations are either non-existent or their sample sizes are too small. Because daily precipitation data is the most accessible and abundant source of rainfall information, it seems natural, at least for the regions where data at higher time resolution are scarce, to develop and apply methods to derive the IDF characteristics of short-duration events from daily rainfall statistics.

Over the last two decades, concepts of scale invariance have come to the fore in both modelling and data analysis in hydrological precipitation research. Gupta and Waymire[1] studied rainfall spatial variability by introducing the concepts of simple and multiple scaling to characterise the probabilistic structure of the precipitation processes. Kuzuha et al.[2] showed the scaling framework and regional flood frequency analysis. Burlando and Rosso[3] showed that both the simple scaling and multiscaling lognormal models can be used to derive Depth Duration Frequency (DDF) curves of point precipitation. Menabde et al.[4] developed a simple scaling methodology to use daily rainfall statistics to infer the IDF curves for rainfall duration less than 1 day. The scaling hypothesis was verified by fitting the model to two different sets of data (from Australia and South Africa). Yu et al.[5] is an example of methodology in which the theories of scaling properties and employed to infer the IDF characteristics of short-duration rainfall from daily data.

Until now, time scaling characteristics have been studied by many researchers, while space scaling properties that will link to derivation of Area-Intensity-Duration-Frequency (AIDF), are not well studied. Thus, in this paper, the properties of time and space scale invariance of rainfall are examined in the Yodo River catchment. Then the IDF relationships for short-duration rainfall from daily data are derived according to Menabde[3] and compared with the obtained from the traditional method.

The paper is organized as follows: The next

section presents the theoretical background as related to the time and space scale invariance properties of short-duration rainfall. In section 3, these properties are verified for data observed at rainfall gauging stations located in the Yodo River catchment in Japan. The IDF relationship, derived from daily rainfall data, then compared to the traditional estimated curve in section 4 and conclusions are given in the last section.

2. SIMPLE SCALING CHARATERISTICS OF RAINFALL IN TIME AND SPACE

In this section, a general theoretical framework for the simple scaling is introduced. The scaling or scale-invariant models enable us to transform data from one temporal or spatial model to another one, and thus, help to overcome the difficulty of inadequate data. A natural process fulfills the simple scaling property if the underlying probability distribution of some physical measurements at one scale is identical to the distribution at another scale, multiplied by a factor that is a power function of the ratio of the two scales. The basic theoretical development of scaling has been investigated by many authors including Gupta and Waymire[1] and Kuzuha *et al.*[2]

Let $X(t)$ and $X(\lambda t)$ denote measurements at two distinct time or spatial scales t and λt, respectively. Definition of scaling of the probability of the $X(t)$ is

$$X(t) \overset{dist}{=} \lambda^{-H} X(\lambda t) \qquad (1)$$

The equality " $\overset{dist}{=}$ " refers to identical probability distributions in both side of the equations, λ denotes a scale factor and H is a scaling exponent. Gupta and Waymire[1] introduced the notions of strict and wide sense simple scaling. The strict sense simple scaling in Eq.(1) implies that $X(t)^q$ and $(\lambda^{-H} X(\lambda t))^q$ have the same probability distribution. The wide sense simple scaling is expressed as they have the same moments, i.e.

$$E[X(t)^q] = \lambda^{-Hq} E[X(\lambda t)^q] \qquad (2)$$

The scaling exponent, Hq can be estimated from the slope of linear regression relationship between the log transformed values of moment, $\log E[X(\lambda t)^q]$ and scale parameters $\log \lambda$ for various order of moment q. This is definition of a "wide sense" simple scaling[1]. A "wide sense" simple scaling with $t=1$ and $\lambda t = \lambda$, is given by

$$E[X(1)^q] = \lambda^{-Hq} E[X(\lambda)^q] \qquad (3)$$

If the scaling exponent H is not constant and changes probabilistically, Eq.(3) are described as

$$E[X(1)^q] = \lambda^{-K(q)} E[X(\lambda)^q] \qquad (4)$$

where $K(q)$ is a function of the moment order. The procedure to test the suitability of scale invariant model to describe the rainfall process is briefly described in **Fig.1**. The moments $E[X(\lambda t)^q]$ are plotted on the logarithmic chart versus the scale λ for different moments' order q. The slope $K(q)$ is plotted on the linear chart versus the moment order q. If the resulting graph is a straight line, the field is simple scaling, while in other cases, the multi-scaling approach has to be considered[1].
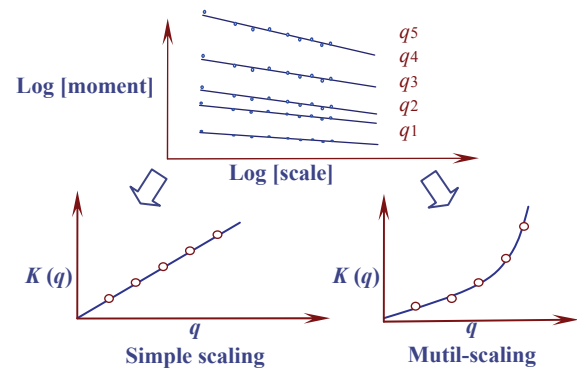

Fig.1 Simple and multiscaling in term of statistical moments. First step, moments of different orders q are plotted as function of scale in a log-log plot. From the slope, values of the function $K(q)$ are obtained. If $K(q)$ is linear, the process is simple scaling. If $K(q)$ is non-linear, the process is multiscaling.

(1) Time scaling of rainfall intensity

Let the random variable $I(d)$ the maximum annual value of local rainfall intensity over a duration d. It is defined as:

$$I(d) = \max_{0 \le t \le 1 year}\left[\frac{1}{d} \int_{t-d/2}^{t+d/2} X(\xi)d\xi \right] \qquad (5)$$

where $X(\xi)$ is a time continuous stochastic process representing rainfall intensity and d is duration. It is supposed that $I(d)$ represents the Annual Maximum Rainfall Intensity (AMRI) of duration d, defined by the maximum value of moving average of width d of the continuous rainfall process. Here, some concepts are introduced about scaling of the probability distribution of random functions. A generic random function $I(d)$ is denoted by simple scaling properties if it obeys the following:

$$I(d) \overset{dist}{=} \left(\frac{D}{d} \right)^{-H_d} I(D) \qquad (6)$$

D is a aggregated time duration, i.e.: 2, 3, ..., 24 hours.

Defining the scaling ratio is $\lambda_d = \dfrac{D}{d}$

$$I(d) \overset{dist}{=} \lambda_d^{-H_d} I(\lambda_d d) \qquad (7)$$

The Eq.(7) is rewritten in terms of the moments of order q about the origin, denoted by $E\left[I(d)^q\right]$. The resulting expression is:

$$E\left[I(d)^q\right] = \lambda_d^{-H_d q} E\left[I(\lambda_d d)^q\right] \qquad (8)$$

If one assumes the wide sense simple scaling exists, the distribution of IDF for short-duration of rainfall intensity can be derived from daily rainfall.

(2) Space scaling of rainfall intensity

Let consider a continuous precipitation process $I(d,a)$ which represents the maximum annual value of average rainfall intensity over a duration d, and an area a. It is defined as

$$I(d,a) = \max_{0 \le t \le 1 year}\left[\frac{1}{da}\int_{t-d/2}^{t+d/2}\int_a X(\xi,\omega)d\omega d\xi\right] \qquad (9)$$

where $X(\xi,\omega)$ is a time-space continuous stochastic process representing rainfall intensity.

Since the probability of extreme events is usually examined, the maximum annual rainfall intensity can be defined as the maximum value of a moving average (with area span) in a given year. The concepts are introduced about scaling of the probability distribution of random functions. A generic random function $I(d,a)$, with fixed duration d, is denoted by the simple scaling properties of space if it obeys the following relationship:

$$I(d,a) \overset{dist}{=} \left(\frac{A}{a}\right)^{-H_a} I(d,A) \qquad (10)$$

Defining the space scaling ratio is $\lambda_a = \dfrac{A}{a}$

$$I(d,a) \overset{dist}{=} \lambda_a^{-H_a} I(d,\lambda_a) \qquad (11)$$

This also implies that the moments of any order are scale-invariance. The resulting expression is

$$E\left[I(d,a)^q\right] = \lambda_a^{-H_a q} E\left[I(d,\lambda_a a)^q\right] \qquad (12)$$

where q denotes the order of the moment. If one assumes the wide sense simple scaling exists, the AIDF curves can be derived by developing a general framework based on the estimation of a common scaling exponent for any frequency level. This approach is used to derive the distribution of the AIDF where data for the spatial scale of interest does not exist.

3. SCALE INVARIANCE PROPERTIES OF RAINFALL IN TIME AND SPACE IN YODO RIVER CATCHMENT, JAPAN

The scaling properties of rainfall data was investigated by computing the moment for each duration and area and then by examining the log-log plots of the moments against their duration and area. **Fig.2** and **Fig.3** illustrate the relationships between moments versus duration (time) and area (space) and a plot of the scaling exponents versus the order of moments for the Yodo River catchment.

In order to examine the time scale invariance of rainfall in the Yodo River catchment, the four stations Hikone, Hirakata, Ieno and Kamo were selected from Yodo River catchment. The record lengths are 21 years and ranged from 1982 to 2002. The analysis was performed on annual maximum rainfall series for storm durations from 1 hour to 24 hours, with λ_d =1, 2...24. **Fig.2a)** shows the relationships between the log-transformed values of moment of various orders against values duration at the Hikone station, the slopes of linear regression define the scaling exponents H_d. **Fig.2b)** shows the scaling exponent decreases with the order of moment and a linear relationship exists between scaling exponents and orders of moment, which implies that the property of wide sense simple scaling of rainfall intensity exists in this station.

In **Table 1**, the results of the scaling exponent factor H_d of the four stations in the Yodo River catchment are shown with the high coefficients of determination for each station ranging from 0.98 to 1. The result indicates a strong validity of the simple scaling property of the extreme rainfall in time series.
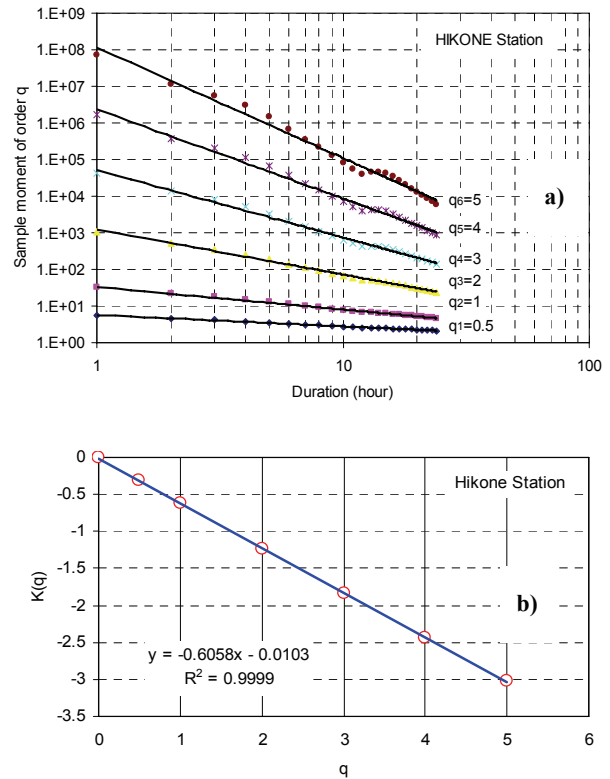

Fig.2 The time scale invariance at the Hikone station, the Yodo catchment, Japan **a)** Relationship between moment of order q and duration (hour) **b)** Relationship between $K(q)$ and the sample moment order q.
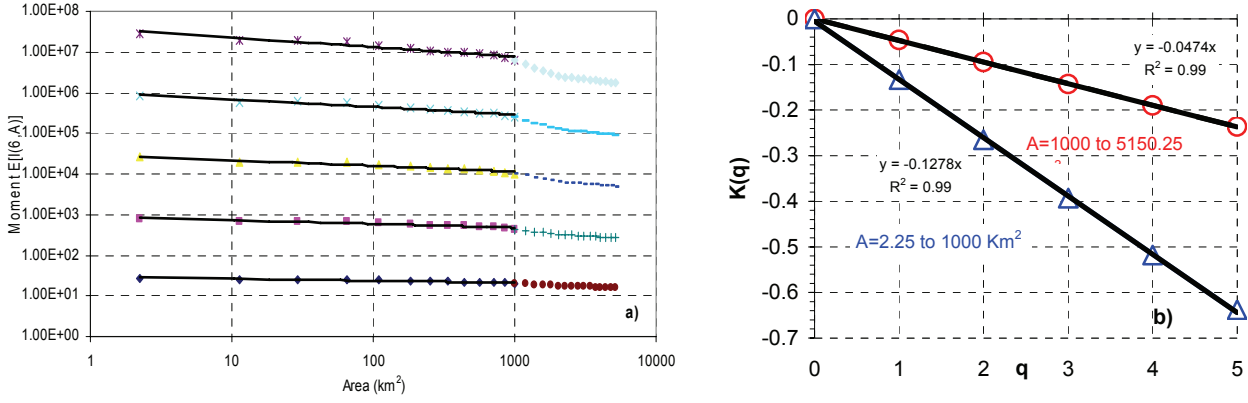
Fig.3 The space scale invariance at the Yodo River catchment for 6 hours. **a)** Relationship between moment of order q and area A (km^2). **b)** Relationship between $K(q)$ and the sample moment order q.

Table 1. The scaling exponent factor of the four stations in the Yodo River catchment

Name of the station	No of years record	Scaling exponent H_d	R^2
Hikone	21	-0.605	0.99
Hirakata	16	-0.678	1.00
Ieno	14	-0.681	0.99
Kamo	21	-0.693	1.00

To examine the space scaling invariance, at first hourly data from 1982 to 2002 with 1.5 x 1.5 km spatial resolution, which is generated by AMeDAS rain gauge data and covers the Yodo River basin are arranged. To obtain spatial mean rainfall intensity the Otorii station is put at the center of the area, and the area is expanded in the form of circle with increased radius by 0.5 km.

The data used for examined spatial scaling factor are annual maximum rainfall series for the area from 2.25 km^2 to 5150.25 km^2, with λ_a =1,5...2289 and fixed duration d. In **Fig.3** a difference of the degree of steepness in the slopes for the area (2.25 km^2 to 1000 km^2) and larger area (1000 km^2 to 5000 km^2) area is found, which indicates that two different scaling space regimes exist for rainfall space scaling. A steeper slope is found in the smaller area compared to the lager area. The plots indicate that the relationships between moments and areas are linear having two different slopes with a breaking point at 1000 km^2. This property suggests an existence of the two different regimes with a transition in storm dynamics from high variability convective storms with less than 1000 km^2 to frontal storms in an area larger than 1000 km^2.

4. DERIVATION OF IDF FOR SHORT DURATION

All forms of the generalized IDF relationships assume that rainfall depth or intensity is inversely related to the duration of a storm raised to a power, or scale factor[6]. There are several commonly used functions found in the literature of hydrology applications[7),8),9),10)]. Koutsoyiannis *et al.*[11] have modified the IDF relationship for a given return period as particular cases, using the following general empirical formula

$$i = \frac{w}{(d + \theta)^{\eta}} \qquad (13)$$

where i denotes the rainfall intensity for duration d and w, θ and η represent non-negative coefficients. In fact, these arguments justify the formulation of the following general model for the IDF relationships:

$$i = \frac{a(T)}{b(d)} \qquad (14)$$

In Equation (14), $b(d) = (d + \theta)^{\eta}$ with $\theta > 0$ and $0 < \eta < 1$, whereas $a(T)$ is completely defined by the probability distribution function of the maximum rainfall intensity. The form of Equation (14) is consistent with most IDF empirical equations estimated for many locations[12]: For example Nhat *et al.*[13] established the IDF curves for precipitation in the monsoon area of Vietnam.

The random variable rainfall intensity $I(d)$ for duration d, has a cumulative probability distribution CDF, which is given by

$$\Pr(I(d) \leq i) = F_d(i) = 1 - \frac{1}{T(i)} \qquad (15)$$

According to the scaling theory by Menabde *et al.* [4], the scaling property in a strict sense can be written explicitly using the CDF:

$$F_d(i) = F_{\lambda_d d}[\lambda_d^{-H_d} i] \qquad (16)$$

For many parametric forms, left hand side of Equation (16) may be expressed in terms of standard variant, as in

$$F_d(i) = F\left[\frac{i - \mu_d}{\sigma_d}\right] \qquad (17)$$

where $F(.)$ is a function independent of d. Under this form, it can be deduced from Equation (16) that

$$\mu_d = \lambda_d^{-H_d} \mu_{\lambda_d d} \qquad (18)$$

$$\sigma_d = \lambda_d^{-H_d} \sigma_{\lambda_d d} \qquad (19)$$

Substituting Equations (17), (18) and (19) into Equation (15) and investing with respect to i, one obtains:

$$i_{d,T} = \frac{\mu_{\lambda_d d}(\lambda_d d)^{-H_d} + \sigma_{\lambda_d d}(\lambda_d d)^{-H_d} F^{-1}(1-1/T)}{d^{-H_d}} \qquad (20)$$

By equaling Equation (20) to the general model for IDF relationship, given by Equation (13), it is easy to verify that

$$\eta = -H_d \qquad (21)$$

$$\theta = 0 \qquad (22)$$

$$b(d) = d^\eta \qquad (23)$$

$$a(T) = \mu_{\lambda_d d}(\lambda_d d)^{-H_d} + \sigma_{\lambda_d d}(\lambda_d d)^{-H_d} F^{-1}(1-1/T) \qquad (24)$$

$$i_{d,T} = \frac{\mu + \sigma F^{-1}(1-1/T)}{d^\eta} \qquad (25)$$

where $\mu = \mu_{\lambda_d d}(\lambda_d d)^{-H_d}$ and $\sigma = \sigma_{\lambda_d d}(\lambda_d d)^{-H_d}$ are constants. It is worthwhile to note that the simple scaling hypothesis leads to the equality between the scale factor and the exponent in the expression relating rainfall intensity and duration.

Application of the simple scaling model for rainfall intensities at the Hikone station, the scale factor can be estimated $H_d = -0.6058$. The IDF relationship for a short duration rainfall can be deduced from daily data by Equation (25) with the estimates of μ_D and σ_D with D=24h. From 24-hour data collected at the Hikone recording station, the sample of 21 years of 24 hours annual maximum rainfall intensity yields, which can be estimated $\mu_{D=24} = 4.615$ and $\sigma_{D=24} = 2.604$.

Back with these estimates and the Gumbel inverse function, the deduced IDF relationship for the location of Hikone may be written as Equation (25) with $\mu = \lambda^\eta \mu_{24} = 31.56$ and $\sigma = \lambda^\eta \sigma_{24} = 17.81$ then

$$i = \frac{31.56 - 17.81\ln(-\ln(1-1/T))}{d^{0.605}} \qquad (26)$$

The rainfall Intensity Duration Frequency curves for the Hikone station can be reconstructed by scaling methods with Equation (26). **Figure 4** shows well matching of the IDF relationships calculated by Equation (26) and plots obtained by the historical data as the return periods from 5 to 100 years. The scale factor H_d along with the parameters σ and μ, may be interpreted as regional climatic characteristics.
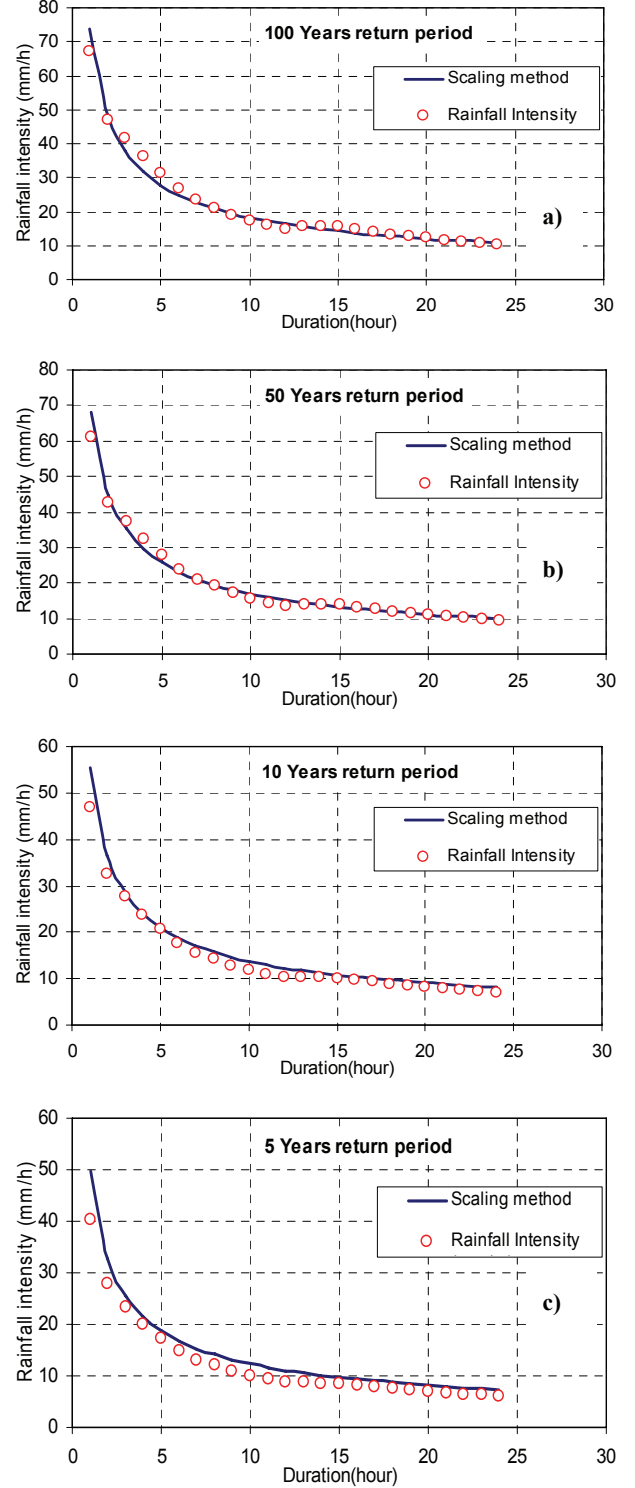

Fig.4 The Rainfall Intensity Frequency Curves at the Hikone station, the Yodo Catchments, Japan by the scaling method.
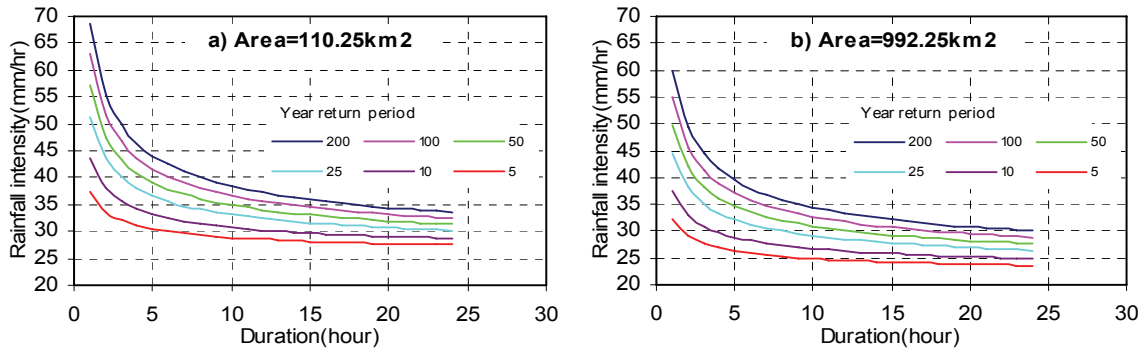
Fig. 5 The Area Rainfall Intensity Frequency Curves at the Yodo River catchment, Japan by the scaling method.
a)Area=110.25Km^2 and b)Area=992.25.25Km^2.

The AIDF curves can be derived by the simple scaling method. For example, with a fixed area equal to 110.25 km^2 and a scaling factor Hd=-0.51; $\mu = \lambda^\eta \mu_{24} = 25.08$; $\sigma = \lambda^\eta \sigma_{24} = 8.22$ are found, the AIDF curves can be derived by:

$$i = \frac{25.08 - 8.22 \ln(-\ln(1-1/T))}{d^{0.514}} \qquad (27)$$

For an area of 992.25 km^2, the IDF curve can be constructed with a scaling factor H_d= -0.46:

$$i = \frac{21.09 - 7.35 \ln(-\ln(1-1/T))}{d^{0.46}} \qquad (28)$$

The **Figure 5** shows a result of the AIDF based on the scaling method in the Yodo River catchment by the equations (27) and (28).

5. CONCLUSION

The major findings of the present study can be summarized as follows: The properties of the time and space scale invariance of rainfall quantiles are examined in the Yodo River basin: For time scaling, rainfall properties follows a simple scaling; For space scaling, a simple scaling process with the two different regimes of 2.25 km^2 to 1000 km^2 and 1000 km^2 to 5000 km^2 are found.

Then, according to Menabde *et al.*[4] the rainfall IDF curves for a short duration (hourly) were derived from 24-hour data. The simple scaling properties are verified by local data; the IDF relationships are deduced from daily rainfall, which shows good results in comparison with the IDF curves obtained from at-site short-duration rainfall data.

Results of this study are of significant practical importance because statistical rainfall inferences can be made with the use of a simple scaling in time and space. Furthermore, daily data are more widely available from standard rain gauge measurements, but data for short durations are often not available for the required site. Further works will focus on a combination between the time scaling and the space

scaling and theoretical description of the AIDF applicable for a less gauged basin.

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