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ENERGY BALANCE SNOWMELT MODELING FOR DATA-POOR BASINS

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Few investigations have been made into modeling snowmelt in data-poor basins; hence, a degree-day method is widely used and is routinely justified under the auspices that energy-balance models require too many input data. To test this claim, we investigated the utility of merely adopting a full energy approach to model snowmelt. This study first developed so-called “full energy balance snow model” to simulate snowmelt at four sites located in Japan and the USA. The results showed very good agreement between observed and predicted snow water equivalent, R^2>0.95. We duplicated the simulations using the approximated version of the model that requires only air temperature and wind speed as input data. Although the original model corresponds better, the performance of its simplified version can be evaluated as good, R^2>0.9. These results provide significant information for the development of appropriate approximations in energy balance snowmelt modeling.

Key Words: *Snowmelt, energy balance, snowmelt modeling, data-poor basins*

1. INTRODUCTION

Snowmelt is a significant surface water input of importance to many aspects of hydrology. Flooding, contaminant transport, water supply recharge, and erosion are a few processes receiving public attention that are directly linked to snow processes. Numerous studies[1)2)] have reported snowmelt-modeling problems as commonly acknowledged weakness in hydro-environmental models. Modeling snowmelt in hydrological models is particularly problematic for data-poor basins where there is a lack of data.

Despite the well-establish accuracy of energy balance snowmelt models, there is a propensity towards using degree-day snowmelt relationships, especially in applications to poorly gauged basins. The most common justification for degree-day models is that energy balance calculations require too many data. Nevertheless, the use of the degree-day method has some precautions, the most significant being the determination of degree-day factors. No single, universally applicable degree-day factor of snowmelt exists. Factors vary with atmospheric conditions, time of year, vegetation cover, topography, physical properties of snow cover, and many other variables. In additional, distributed snowmelt modeling, which integrates several energy exchange processes, is dependent on how the relevant processes are spatially distributed and it is unlikely that a degree-day model will meaningfully capture this heterogeneity.

Walter et al.[3)] investigated the feasibility of “estimating missing data” to facilitate energy balance snowmelt modeling and estimating, by straightforward methods, the required parameters that are seldom available. In spite of this study improved our knowledge of simple adopting of full energy balance to simulate snowmelt, further work is needed to better understand how well this type of modeling approach works.

In this study, we develop a simple energy balance snowmelt model that requires only maximum and minimum daily air temperature and wind speed as input data and examined its accuracy in comparison with so-called “full energy balance model” that also is proposed in this paper.

2. DAILY SNOW ENERGY BUDGET

We used the following simple, but all components, energy balance (hereafter denoted as proposed_EB) for a one-layer snowpack:

$$\lambda_f \rho_w \Delta SWE = Q_m = Q_n + Q_h + Q_e + Q_g + Q_a - \Delta Q_i \quad (1)$$

where λ_f is the latent heat of fusion (0.334 MJ kg^{-1}), ρ_w is density of water (kg m^{-3}), ΔSWE is the change in the snowpack's water equivalent (mm), Q_m is the energy available for snowmelt, Q_n is the net all-wave radiation to snow, Q_h is the sensible convective heat flux from the atmosphere, Q_e is the latent heat flux to the surface due to vaporization and condensation, Q_g is ground heat conduction to the bottom of the snowpack, Q_a is the energy transported to the snowpack by rainfall, and ΔQ_i is the rate of change in the internal energy stored in the snowpack. Units for each energy balance term are in MJ m^{-2} day^{-1}.

(1) Net Radiation

Net radiation Q_n is the sum of the net short-wave Q_{sw} and net long-wave Q_{lw} fluxes. Since the net long-wave is the amount of long-wave radiation emitted downward by the atmosphere $Q_{lw}^{\downarrow}$, and the component emitted upward by the earth's surface $Q_{lw}^{\uparrow}$, Q_n can be expressed as

$$Q_n = Q_{sw} + Q_{lw}^{\downarrow} - Q_{lw}^{\uparrow} \quad (2).$$

The amount of energy available for snowmelt from the absorption of short-wave radiation is

$$Q_{sw} = K_{ET} \cdot f(C) \cdot (1 - A) \quad (3)$$

where K_{ET} is the daily extraterrestrial solar radiation, $f(C)$ is the function expressing the effect of cloud cover, and A is the surface albedo.

The extraterrestrial radiation, K_{ET}, is readily estimated from the solar constant and declination, which are the functions of the location latitude and the date in the year, based on well establish astronomical relationships proposed by Digman[4].

The effect of cloud cover can be estimated using an empirical relation as[4]

$$f(C) = 0.355 + 0.68 \cdot (1 - C) \quad (4)$$

where C is the fraction of cloud cover.

The snow albedo is modeled using a function considering the age and the surface temperature of the snowpack as[5]

$$A = A_{min} + A_{add} \cdot e^{-kn} = 0.3 + 0.6 \cdot e^{-kn} \quad (5)$$

where A_{min} is the lowest possible albedo of snow, A_{add} is a constant that added to A_{min}, is the initial albedo, k is a recession factor depending on the snow surface temperature and n is the number of days since the last considerable snowfall; each time such a snowfall occurs the snow albedo is reset to its maximum value. We chose the following recession factors: k=0.05 and k=0.12[5] for positive and negative snow temperature, respectively.

Incoming long-wave radiation for clear sky is estimated based on air temperature, T_a (ºC), using the Stefan-Boltzmann equation

$$Q_{lwcls}^{\downarrow} = \varepsilon_{acls} \cdot \sigma \cdot (T_a + 273.2)^4 \quad (6)$$

where σ is the Stefan-Boltzmann constant (σ = 4.90 · 10^{-9} MJ m^{-2} day^{-1} K^{-4}) and ε_{acls} is the air emissivity, which can be calculated by a variety of methods. For example Idso[6]

$$\varepsilon_{acls} = 0.70 + 5.95 \cdot 10^{-5} e_a \cdot \exp\left(\frac{1500}{T_a + 273.2}\right) \quad (7)$$

where e_a is vapor pressure (kPa).

When cloud is presented, incoming long-wave radiation is often expressed

$$Q_{lw}^{\downarrow} = N \cdot Q_{lwcls}^{\downarrow} \quad (8)$$

where N is a coefficient depending on cloud amount, which can be expressed using the following equation Kustas[7]

$$N = 1 + 0.22 \cdot C^2 \quad (9).$$

Outgoing long-wave radiation is

$$Q_{lw}^{\uparrow} = \varepsilon_s \cdot \sigma \cdot (T_s + 273.2)^4 \quad (10)$$

where ε_s is the surface emissivity for snow surface and T_s is the snow temperature (ºC). We assumed ε_s=0.98 being the mean value of the long-wave emissivity of snow.

(2) Turbulent fluxes

Sensible and latent heat fluxes between the snow surface and air above are modeled using a simple empirical equation proposed by Kuzmin[8]

$$Q_h + Q_e = D_{he} \cdot \left[(T_a - T_s) + 1.75 \cdot (e_a - e_s)\right] \cdot (1 + 0.547 \cdot v_a) \quad (11).$$

Here, D_{he} is the bulk transfer coefficient (0.293 MJ m^{-3} day^{-1} ºC^{-1} kPa^{-1} s) and v_a is wind speed (m s^{-1}).

(3) Ground heat conduction

Heat conduction from the ground into a snowpack tends to be small. A constant value of 0.02 MJ m^{-2} day^{-1} was assumed based on US Army Corps of Engineers[9] melt estimation.

(4) Precipitation heat

If the temperature of the rain is assumed to be equal to the air temperature, heat from precipitation can be calculated as

$$Q_a = \rho_w \cdot c_w \cdot r \cdot T_{rt} \quad (12)$$

where ρ_w is density of water (1000 kg m^{-3}), c_w is heat capacity of water (4.19 × 10^{-3} MJ kg^{-1} ºC^{-1}), r is the rainfall rate (m day^{-1}), and T_{rt} is the temperature of the rain (ºC).

(5) Snow fall accumulation

Measured precipitation rate P, is partitioned into rain P_r, and snow P_s, (both in terms of water equivalence depth, mm day^{-1}) using the following rule based on air temperature T_a,

$$P_r = P \quad \text{if} \quad T_a \geq T_r$$
$$P_s = P \quad \text{if} \quad T_a < T_r \qquad (13)$$

where T_r is a threshold air temperature (ºC) below which all precipitation is snow.

(6) Stored snowpack energy

In the model, the snowpack is treated as one layer. During period of net energy loss from the snow pack, the snow temperature will decrease proportionally and will rise during periods of net energy gain, but will not exceed the freezing point, T_{frz}. If the condition is right for melt then all heat added to the snowpack will produce liquid melt. The meltwater outflow from the snowpack R_w (mm day^{-1}) during the melting season is determined as[8]

$$R_w = \begin{cases} \dfrac{M}{1-\varphi} + P_r \\ 0, \quad \sum (M + P_r) \leq \varphi (SWE + P_s) \end{cases} \qquad (14)$$

where M is the liquid melt (mm) produced by Eq.(1) and φ is a parameter characterizing the maximum liquid water holding capacity of snow. We assumed the liquid water holding capacity as 5%[8] of the SWE and during period of net loss from the pack, this water was refrozen.

(7) Calibration factor, T_{thr}

In order to simplify model calibration, we made an assumption that the freezing point and the threshold air temperature are equal. The relationships between T_r and T_{frz} can expressed through a calibration factor, T_{thr}, as

$$T_{thr} = T_r = T_{frz} \quad \text{if} \quad T_a < 0\ ºC$$
$$T_{thr} = T_r,\ T_{frz} = 0\ ºC \quad \text{if} \quad T_a \geq 0\ ºC \qquad (15).$$

3. METHODOLOGY AND DATA

(1) Model approximations.

We first assumed that cloud cover (C) can be sufficiently substituted for an atmospheric transmittance factor, T_f, that can be calculated with an equation originally proposed by Bristow and Campbell[10] and latter modified by Thornton and Running[11]:

$$C = 1 - T_f = 1 - a \cdot \left(1 - \exp\left(-b \cdot \Delta T^c\right)\right) \qquad (16)$$

where ΔT is the diurnal temperature range, a (=0.8) is the maximum clear sky transmittance, and c (=2.4) is an empirical parameter that Bristow and Campbell calibrated. b is a parameter dependent on the monthly mean diurnal temperature range, $\Delta \overline{T}$,

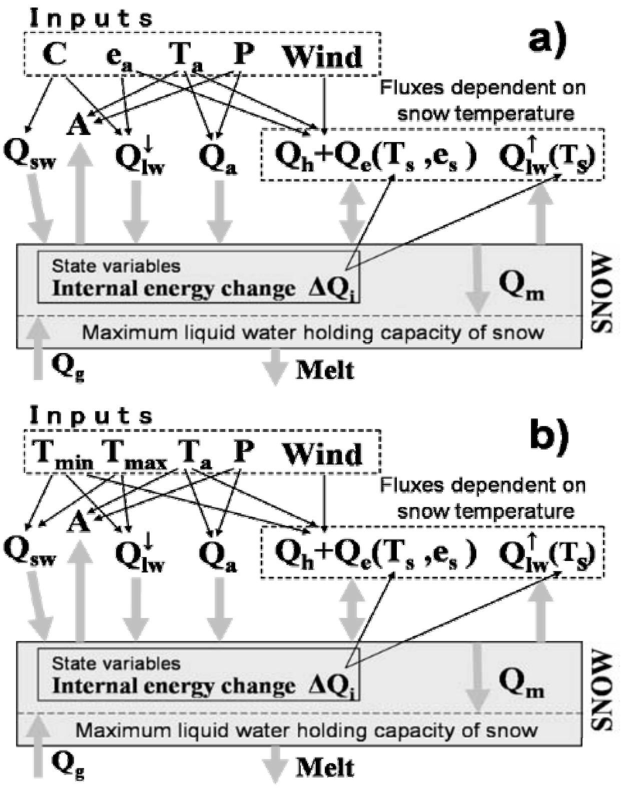

Fig.1 Snow models physics and parameterization: a) proposed_EB, b) simplified_EB.

which can be obtained using a three-parameter exponential decay curve:

$$b = b_0 + b_1 \cdot \exp\left(-b_2 \cdot \Delta \overline{T}\right) \qquad (17)$$

where b_0 (=0.031), b_1(=0.201) and b_2(=0.185) are parameters calibrated by Thornton and Running using data from 40 stations in contrasting climates of the United States.

Secondly, we relied on a well-known assumption that air vapor pressure (e_a) can be generated as a function of the daily minimum temperature (T_{min})[11]

$$e_a = 6.1078 \cdot \exp\left[\frac{17.269 \cdot T_{\min}}{237.3 + T_{\min}}\right] \qquad (18).$$

These above approximations allowed avoiding the use in the simplified version of the model (hereafter denoted as simplified_EB) both air vapor pressure and cloudiness as inputs (see **Fig.1**).

(2) Data

We used four data sets to test the applicability of both the proposed_EB model and its simplified version here to a wide range of environments. Of four sites used in the study, three (Nagaoka, Shinjiyo and Myoko) are located in Japan and one, Reynolds Creek, in the USA. The data set of each site includes both hourly meteorological and daily snow water equivalent data. All the Japanese sites' data were provided by Snow and Ice Research Center, National Research Institute for Earth

Science and Disaster Prevention, Japan[13]. The Reynolds Creek Experimental Wateshed, located in the Owyhee Mountains in southwestern Idaho, was established in 1960 as a field laboratory for hydrologic research. Historical data monitored in the watershed can be downloaded from ftp.nwrc.ars.usda.gov.

(3) Method.

In order to evaluate the performance of the proposed_EB and to test the feasibility of its approximated version to facilitate snowmelt modeling, we explored two options for simulating snow water equivalent (SWE).

In the first, the proposed_EB was applied at each site. All the model parameters were unalterable, as described above, except one parameter, T_{thr}, that was used to conduct model calibration.

The second option we explore were subsequent applications of the simplified_EB using the same fixed parameters along with the calibrated T_{thr} values to see how well this type of modeling approach works.

(4) Validation statistics.

Four standard quantitative tests were used to evaluate model performance and the goodness of fit for the model applications; namely, the coefficient of determination, the Nash-Sutcliffe coefficient, the mean bias difference (MBD), and the standard error (Ste)[12]. The coefficient of determination or the R-squared value (R^2) is an indicator that reveals how closely the estimated values for the trendline correspond to the actual values. The Nash-Sutcliffe coefficient or "model efficiency" (ME) describe the variation in the observed parameter accounted for by the simulated values.

4. RESULTS AND DISCUSSION

Since we developed a new modification of existing energy budget snowmelt models, the ability of the proposed_EB to simulate SWE change was examined first. By applying the model, the following was found out:

(1) The model requires calibration. **Fig. 2** shows predicted and simulated SWE at all the locations. It can be seen from this figure that the proposed_EB was calibrated against observed SWE at each site in order to match better agreements.

(2) Model calibration can be sufficiently fulfilled by the calibration factor only while the other model parameters are fixed. **Fig.2** displays the values of T_{thr} corresponding to the best fit for each site we could achieve. There is only one site, Myoko,

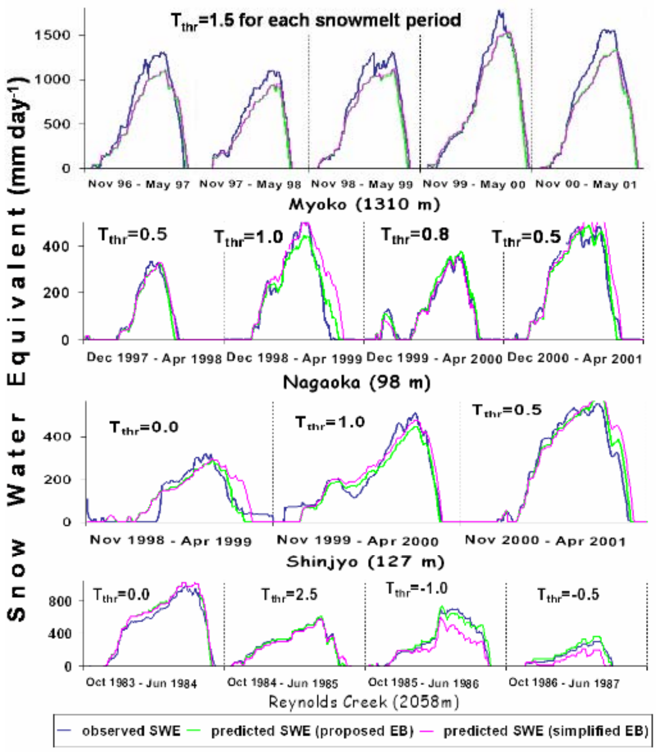

Fig.2 Comparison between predicted and observed SWE for simulations at each site used in this study. T_{thr}, calibration factor.

where the model showed good performance with the permanent value of T_{thr} applied through all of five winter seasons. However, for the other sites, such a method failed, and that required calibration by adjusting T_{thr} separately for each site and snow season. The calibrated values of T_{thr} range from -1.0 for Reynolds Greek (Oct 1985 – Jun 1986 snow season) to 2.5 for the same site (Oct 1984 – Jun 1985 snow season). It is important to note that model results are very sensitive to T_{thr} tuning for SWE estimates.

After calibrating the proposed_EB, we tested the performance of its simplified version, the simplified_EB, using the derived values of T_{thr}. In **Fig.2**, in additional to the SWE predicted by proposed_EB, the ones resulted from the applications of the simplified_EB are also shown. In this figure both the proposed_EB and simplified_EB models show good agreements with the proposed_EB predicted SWE being slightly better.

In order to clarify the difference among the proposed_EB and simplified_EB SWE results, we calculated snow seasons daily average values of both of simulated and observed SWE for each site as shown in **Fig.3**. It can be seem from this figure that even though the proposed_EB in general performed better for almost all the simulations, except at Myoko where both SWE results are nearly similar. There are no significant differences between both predicted SWE.

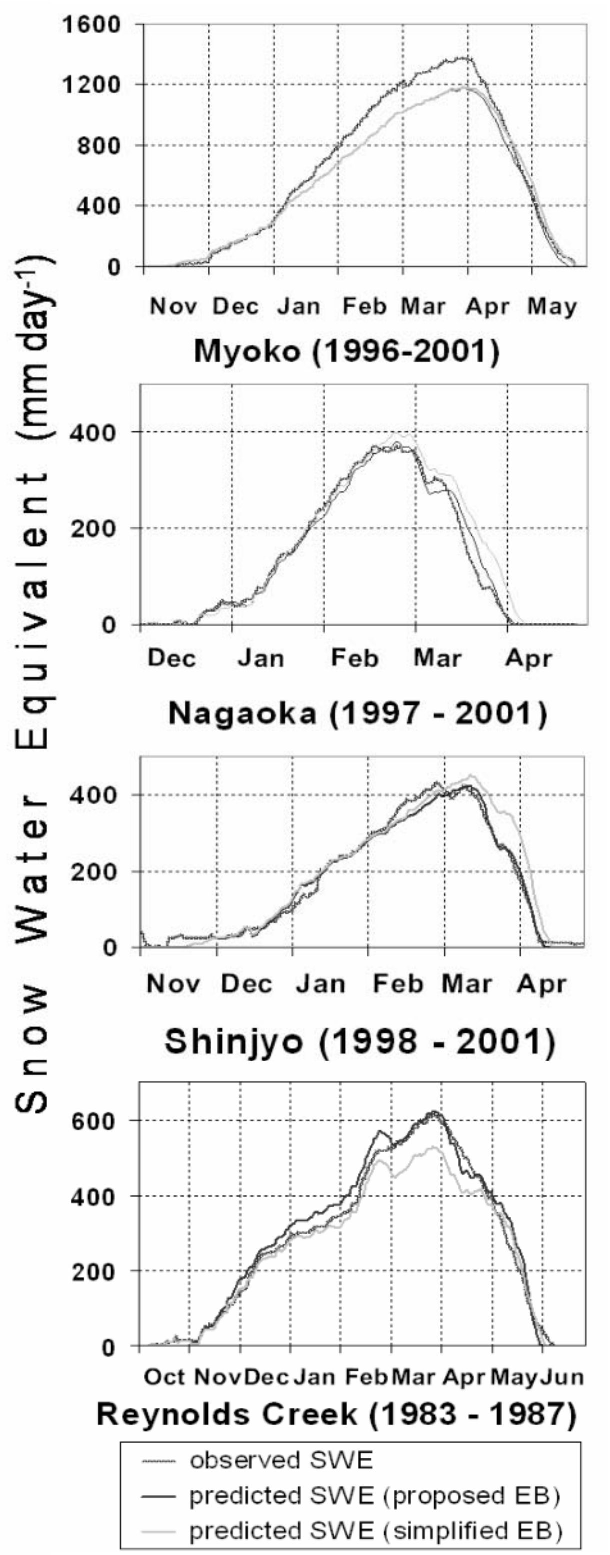

Fig.3 Predicted and observed SWE: average values over the snow periods for each site.

There is an analogous underestimated shape of the average SWE results at Myoko site, probably due to underestimation of winter precipitation in source data. Perhaps, this site precipitation records should be corrected with catch efficiently of precipitation gauge as a function of wind speed that was neglected in this study.

Table 1. Statistical evaluation of the models.

Snow season	R^2	ME	MBD (mm)	Ste (mm)	Max[a] (mm)
Myoko					
Nov-96 – May-97	0.96/0.95	0.92/0.91	-71/-59[b]	76/87	1309
Nov-97 – May-98	0.99/0.99	0.93/0.95	-76/-60	29/28	1107
Nov-98 – May-99	0.99/0.99	0.93/0.95	-88/-65	48/38	1310
Nov-99 – May-00	0.98/0.99	0.96/0.98	-53/-25	76/66	1782
Nov-00 – May-01	0.99/0.98	0.93/0.93	-94/-86	47/69	1561
average[c]	0.99/0.99	0.95/0.95	-77/-59	28/48	1375
Nagaoka					
Dec-97 – Apr-98	0.99/0.98	0.97/0.98	-7/-2	18/16	337
Dec-98 – Apr-99	0.96/0.85	0.95/0.77	-2/40	33/71	525
Dec-99 – Apr-00	0.98/0.98	0.97/0.98	9/-5	17/13	362
Dec-00 – Apr-01	0.97/0.93	0.97/0.92	-6/19	33/54	484
average	0.98/0.95	0.98/0.94	-1/13	16/30	370
Shinjyo					
Nov-98 – Apr-99	0.86/0.75	0.86/0.73	-3/13	38/52	322
Nov-99 – Apr-00	0.95/0.93	0.92/0.91	-19/4	33/42	515
Nov-00 – Apr-01	0.97/0.92	0.97/0.90	7/19	37/63	558
average	0.98/0.94	0.98/0.93	-6/12	18/38	431
Reynolds Greek					
Oct-83 – Jun-84	0.99/0.98	0.97/0.97	40/43	38/34	1012
Oct-84 – Jun-85	0.99/0.97	0.98/0.97	-6/12	23/28	589
Oct-85 – Jun-86	0.97/0.94	0.98/0.87	10/-74	44/41	702
Oct-86 – Jun-87	0.97/0.83	0.96/0.80	14/-42	21/28	309
average	0.99/0.98	0.98/0.97	12/-24	15/15	616

[a] Peak value of observed SWE
[b] Results expressed as proposed_EB/simplified_EB
[c] Snow seasons average predicated SWE as shown in fig.2

At two sites, Nagaoka and Shinjyo, there is a later snowmelt event simulated by the simplified_EB in comparison with one predicted by the proposed_EB. Oppositely, at Reynolds Greek, the SWE simulated by the simplified_EB is underestimated due to more earlier beginning of melt. We tried to reveal the source of these differences in simulation results. The main reason for this is that the atmospheric transmittance factor calculated under Eq. (16) cannot entirely replace cloud cover causing some uncorrected prediction in the radiation fluxes. That was found out after examining Eq. (18) which showed good ability to predict vapor pressure as a function of the daily minimum temperature with $R^2 > 0.95$ for all the site (figure is omitted). Nevertheless, it should be concluded that the shape of time-series and the period of snow seasons for both predicted SWE are in general well simulated.

Table 1 presents values of the validation statistics calculated for all applications. All the results have low standard errors, Ste, generally <10% of range of observed SWE, and very high R^2 (for average results: >0.98/0.94) and ME (for average results: >0.95/0.93). The magnitudes of MBD in all cases are small and much more beyond the sensitivity of the snowpack. Low values of R^2 and ME are apparent at Shinjyo for Nov-98 – Apr-99 simulations due to the lack of observed SWE data at Shinjyo for that period. The other simulation results agree well with observations. In fact, the proposed_EB results are in general better than the simplified_EB ones. Nevertheless, although some of the simplified_EB results don't indicate good agreement between predicted and observed SWE, for instance, at Reynolds Greek for the winter periods of Oct-85 – Jun-86 (R^2=0.94, ME=0.87) and Oct-86 – Jun-87 (R^2=0.83, ME=0.80) results as well as at Nagaoka for the Dec-98 – Apr-99 period (R^2=0.85, ME=0.77), the simplified_EB model results can be evaluated as good according to the small difference in its simulations in comparison with those simulated by the proposed_EB.

5. CONCLUSIONS

The objectives of this investigation were to assess how snowmelt can be reliably modeled with a simplified energy balance method using only maximum and minimum daily air temperature and wind speed as data input. In obtaining these results, we compared one model to another. We first developed an energy balance model all parameters of which was reasonably estimated from simple publish relationships and were fixed while testing the model. We assumed that only one parameter is in charge of this model calibration. The proposed model was then calibrated against continuous records of SWE collected at four sites located in a wide range of geographic conditions and showed very good performance. We further approximated the given model in order to reduce input data required to drive the model and to make it more transportable and acceptable for wider applications, for example, for data-poor basins. Using the parameter values calibrated by the first model, its simplified modification was tested on the same data. Although the proposed model indicated better agreement between predicted and observed SWE, the simplified model performed very well.

The analysis reported in this paper may be very useful to improve the applicability of the energy balance based snowmelt models in data-poor basins. Further work has to be done to test the models developed in this study against additional data.

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