Annual Journal of Hydraulic Engineering, JSCE, Vol.51, 2007, February

IDENTIFICATION OF MODEL STRUCTURAL STABILITY THROUGH COMPARISON OF HYDROLOGIC MODELS

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No matter how sophisticated and accurate hydrologic models may be, prediction uncertainty is unavoidable problem in rainfall-runoff modeling and it stems from various components. Therefore, it is essential to identify and reduce sources of uncertainty for more precise agreement between model prediction and observations in the real system. In the context of uncertainty issues, the problem of model structural uncertainty or stability is an issue of increasing interest in recent research. This paper examines a nature of model structural inadequacy using a single-objective global optimization method in hydrological modeling and proposes a framework to assess model structural stability through a comparison of two hydrologic models.

Key Words: *prediction uncertainty, model structural stability, single-objective global optimization*

1. INTRODUCTION

The principal and intense task of modelers (*e.g.*, hydrologists and engineers) is to identify a hydrologic model through the estimation of an optimal parameter set within a specific model structure, which is suitable for given catchment characteristics, historical data and intended modeling purpose. The model identification process therefore consists of two steps generally[1)]. The first step is the selection of a proper hydrologic model structure and the second one is the identification of an appropriate parameter set. However, this work is not easy due to an influence of uncertainties involved in modeling procedure that are also unavoidably propagated into the prediction of model output variables (*e.g.*, stream flow). This model output uncertainty originates from various sources: input uncertainty; measurement errors; parameter uncertainty; and model structure uncertainty.

A great deal of research has been proposed to recognize the propagation of the uncertainties which appear in the different components of the rainfall-runoff modeling into model prediction[2),3),4)]. Beven and Binely[2)] suggest a method of estimating parameter uncertainty and its propagation, Generalized Likelihood Uncertainty Estimation (GLUE) approach. On the other hand, Kavetski *et al.*[4)] review the influence of forcing input uncertainty on model output in hydrological modeling.

Especially, the problem of model structural uncertainty with advanced automatic calibration methods is an issue of increasing interest in recent researches[3),5),6)]. Structure error is unavoidable problem in hydrological modeling since hydrologic models are conversion and simplification of reality, thus no matter how sophisticated and accurate they may be those models only represent aspects of conceptualization or empiricism of modelers. In consequence, output time series of hydrologic models are as reliable as hypothesis, structure of models, quantity and quality of available forcing data, parameter estimates[7)].

Gupta *et al.*[3)] pointed out that one parameter set may be insufficient to represent the behavior of the catchment due to the inadequacy of model structures. In other words, a subjective selection of objective functions for calibration of conceptual hydrologic models results in an overemphasis on different response modes such as low and high flow periods. This fact implies different parameter combinations can be existent according to various objective functions due to the presence of structural uncertainty.

Hydrologists have concentrated their effort on development of more rigorous and efficient scheme to assess the suitability of model structure for representing the natural system and identifying model structural inadequacy[3),6),8)]. However, their research is limited to improve a classical calibration strategy, single-objective optimization algorithm, coupled with their own conceptual model (*e.g.*, SAC-SMA model). Hence, a method to identify the model structural stability, not only limited to lumped models but also applicable to various type of distributed models is required.

As reported in previous studies[3),5),6),8)], the result of variation of optimal parameter combination calibrated by a single-objective optimization method can be employed as one of the well-founded indicators to account for model structural stability. Accordingly, we think that a more reliable model structure leads to the identical optimal parameter set without regard of any objective functions selected subjectively. Moreover, such model structure maintains high degree of accuracy for simulated hydrographs when applying parameter set for various type and magnitude of floods in the same study site. It means that a structurally-stable model has high parameter transferability from event to event. As a result, model structural stability can be estimated as a degree of capability which enables to reduce the influence of objective functions and flood events on model parameter sets.

From this point of view, this study is conducted to investigate answers to the following questions: 1) What kinds of models are stable in terms of model structure for description of rainfall-runoff processes? 2) How can modelers identify model structural stability and suitability? In section 2, a framework to assess the model structural stability is proposed and different kinds of two models are evaluated through the framework. In section 3, the comparison results with respect to the assessment of model structural stability are discussed and conclusions are presented in section 4.

2. METHODOLOGY FOR MODEL STURCTURE ANALYSIS

Our purpose of this study is to establish a framework for how to assess the model structural stability. This work is summarized by two main procedures. The first step is an identification of model stability according to selection of objective functions. The second procedure is an assessment of model structural stability through the analysis of parameter transferability. Figure 1 illustrates the schematic process of the framework for assessment of model structural stability.

This assessment procedure is based on the following ideas:

1) If hydrologic observed data (*e.g.*, rainfall, stream flow) used for model calibration are not erroneous, the calibrated parameter set can reflect the structure of hydrologic model.
2) An ideally-structured model can be regarded as a stable model which has the identical best parameter set regardless of objective functions and various flood events.

Therefore, analysis of the variation of single optimal parameter sets calibrated by the single -objective global optimization method for various flood events can be used as an indicator of the degree of structural stability.

The Shuffled Complex Evolution (SCE-UA) algorithm with three different objective functions (SLS, HMLE, MIA) is used to calibrate a conceptual lumped model (Storage Function Method, SFM) and a physically based distributed model (Cell Distributed Runoff Model Version 3, CDRMV3, http://fmd.dpri.kyto-u.ac.jp/~flood/product/cellModel.html)[9)]. Five historical flood events at Kamishiiba catchment (210㎢) located in the Kyushu area are used to compare model structural stability for verification of our framework. Applied hydrologic models, optimization method and objective functions are introduced in following sub-sections.

(1) Hydrologic Models

a) Conceptual lumped model, Storage Function Method (SFM)

This model is a lumped model with the reflection of nonlinear characteristics of hydrologic response behavior. SFM is used for the rainfall-runoff simulation in a small watershed usually less than five hundred square kilometers. The form of SFM is expressed as:

$$\frac{dS}{dt} = r_e(t - T_l) - q, \qquad S = kq^p \tag{1}$$

$$r_e = \begin{cases} f \times r, & if \ \sum r \le R_{SA} \\ r, & if \ \sum r > R_{SA} \end{cases} \tag{2}$$

where S = water storage; r_e = effective rainfall intensity; r = rainfall intensity; q = runoff; t = time; k = storage coefficient; p = coefficient of nonlinearity; f = primary runoff ratio; T_l = lag time; and R_{SA} = cumulative observed rainfall from the beginning of the studied storm. Four parameters (k, p, f and R_{SA}) are optimized in this model.

b) Physically based distributed model, CDRMV3

CDRMV3 is a physically based distributed hydrologic model developed by Kojima *et al.*[9)] including discharge-stage relationship with saturated -unsaturated flow[10)]. The model solves the one dimensional kinematic wave equation with the discharge-stage equation using the Lax-Wendroff finite difference scheme according to the flow

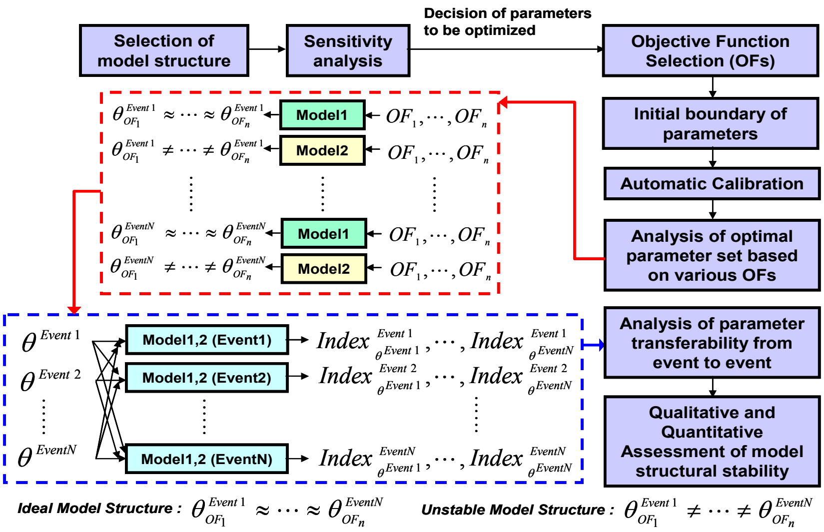


Fig.1 Schematic illustration of a framework to assess model structural stability; θ_i^j = optimal parameter set; $Index_k^j$ = measurement value for assessment of parameter transferability; i = Objective Function; j = storm event; k = optimal parameter set of each event.

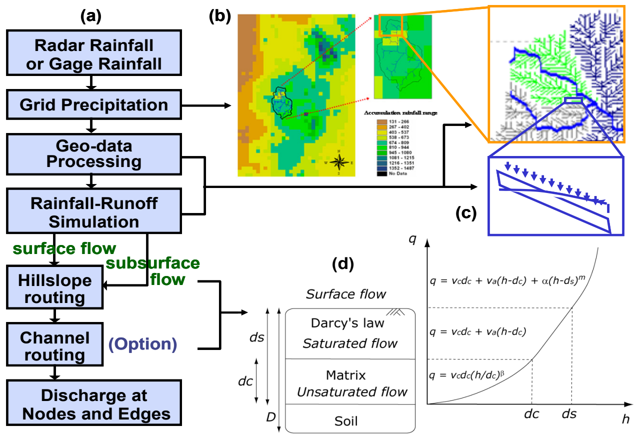


Fig.2 Schematic representation of CDRMV3 (a) Modular structure of CDRMV3 (b) Distributed grid rainfall data (c) Slope and channel components extracted from DEM (d) Model structure for the hillslope soil layer and discharge -stage relationship.

direction map (see Figure 2). All geomorphologic information are extracted from 250m based DEM data. Channel routing is also carried out by the kinematic routing scheme as well as calculation of slope elements reflecting contributing areas.

The model assumes that a permeable soil layer covers the hillslope as illustrated in Figure 2(d). The soil layer consists of a capillary layer in which unsaturated flow occurs and a non-capillary layer in which saturated flow occurs. According to this mechanism, if the depth of water is higher than the soil depth, then overland flow occurs. The discharge-stage relationship is expressed by equation (3) corresponding to water levels (see Figure 2(d)) defined as:

$$q = \begin{cases} v_c d_c (h/d_c)^{\beta}, & 0 \le h \le d_c \\ v_c d_c + v_a (h-d_c), & d_c \le h \le d_s \\ v_c d_c + v_a (h-d_c) + \alpha (h-d_s)^m, & d_s \le h \end{cases} \qquad (3)$$

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = r(t) \qquad (4)$$

Flow rate of each slope segment are calculated by above governing equations combined with the continuity equation (4), where $v_c = k_c i$; $v_a = k_a i$; $k_c = k_a / \beta$; $\alpha = \sqrt{i}/n$; i is slope gradient, k_c is saturated hydraulic conductivity of the capillary soil layer, k_a is hydraulic conductivity of the non-capillary soil layer, n is roughness coefficient, d_c is the depth of the capillary soil layer and d_s is soil depth. Detailed explanations of model structure appear in Tachikawa *et al.*[10]. There are 5 parameters (n, k_c, β, d_c and d_s), which are assumed to have homogeneous values spatially to be optimized in CDRMV3.

(2) Shuffled Complex Evolution (SCE-UA) Alogrithm

The Shuffled Complex Evolution Algorithm (SCE)[11),12)] is used to identify the best fitted parameter set, which is a single-objective global optimization method designed to handle high -parameter dimensionality encountered in calibration of a nonlinear hydrologic simulation models. This evolutionary approach method has been performed by a number of researchers on a variety of models with outstanding positive results[7)] and has proved to be an efficient, powerful method for the automatic optimization[13),14)]. SCE algorithm is basically synthesized by following three concepts: (1) combination of a simplex procedure with the concepts of controlled random search approaches; (2) competitive evolution; and (3) complex shuffling. The integration of these steps above mentioned makes the SCE method effective and flexible[12)]. Initial state variables of both models are determined by initial observed discharge assuming steady-state condition.

(3) Applied Objective Functions

The aim of computer-based automatic calibration is to find the values of the model parameters that minimize or maximize the numerical value of the objective functions[15]. In general, the most commonly utilized objective functions in hydrological modeling are variations of the Simple Least Squares (SLS) function defined as:

$$SLS = \sum_{t=1}^{N} (q_t^{obs} - q_t(\theta))^2 \quad (5)$$

where q_t^{obs} is observed stream flow value at time t; $q_t(\theta)$ is model simulated stream flow value at time t using parameter set θ; N is the number of flow values available. SLS has a feature that large discharge is emphasized due to squared errors while low flows are neglected[16], thus the parameter set fitting around peak discharge value is likely to obtain.

Krause *et al.*[17] proposed the Modified Index of Agreement (MIA)[18] to reduce the influence of the squared term during high flows as putting an weight on flow values. This objective function is calculated as:

$$MIA = 1 - \frac{\sum_{t=1}^{n} \left(\frac{q_t^{obs} - q_t(\theta)}{q_t^{obs}} \right)^2}{\sum_{t=1}^{n} \left(\frac{\left| q_t(\theta) - q_t^{mean} \right| + \left| q_t^{obs} - q_t^{mean} \right|}{q_t^{mean}} \right)^2} \quad (6)$$

where q_t^{mean} is mean value of observed time series.

Sorooshian and Dracup[19] proposed a different objective function to consider entire behavior of hydrograph, the Heteroscedastic Maximum Likelihood Estimator (HMLE), which enables to estimate the most likely weights through the use of the maximum estimation theory. This new measure can eliminate some of the subjectivity involved in the selection of transformation and/or a weighting scheme by handling heteroscedastic error, so that it yields a more balanced performance over the entire hydrograph[3),5)]. It is calculated as:

$$\min_{\theta,\lambda} \; HMLE = \frac{1}{N} \sum_{t=1}^{N} w_t \varepsilon_t \bigg/ \prod_{t=1}^{N} w_t \quad (7)$$

where $\varepsilon_t = q_t^{obs} - q_t(\theta)$ is the model residual at time t; w_t is the weight assigned to time t, computed as $w_t = f_t^{2(\lambda-1)}$; $f_t = q_t^{true}$ is the expected true flow at time t; λ is the transformation parameter which stabilizes the variance.

In this study, above mentioned three objective functions are used for the calibration trials and the analysis of model structural stability.

3. RESULTS AND DISCUSSIONS

(1) The Influence of Objective Functions on Model Performance

The plots of comparisons between the simulated and the observed hydrographs according to the three objective functions (SLS, MIA and HMLE) are illustrated in Figure 3. From Figure 3, we notice that:

1) In SFM cases, the simulated hydrographs based on the parameters calibrated by SLS are close to the observed ones over all events while other parameters optimized by HMLE, MIA lead to less magnitude than the measured stream flow in large flood, Event 4 and 5 (peak flow > 500㎥/s). The results show that the optimized parameter set is dependent on objective functions. On the other hand, the computed hydrographs in small flood, Event 3 (peak flow < 500㎥/s) have a good "goodness-of-fit" measurement value regardless of objective functions.
2) In CDRMV3 cases, there is no influence of objective functions on model performance. In other words, all predicted hydrographs shown in Figure 3(b) are close to observed discharge for any objective functions. However, constant parameter set is not observed for small flood events (Event 2, 3) while approximate values of parameter sets are obtained for large flood events (Event 1, 4, 5), *i.e.*, $\theta_{OF_1} \approx \ldots \approx \theta_{OF_n}$.

This result implies that the lumped model used in this study is structurally unstable in terms of dependency of objective function, so that we need to change the model parameter set according to the modeling purpose. On the other hand, the problem of subjectivity related with selection of objective functions for automatic calibration can be ignored for the distributed hydrological modeling used here.

(2) The Assessment of Parameter Transferability from Event to Event

From event to event, transferability of the identified parameter set also can be guideline to assess model stability. If optimal parameter sets obtained from various flood events are occupied in a similar location on feasible parameter space, *i.e.*, $\theta^{Event1} \approx \ldots \approx \theta^{EventN}$, undoubtedly, each optimal parameter set could be applicable for different flood events and model performances would be good. We can regard that such model structure has high parameter transferability. Each model performance with transferred parameter sets is evaluated by Peak Discharge Ratio (PDR) and Nash-Sutcliffe (NS) statistics of the residuals as guideline indexes for measurement of parameter transferability, defined as:

$$PDR = Peak_{sim} / Peak_{obs} \quad (8)$$

$$NS = 1 - \frac{\sum_{t=1}^{N} (q_t^{obs} - q_t(\theta))^2}{\sum_{t=1}^{N} (q_t^{obs} - q_t^{mean})^2} \quad (9)$$

where $Peak_{sim}$ is the simulated peak discharge, $Peak_{obs}$ is the observe peak discharge. PDR measures tendency of the simulated peak discharge to be larger or smaller than the observed peak discharge. NS measures a relative magnitude of the residual

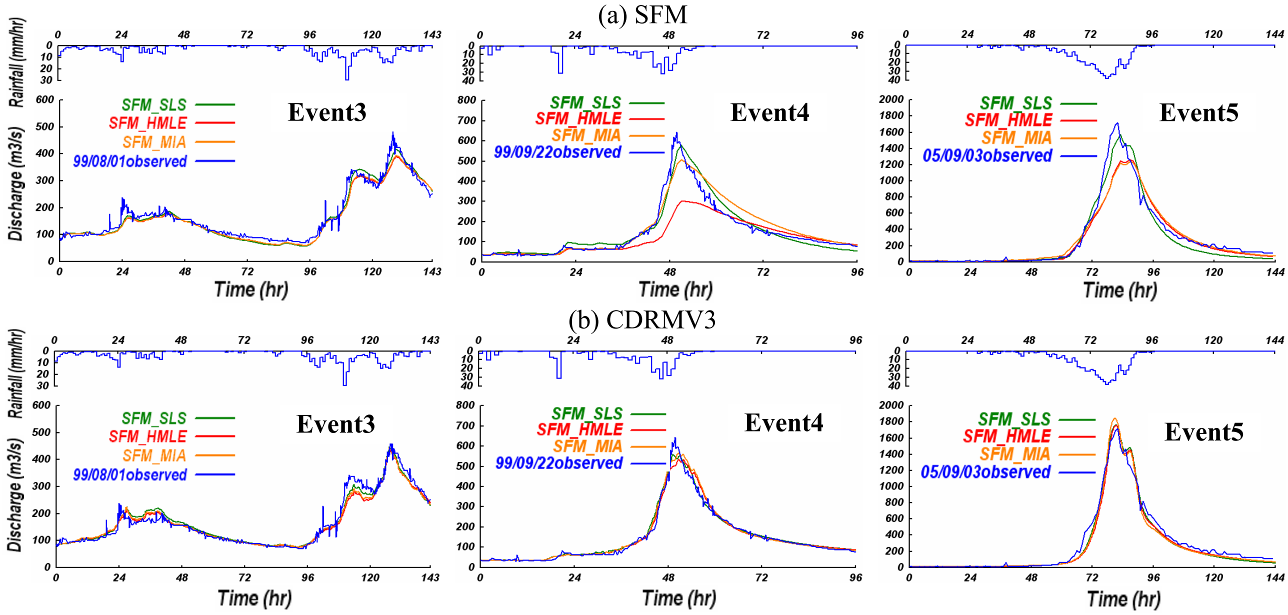


Fig.3 Comparison between the simulated and the observed hydrographs according to three objective functions; (a) SFM cases, (b) CDRMV3 cases.

variance to the variance of the observed stream flows; the optimal value of both measures is 1.0.

The identified parameter set for each event is applied to different events and their model performances are plotted in the Figure 4. This figure illustrates the quantified results of parameter transferability. Each optimal parameter set of SFM is not applicable for different flood events (*i.e.*, low parameter transferability; NS, PDR are scattered far from 1.0) while those of CDRMV3 are applicable for simulations of different flood events (*i.e.*, high parameter transferability; NS, PDR are plotted near optimal value 1.0). Interestingly, in CDRMV3, the results evaluated by optimal parameters of Event 2 over entire cases tend to be inapplicable and inaccurate; NS values are usually less than approximately 0.75 and PDR values are underestimated or overestimated irregularly. Simulated results for Event 2 with the optimal parameter sets of other events are also inapplicable. This result indicates that the observed data for Event 2 is unreliable, so that it is impossible to extract useful information from this kind of uninformative data. This finding shows that the proposed framework for assessment of model stability is also possible to detect a low quality observed data.

The results of model stability assessment show that CDRMV3 is more stable than SFM with respect to independency of objective functions and flood magnitudes because the former model has physically -based structure which reflects more reliable geomorphological characteristics of watershed.

Interesting finding is that even though the physically-based distributed model has high parameter transferability (*e.g.*, $I^{Event1}_{\theta^{Event1}} \approx \cdots \approx I^{Event1}_{\theta^{EvnetN}}$), the identical single optimal parameter set over all storm events is not observed (*i.e.*, $\theta^{Event1} \neq \cdots \neq \theta^{EventN}$). Instead, the different identified parameter sets are applicable to reproduce other flood events. It means that the different parameter combinations can lead to acceptable model performances with proper values of NS or PDR. This effect is often called “equifinality”[2]. Equifinality makes it difficult to identify a suitable model parameter set for distributed hydrological modeling. Therefore, analysis of equifinality is an urgent problem to be solved for more reliable rainfall-runoff simulation.

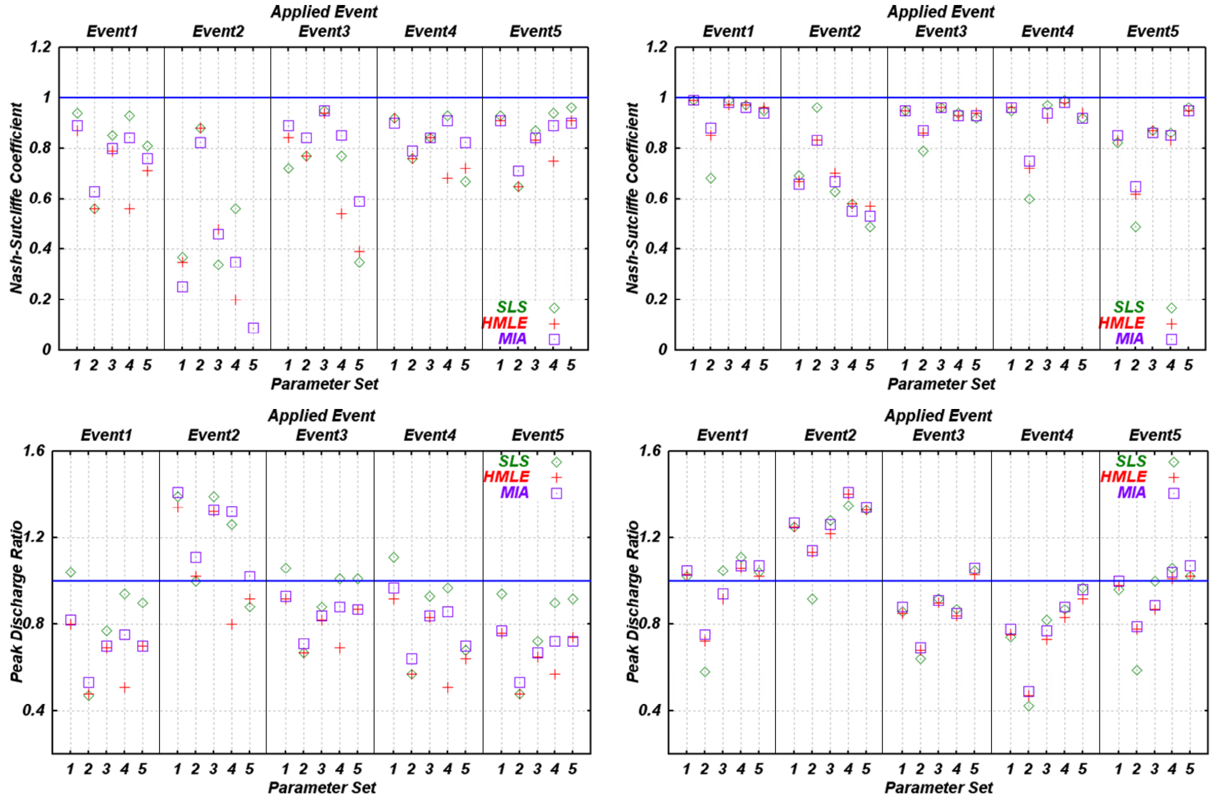
4. CONCLUSION

In this paper, we have demonstrated a framework for assessment of model structural stability through a single-objective global optimization (SCE-UA) method with three objective functions for various flood events and compared two hydrologic models (SFM, CDRMV3) as an example. The results under our framework lead to following conclusions:

1) The simulated results of CDRMV3 are not affected by objective functions while the predicted hydrographs of SFM depend on the objective functions and magnitude of flood.
2) The structural stability of CDRMV3 is superior to SFM with respect to parameter transferability.
3) Our framework can be an indicator to detect a low quality observed data.

Through the analysis, we found that the equifinality problem still remains despite of a remarkable improvement of hydrologic modeling with respect to model structure. The analysis of equifinality and a development of a method to distinguish a suitable model parameter set which has an appropriate physical meaning are in progress.

ACKNOWLEDGMENT: This study was supported



(a) SFM (b) CDRMV3

Fig.4 Plots of parameter transferability from event to event; each point indicates the evaluated NS, PDR due to parameter transfer; bottom label = applied optimal parameter set of each event; upper label = target event.

by Grant-in-Aid for Scientific Research (C) 18560497 (PI: Assoc. Prof. Y. Tachikawa, Kyoto Univ.).

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(Received September 30, 2006)