Annual Journal of Hydraulic Engineering, JSCE, Vol.50, 2006, February

COMPUTATION OF FLOW, TURBULENCE AND BED EVOLUTION WITH SAND WAVES

Sanjay GIRI[1] and Yasuyuki SHIMIZU[2]

[1]Student member of JSCE, JSPS Postdoc Research Fellow, Dept. of Hydraulic Research, Hokkaido University (North 13, West 8, Kita-Ku, Sapporo 060, JAPAN)

[2]Member of JSCE, Dr. Eng., Professor, Dept. of Hydraulic Research, Hokkaido University (North 13, West 8, Kita-Ku, Sapporo 060, JAPAN)

A vertical two-dimensional morphodynamic model with non-hydrostatic, free surface flow is reported herein. This numerical model can simulate flow and turbulence characteristics over sand waves. Likewise, model can reproduce the sand wave formation and migration process as well as free surface oscillation simultaneously in a moving boundary domain with the implication of generalized coordinate system. A nonlinear k-ε model is employed as a turbulence closure. CIP numerical technique is used to resolve advection term of momentum equations. An Eulerian stochastic formulation of sediment exchange process in terms of pick up and deposition function is incorporated to simulate non-equilibrium sediment transport that explicitly considers the flow variability during morphodynamic computation. The numerical model is validated with experimental data on flow and turbulence over fixed dunes as well as laboratory measurement and visualization of dune geometry and migration.

Key Words**:** *Numerical simulation, free surface flow, sediment transport, morphodynamics*

1. INTRODUCTION

The microscale sand waves in alluvial streams are usually formed in lower flow regime under rough turbulent condition. Several investigations have been carried out so far that have made invaluable contributions to improve understanding on morphodynamic features of dunes as well as flow and turbulence characteristics induced by sand waves. The complex hydraulic and morphodynamic aspects associated with bed form evolution have an important bearing on problems; however still remain poorly understood despite numerous efforts that have been made since long.

Understanding flow and turbulence induced by sand waves is of importance to explicate their formation mechanism and further development. Number of physical studies in this regard was made in the past[1)]. For instance, Nelson et al.[2)] conducted flow and turbulence measurement over two-dimensional fixed dunes, in which along with the coupling of mean flow and turbulence, effects on bed form instability and finite amplitude stability were examined. Schmeeckle et al.[3)] conducted laboratory visualization of turbulence and suspended sediment transport over two-dimensional dune. Furthermore, some notable numerical studies were also carried out. Shimizu et al.[4)] performed a three-dimensional direct numerical simulation of flow and turbulence over two-dimensional fixed dunes. The numerical model was able to reproduce the coherent structure induced by dune crest adequately. Most observations yielded strong correlation between coherent turbulence structure induced by flow separation, bed resistance and sediment transport mechanism. The flow field and bed configuration are found to be interdependent having direct and converse effect on each other that creates difficulties in appropriate physical interpretation and quantitative determination of the said problem.

Numerous approaches were developed in order to quantify sand wave induced morphology of alluvial channels. Most early investigations were conducted using theoretical, semi-empirical and empirical approaches[5), 6)]. Among them, theoretical studies are largely based on linear stability theory which is, in effect, not appropriate approach for the determination of natural phenomena having finite amplitude features[2)]. Scant attention has been given in regard to numerical modeling of sand waves' morphodynamics[7], probably because of numerical complexity and limitations on its widespread application to real-world problem. Nakayama and Shimizu[8] carried out a numerical study on suspended sediment transport over sand waves. They compared the computation result of time averaged concentration of suspended sediment with Rouse distribution curve as well as laboratory measurements. One of the significant studies was made by Onda and Hosoda[9], in which a depth-integrated flow model was proposed with an allowance for vertical acceleration. They described bottom shear stress based on potential flow analysis considering acceleration-deceleration effect near the bed. In order to compute water surface profile, a numerical technique was used to simulate undular bore and flow over dunes with an attempt to eliminate the deficiency of hydrostatic assumption by introducing a reduction factor to the vertical acceleration terms in case of high depth gradient. However, this flow model cannot reproduce separation behind dune crest and, thereby, its effect on flow field and sediment transport. Furthermore, they used sediment pick up and deposition function[10] to simulate sand dune formation. Finally, they analyzed temporal variation of sand wave spectrum as well as dune geometry using their numerical results.

Present research has been prompted by the need to develop a more complete and reliable computational model to simulate flow and morphodynamic features of sand waves. An earlier proposed three-dimensional model[4] seems to be more complete approach for computation of flow and turbulence over dunes. However, coupling this flow model with a morphodynamic module may inhibit it from being applied efficiently because of rather high computational cost. Consequently, aforesaid three-dimensional hydrodynamic model is simplified to be a vertical two-dimensional model and enhanced by imposing non-hydrostatic, free surface flow condition as well as coupling with morphodynamic module. The model is verified with flow and turbulence experiment over fixed dunes as well as laboratory measurement of sand dune geometry and visualization of dune migration process.

2. COMPUTATION OF FLOW AND TURBULENCE

(1) Governing equations

The governing equations for vertical two-dimensional flow in cartesian coordinate system reads as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(-\overline{u'u'}\right)+\frac{\partial}{\partial y}\left(-\overline{u'v'}\right) \qquad (2)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\partial}{\partial x}\left(-\overline{u'v'}\right)+\frac{\partial}{\partial y}\left(-\overline{v'v'}\right)-g \qquad (3)$$

where x and y = coordinates in horizontal and vertical direction respectively; u and v = components of velocity in horizontal and vertical direction respectively, $-\overline{u'u'}$, $-\overline{u'v'}$ and $-\overline{v'v'}$ = Reynolds stress tensors, ρ = fluid density, g = gravitational acceleration, p = pressure.

Eq. (1)-(3) are transformed from x, y, t cartesian coordinate system to a moving boundary fitted ξ, η and τ coordinate system[4].

Pressure term in momentum equations can be computed considering non-hydrostatic component as follows:

$$p=p_0+p'=g\int_y^H \rho\, dy+p' \qquad (4)$$

Eq.(4) can be rewritten as:

$$p=\rho g(H-y)+p' \qquad (5)$$

in which, p'= non-hydrostatic component of pressure, H = the location of free surface. Calculation is performed substituting pressure term (p) by Eq.(5) in momentum equations, consequently, computing the non-hydrostatic component.

(2) Calculation of water surface fluctuation

Computation of time-dependent water surface change is of importance for the realistic reproduction of free surface flow over migrating bed forms. Basically, majority of morphodynamic numerical models developed earlier have assumed the rigid lid water surface condition to achieve numerically stable solution. In present study, the kinematic condition is imposed along the free surface (at y = H) to compute the temporal water surface elevation. The kinematic condition, which constraints fluid particles to remain on the water surface at any time following the local flow velocity, is expressed as follows:

$$\frac{\partial H}{\partial t}+u\frac{\partial H}{\partial x}=v \qquad (6)$$

$$H=y_b+h \qquad (7)$$

where y_b = bed elevation and h = local flow depth.

(3) Turbulence closure

In conventional k-ε model, turbulence stress tensors are evaluated using linear relationship. In order to reproduce turbulence characteristics more accurately in shear flow with separation zone, a nonlinear term is added to the standard k-ε model as follows[11]:

$$-\overline{u_i' u_j'} = \nu_t S_{ij} - \frac{2}{3} k\delta_{ij} - \frac{k}{\varepsilon}\nu_t \sum_{\beta=1}^{3} C_\beta \left(S_{\beta ij} - \frac{1}{3} S_{\beta\alpha\alpha}\delta_{ij} \right) \tag{8}$$

$$\nu_t = C_\mu \frac{k^2}{\varepsilon} \tag{9}$$

where ν_t= eddy viscosity coefficient, δ_{ij}= Kronecker delta, k= turbulent kinetic energy, ε= dissipation rate of k. The detail description and formulation of strain tensors S_{ij}, $S_{\beta ij}$, $S_{\beta\alpha\alpha}$, as well as coefficient C_β can be found elsewhere[11].

k-ε equations can be expressed as follows:

$$\frac{\partial k}{\partial t} + u\frac{\partial k}{\partial x} + v\frac{\partial k}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial y}\right) + P_h - \varepsilon \tag{10}$$

$$\frac{\partial \varepsilon}{\partial t} + u\frac{\partial \varepsilon}{\partial x} + v\frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial x}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right) + C_{1\varepsilon}\frac{\varepsilon}{\kappa}P_h - C_{2\varepsilon}\frac{\varepsilon^2}{k} \tag{11}$$

$$P_h = -\overline{u_i' u_j'}\frac{\partial u_i}{\partial x_j} \tag{12}$$

where σ_k, σ_ε, $C_{1\varepsilon}$ and $C_{2\varepsilon}$ are standard model constants.

(4) Boundary conditions

The boundary conditions are no slip at the bed and free flow at the surface. The following expression for near-bed region is adopted:

$$\frac{u_p}{u_*} = \frac{1}{\kappa}\ln\frac{y_p}{y_0} \tag{13}$$

where u_p = velocity at near-bed grid point, u_*= bed shear velocity, κ = Karman constant, y_p= distance from bed to nearest grid, y_0= $k_s/30$, k_s = roughness height (= $2.5d$), d = sediment diameter.

The periodic boundary condition is employed in this computation. Turbulent energy and its dissipation in near-bed region are evaluated as follows:

$$\frac{k}{u_*^2} = \frac{1}{\sqrt{C_\mu}} \tag{14}$$

$$\varepsilon = \frac{u_*^3}{ky} \tag{15}$$

In order to account for the diminution of turbulence length scale at the free surface layer, following damping function to the eddy viscosity is used:

$$f_s = 1 - \exp\left(-B\frac{(h-y)\varepsilon_s}{k_s^{3/2}} \right) \tag{16}$$

where B=10.

The dissipation rate of turbulent energy at the surface layer is evaluated using following equation:

$$\varepsilon_s = \frac{C_{\mu 0}^{3/4} k_s^{3/2}}{0.4\Delta y_s} \tag{17}$$

where Δy_s = distance from water surface to first grid and coefficient $C_{\mu 0}$= 0.09

$$\frac{\partial k}{\partial y} = 0 \tag{18}$$

(5) Numerical algorithm

The transformed equations are numerically solved by splitting them into non-advection and pure advection phase. The non-advection phase is computed using central difference method. The pressure field is resolved using SOR method. The advection phase is calculated using a high-order Godunov scheme known as CIP method[12]. In this scheme, at very small time increment, the change in time of velocity components at a point in space can be split into the time evolution of the inhomogeneous terms and the time evolution at a point due to the advection of the field. A brief description of CIP numerical scheme to solve advection terms in generalized coordinate system can be found elsewhere[4].

3. SEDIMENT TRANSPORT MODEL

Some earlier investigations[13] have revealed that spatially or/and temporally averaged sediment transport models are inadequate to describe the extremely inconsistent nature of sediment particle motion particularly in the region with high near-bed turbulence. In effect, a direct computation of sediment particle motion, viz. Discrete Element Model (DEM), which responds to the local and temporally variable flow field, is physically more complete approach. However, coupling this approach with an advanced hydrodynamic model, at present, seems to be rather sophisticated in view of computational complexity and expenses, in particular, for solving comparatively big scale problem. Computationally efficient and relatively comprehensive approach is believed to be the modeling of pick up and deposition of sediment particles in terms of a stochastic formulation. Consequently, an Eulerian stochastic formulation of sediment transport proposed by Nakagawa and Tsujimoto[10] is incorporated in our numerical code. This method yielded one of the best predictions of pick up function for fine sediment grain. Moreover, this approach was effectively used earlier[9] as well for bed form development computation.

The dimensionless pick up rate is expressed as follows:

$$p_s \sqrt{d/(\rho_s/\rho - 1)g} = 0.03\tau_*(1 - 0.035/\tau_*)^3 \quad (19)$$

Where p_s =sediment pick up rate, ρ and ρ_s = fluid and sediment density respectively and τ_*= dimensionless local bed shear stress.

The sediment deposition rate reads as:

$$p_d = p_s f_s(s) \quad (20)$$

where p_d = sediment deposition rate and $f_s(s)$ = distribution function of step length.

Distribution function of step length is found to be exponential as follows:

$$f_s(s) = \frac{1}{\Lambda}\exp\left(-\frac{s}{\Lambda}\right) \quad (21)$$

where Λ = the mean step length and s = the distance of sediment motion from pick up point.

The mean step length can be calculated as $\Lambda = \alpha d$, in which α is an empirical constant and proposed to be 100.

In order to consider the instability of bed forms in upper flow regime, a module to compute suspended sediment transport is also incorporated in proposed numerical model. Considering suspended sediment transport, the bed deformation rate can be computed using following sediment continuity equation:

$$\frac{\partial y_b}{\partial t} + \frac{1}{1-\lambda}\left[\frac{A_3}{A_2}(p_d - p_s)d + (q_{sui} - w_f c_b)\right] = 0 \quad (22)$$

where y_b = bed elevation, λ = porosity of sediment particle, A_2 and A_3 = shape coefficients of sand grain with two- and three-dimensional geometrical properties, q_{sui} = upward suspended sediment flux per unit area, w_f = falling velocity and c_b = reference concentration of suspended sediment.

In case of equilibrium suspended sediment transport or its absence, the third term of Eq. (22) vanishes.

q_{sui} is calculated using equation proposed by Itakura-Kishi:

$$q_{sui} = K\left(\alpha_* \frac{\rho_s - \rho}{\rho_s}\frac{gd}{u_*}\Omega - w_f\right) \quad (23)$$

in which,

$$\Omega = \frac{\tau_*}{B_*}\frac{\int_a^\infty \xi \frac{1}{\sqrt{\pi}}\exp(-\xi^2)d\xi}{\int_a^\infty \frac{1}{\sqrt{\pi}}\exp(-\xi^2)d\xi} + \frac{B_*\eta_0}{\tau_*} - 1 \quad (24)$$

$$a = \frac{B_*}{\tau_*} - \frac{1}{\eta_0} \quad (25)$$

where B_*, η_0, α_* and K are constants, the values of which are adopted, namely B* = 0.143, η0 = 0.5, α* = 0.14 and K = 0.008.

The falling velocity is calculated using well-known Rubey's formula:

$$w_f = \sqrt{\frac{2}{3}Sgd + \frac{36\nu^2}{d^2}} - \frac{6\nu}{d} \quad (26)$$

where S = relative density of sediment in water, namely 1.65, ν = kinematic viscosity of water.

4. DUNE MIGRATION EXPERIMENT

An experimental investigation on sand dune formation and migration process was carried out under the department of hydraulic research of Hokkaido University. Experiments were performed in laboratory flume of CERI of Hokkaido. The total length of the flume was 30m, width was 10cm and height was 30cm having 1/500 slope, regulated by automatic control system. The middle part of flume (10m) was covered with 5cm thick sediment layer with 0.28mm median diameter, which was trapped by using sediment stoppers at the part of upstream and downstream having length of 10m each. The sand feeding was done simply by filling up manually the space between movable bed and sand stopper in the upstream boundary, formed because of transported sand. The migration process of sand dunes was visualized using high resolution video camera. Maximum likelihood evaluation of sand dune geometry was made at the end of each experiment. The experimental cases and condition are listed in **Table 1**. It is to be noted that dune migration feature (particularly, length of dune) seems to be irregular for run A-4 and A-5.

5. MODEL VERIFICATION

(1) Mean flow and turbulence characteristics

In order to evaluate the performance of hydrodynamic model, an experiment[2)] (along with data obtained by personal correspondence with the author of said work) on flow and turbulence over fixed dunes is reproduced by using proposed numerical model. A typical example of instantaneous flow and vorticity field, simulated by numerical model, is depicted in **Fig.1**. From this, it can be seen that the numerical model is able to reproduce the realistic feature of flow separation and vortices generation behind the dune crest.

The time-averaged streamwise and vertical velocity profiles in all regions over dunes are reproduced quite well by numerical model (**Fig.2**). The turbulence characteristics, particularly in the separation region behind the dune crest, are found to be under-predicted in some points (**Fig.3**). It seems that proposed model is still unable to resolve turbulence in small scales with compare to three-dimensional model[4)]. However, on the whole, result can be regarded as satisfactory.

Table 1 Experimental cases and condition

Run	Time [min]	Discharge [$cm^3/sx10^3$]	Flow depth [cm]
A-1	250	2.0	7.0
A-2	105	4.0	9.9
A-3	90	6.0	12.1
A-4	75	8.0	15.2
A-5	60	10.0	16.7

(2) Water surface fluctuation

Proposed numerical model is found to be able to reproduce realistically the water surface fluctuation due to form drag induced by migrating dunes. The increase in depth after dune formation is found to be in same order as observed in our experiments. However, for run A-1 and A-2, average water depth seems to be under-predicted (**Table 2**). The reason is not so clear. Nonetheless, it is thought to be associated with under-prediction of dune geometry in numerical computation as well as some experimental discrepancies.

(3) Dune migration simulation

An attempt has been made to simulate sand wave formation and migration by adding an initial perturbation on sand bed. The compared shape and size of dunes after analogous time increment for experiment and computation has been listed in **Table2**. The dune geometry is found to be poorly predicted by numerical model, particularly for low flow rate, i.e. run A-1 and A-2. On the other hand, the dune characteristics are not so regular, especially for run A-4 and A-5, which is reproduced by numerical model in the same manner as in experiments. Overall, computed results show acceptable prediction capability even for the migration speed of dunes. Furthermore, simulated dune migration process is compared with the experimental visualization for the qualitative assessment of model performance. Some qualitative comparison between simulated and visualized dune geometry is depicted in **Pic.1**. The numerical simulation of dune migration reveals identical qualitative characteristic as in laboratory observation.

Dots indicate particle simulation of flow field

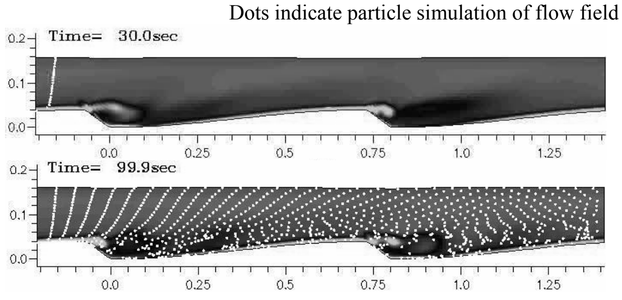


Fig.1 Numerical simulation of flow separation and vorticity

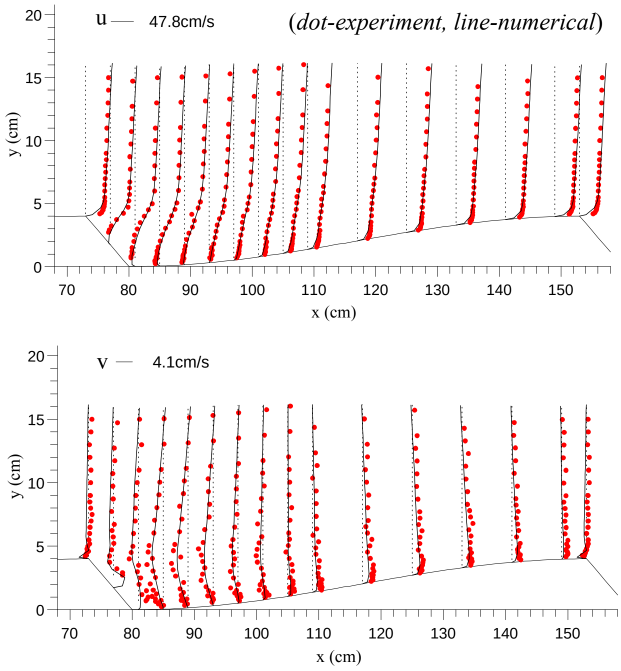


Fig.2 Mean flow comparison (Nelson's experiment, 1993)

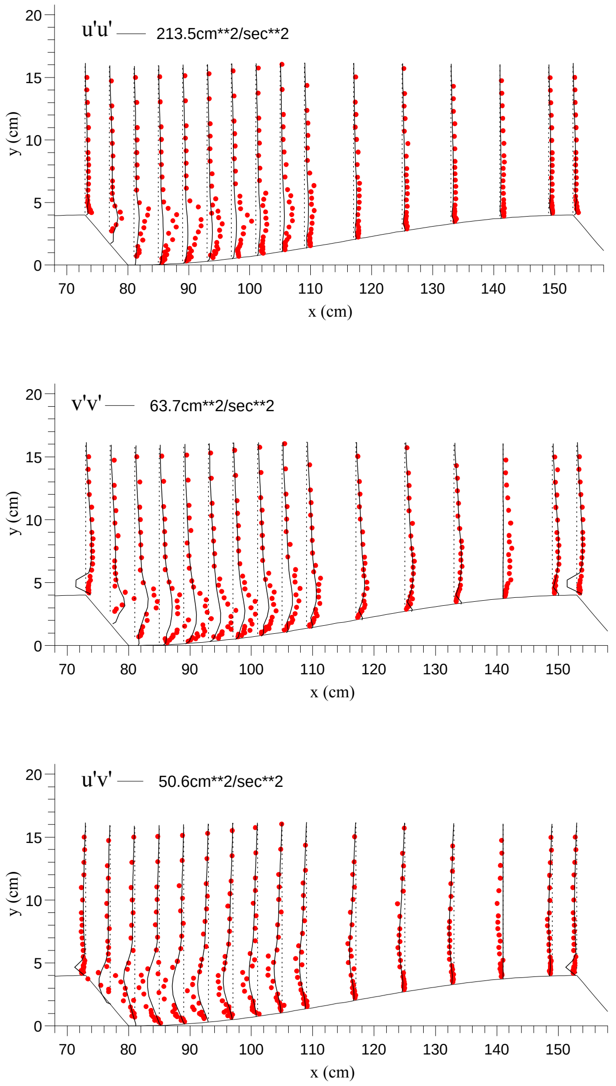
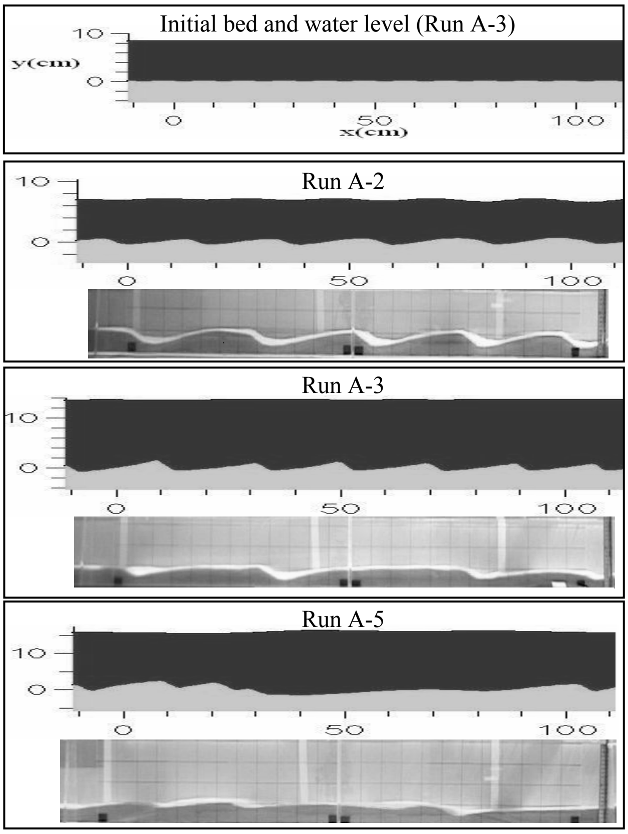


Fig.3 Turbulent characteristic comparison

Table 2 Comparison of measured quantities with computed

Run	Flow depth [cm]	Dune length [cm]	Dune height [cm]	Migration speed [cm/sec]
A-1	7.0/4.9	23/15	3.0/1.0	0.2/0.5
A-2	9.9/7.0	25/20	3.5/2.0	2.8/3.0
A-3	12.1/13.6	40/25	2.4/2.0	4.7/3.5
A-4	15.2/15.8	irregular	2.2/2.5	6.3/6.5
A-5	16.7/16.0	irregular	3.6/3.0	9.4/7.0

Note: values in numerator and denominator denote measured and computed respectively.



Pic.1 Comparison between numerical simulation and experimental visualization of sand dunes

6. GENERAL CONCLUSION

Apparently, a significant attempt has been made here to embody a reliable and physically based modeling approach relating to the flow, turbulence and non-equilibrium sediment transport. The proposed approach is believed to be more appropriate, for computing flow-field and non-equilibrium sediment transport mechanism with bed forms, than generally used morphologic modeling approach. Additionally, module to compute suspended sediment flux could also be relevant to consider the bed form instability during upper flow regime, though still remains to be verified. In general, computation results seem to be realistic and promising, which demonstrate scope and usability of numerical simulation technique that could be a supplementary tool to comprehend a sophisticated river engineering phenomenon.

ACKNOWLEDGEMENT: This study is supported by JSPS Fellowship Program. We gratefully acknowledge Kazutake Asahi who carried out experiments as a part of his graduate study. Likewise, we acknowledge Jonathan Nelson for providing experimental data.

REFERENCES

1) Bennet, J.S. and Best, J.L.: Mean flow and turbulence structure of fixed, two-dimensional bed forms; implication for sediment transport and bed form stability, *Sedimentology*, Vol.42, pp.491-513, 1995.

2) Nelson, J.M., McLean, S.R. and Wolfe, S.R.: Mean flow and turbulence over two-dimensional bed forms, *Water Resour. Res.*, Vol.29 (12), pp.3935-3953, 1993.

3) Schmeeckle, M.W., Shimizu, Y., Hoshi, K. and Tateya, K.: Turbulent structures and suspended sediment over two-dimensional dunes, *Proc. Int. Conf. Riv. Coast. Morph. Dyn., Genova,* pp.261-270, 1999.

4) Shimizu, Y., Schmeeckle, M.W. and Nelson, J.M.: Direct numerical simulation of turbulence over two-dimensional dunes using CIP methods, *J. Hydrosci. Hydr. Eng.*, Vol. 19 (2), pp.85-92, 2001.

5) Engelund, F.: Instability of erodible beds, *J. Fluid Mech.*,Vol.42, pp.225-244, 1970.

6) Engelund, F. and Fredsoe, J.: Sediment ripples and dunes, *Ann. Rev. Fluid Mech.*, Vol.14, pp.13-37, 1982.

7) Fredsoe, J. and Tjerry, S.: Morphological computation of dunes, *Proc. Int. Conf. Riv. Coast. Morph. Dyn.*, Obihiro, Japan, pp. 225-231, 2001.

8) Nakayama S., and Shimizu, Y.: Numerical calculation of suspended sediment over sand waves, *Proc. Int. Conf. Riv. Coast. Morph. Dyn.*, Obihiro, Japan, pp.217-224, 2001.

9) Onda, S. and Hosoda, T.: Numerical simulation on development process of micro scale sand waves and flow resistance, *J. Hydrosc. Hydr. Eng*, pp.13-26, 2005.

10) Nakagawa, H. and Tsujimoto, T.: Sand bed instability due to bed-load motion, *J. Hyd.Div., ASCE*, 106, pp. 2029-2051.

11) Kimura, I. and Hosoda, T.: A nonlinear k-e model with realizability for prediction of flows around bluff bodies, *Int. J. Num. Meth. Fluids*, Vol.42, pp.813-837, 2003.

12) Yabe, T., Ishikawa, T., Kadota, Y. and Ikeda, F.: A multidimensional cubic-interpolated pseudoparticle (CIP) method without time splitting technique for hyperbolic equations, *PSJ*, Vol.59, pp.2301-2304, 1990.

13) Schmeeckle, M.W. and Nelson, J. M.: Direct numerical solution of bedload transport using local, dynamic boundary condition, *Sedimentology*, Vol.50, pp. 279-301, 2003.

(Received September 30, 2005)