

CHARACTERISTICS OF TURBULENT BOUNDARY LAYER OVER A ROUGH BED UNDER CNOIDAL WAVES MOTION

SUNTOYO¹, Hitoshi TANAKA², Ahmad SANA³ and Hiroto YAMAJI⁴

¹Member of JSCE, M. Eng. Dept. of Civil Engineering, Tohoku University (6-6-06 Aoba, Sendai 980-8579, Japan)

²Fellow Member of JSCE, Professor, Dept. of Civil Eng., Tohoku Univ. (6-6-06 Aoba, Sendai 980-8579, Japan)

³Assistant Professor, Dept. of Civil and Architectural Eng., Sultan Qaboos Univ. (Muscat 123, Sultanate of Oman)

⁴Member of JSCE, Res. Associate, Dept. of Civil Eng., Tohoku Univ. (6-6-06 Aoba, Sendai 980-8579, Japan)

Ocean waves in reality often have a strong non-linear shape when propagating to shallow water, and the gap from the sinusoidal wave become remarkable. Therefore, it is generally believed that wave boundary layers, bottom shear stress and sediment transport behaviors actualizing the effect of non-linearity are different from sinusoidal wave. However, the example of research treating such turbulent boundary layer and sediment transport characteristic is very few. Therefore, the accuracy of bottom shear stress and amount of sediment transport used to evaluate the beach morphological change obtained from the wave motion model of sinusoidal is necessary to be clarified by that of non-linear.

In this present study, turbulent boundary layer characteristics for asymmetric or non-linear waves according to the non-linearity effect is examined through both experimental and BSL turbulence model. Moreover, a new calculation method of bottom shear stress based on incorporating acceleration and velocity terms is used to examine both the experimental and BSL model results of bottom shear stress.

Key Words : *Boundary layer ,bottom shear stress, asymmetric waves, turbulence model*

1. INTRODUCTION

Turbulent structure in the bottom boundary layer under wave motion has crucial important in the analyses and modeling of near-shore sediment transport. It is one of the most important aspects for predicting coastal morphology and cross-shore profile evolution. Nevertheless, mechanism through which waves transport sediment onshore to counteract the effects of gravity was not relatively unknown well. The shape of the near-bed wave orbital velocity is a key parameter for cross-shore sediment transport under breaking and near-breaking waves. Realistic waves in nature often have a shape of asymmetric waves when propagating to near shore. Their height increase and their length decrease, they further become remarkably non-linear waves. The wave asymmetric plays an important role for the occurrence of the net onshore-directed transport rate causing accretion of beaches and that of the net offshore-directed transport rate causing erosion of beaches.

Many researchers have done study on turbulent boundary layer and bottom friction through numerical model as well as laboratory experiments related with the sediment transport movement for sinusoidal wave (e.g., Fredsøe and Deigaard¹). Studies involving the sediment transport rate under sinusoidal waves have shown that the net sediment transport over a complete wave cycle is zero. In reality, however waves are non-linear having asymmetric of the near-bottom velocity between wave crest and trough actualized in which the net sediment transport over a complete wave cycle can be produced or non-zero.

Tanaka et al.²) studied the properties of asymmetric oscillatory boundary layers on the smooth bottom and comparison has been made with *k-e* model prediction. Moreover, Tanaka³) estimated the bottom shear stress under non-linear wave by modified stream function theory and proposed formula to predict bed load transport except near the surf zone in which the acceleration effect plays an important role. Bottom shear stress estimation is the

crucial step, which is required as an input to most of sediment transport model. Therefore, the accuracy of bottom shear stress estimation used to evaluate the amount of sediment transport obtained from the sinusoidal wave is necessary to be clarified with the sediment transport estimation incorporating the acceleration effect term in its calculations. Recently, Suntoyo et al.⁴⁾ have proposed a new estimation method of the instantaneous bottom shear stress under saw-tooth waves based on incorporating both velocity and acceleration terms all at once and a good agreement between the new calculation method and experimental results of bottom shear stress under skew waves has been obtained. More recently, Suntoyo and Tanaka⁵⁾ have applied the new calculation method of bottom shear stress into sheet flow sediment transport rate calculation induced by skew waves and a good agreement with the sediment transport rate data from Kouchi et al.⁶⁾ has been obtained.

In the present paper, the new calculation method of bottom shear stress incorporating both velocity and acceleration terms all at once is proposed for cnoidal waves. Comparison among the two existing calculation methods, the new calculation method, the baseline numerical model and the experimental results of bottom shear stress is examined. The experiments of turbulent boundary layer flow for cnoidal waves were conducted in an oscillating wind tunnel over rough bed by mean of Laser Doppler Velocimeter (LDV) to measure flow velocity distribution. Moreover, the turbulent boundary layer characteristics of cnoidal waves are examined according to the non-linearity effect for experimental result as well as for the baseline (BSL) turbulence model results.

2. EXPERIMENTAL STUDY

Turbulent flow experiments under cnoidal waves over rough bed were carried out in oscillating tunnel by using air as the working fluid. The velocity was measured in the center part of wind tunnel at 20 points in the vertical direction by means of LDV. Triangular elements of roughness were chosen in order to the roughness elements protrude out of the viscous sub-layer. Thus, the velocity distribution near a rough bed is logarithmic. It can be therefore assumed that log-law can be used to estimate bottom shear stress over rough bed. But this usual log-law may be underestimated by up to 60 % in acceleration flow and overestimated by up to 80% in decelerating flow for unsteady flows, respectively, as shown by Soulsby and Dyer⁷⁾.

The experimental conditions are given in **Table 1**. The definition sketch for cnoidal wave is shown

Table 1 Experimental conditions

Case	$T(s)$	$U_c(cm/s)$	N_i	a_m/k_s	R_e
1	3	363	0.67	115.6	4.34×10^5
2	3	360	0.60	114.6	4.27×10^5
3	3	352	0.58	112.0	4.08×10^5

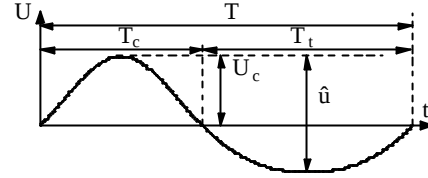


Fig. 1 Definition sketch for cnoidal wave

in **Fig. 1**. Here, a_m/k_s is the roughness parameter, k_s is the Nikuradse's roughness equivalent defined as $k_s=30z_o$ in which z_o is the roughness height and T is wave period and $a_m=U_c/s$, where, U_c is the velocity at wave crest, $N_i=U_c/\hat{u}$ is the non-linearity index and $\hat{u}$ is the total velocity amplitude. Higher N_i indicates more remarkable wave non-linearity, while the symmetric wave has non-linearity index, $N_i=0.50$.

3. NUMERICAL MODEL

In the present study, a two-layer $k-w$ model called as the baseline (BSL) model as proposed by Menter⁸⁾ was used to clarify turbulent boundary layer properties of experimental result. Moreover, the bottom shear stress calculation and experimental results also is examined by mean of this numerical model. The idea BSL model is to retain the robust and accurate formulation of the Wilcox $k-w$ model in the near wall region, and to take advantage of the free stream independence of the $k-e$ model in the outer part of boundary layer.

Equation of turbulent flow motion in the bottom boundary layer is given, as follow

$$\frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + \frac{1}{r} \frac{\partial}{\partial z} \left((n + n_t) \frac{\partial u}{\partial z} \right) \quad (1)$$

where, u : streamwise velocity, U : the velocity at the axis of symmetry, p : pressure, t : time, n : kinematics viscosity, n_t : the eddy viscosity and r : fluid density.

The governing equations of a transport equation for turbulent kinetic energy k and the dissipation of the turbulent kinetic energy w from BSL model are,

$$\frac{\partial k}{\partial t} = \frac{\partial}{\partial z} \left\{ (v + v_t s_{kw}) \frac{\partial k}{\partial z} \right\} + v_t \left(\frac{\partial u}{\partial z} \right)^2 - b^* w k \quad (2)$$

$$\frac{\partial w}{\partial t} = \frac{\partial}{\partial z} \left\{ (v + v_t s_w) \frac{\partial w}{\partial z} \right\} + g \frac{v + v_t}{v_t} \left(\frac{\partial u}{\partial z} \right)^2 - b w^2 + 2(1 - F_1) s_{w2} \frac{1}{w} \frac{\partial k}{\partial z} \frac{\partial w}{\partial z} \quad (3)$$

$$n_t = \frac{k}{w} \quad (4)$$

where, s_{kw} , b^* , s_w , g and b are model constants, F_1 is a blending function.

The boundary condition at the wall which is used are no-slip boundary condition for velocities and turbulent kinetic energy, i.e. at $z = 0$, $u = k = 0$, and at the axis of symmetry of the oscillating tunnel, the gradients of velocity, turbulent kinetic energy and specific dissipation rate are equal to zero, i.e. at $z = z_h$, $\nabla u / \nabla z = \nabla k / \nabla z = \nabla w / \nabla z = 0$. The effect of roughness was introduced through the wall boundary condition of Wilcox⁹⁾, as follow,

$$w = U^* S_R / n \quad (5)$$

where, $U^* = \sqrt{t_o / r}$ is friction velocity and the parameter S_R is related to the grain-roughness Reynold number, $k_s^+ = k_s U^* / \nu$,

$$S_R = \left(\frac{50}{k_s^+} \right)^2 \text{ for } k_s^+ < 25 \text{ and } S_R = \frac{100}{k_s^+} \text{ for } k_s^+ \geq 25 \quad (6)$$

In the present model, the non-linear governing equations were solved by using a Crank-Nicolson type implicit finite-difference scheme. In order to achieve better accuracy near the wall, the grid spacing was allowed to increase exponentially. In space 100 and in time 7200 steps per wave cycle were used. The convergence was achieved through two stages; at first stage of convergence was based on the dimensionless values of u , k and w at every time instant during a wave cycle. Second stage of convergence was based on the maximum wall shear stress. Moreover, the convergence limit was set to 1×10^{-6} for both the stages.

4. BOTTOM SHEAR STRESS CALCULATION METHODS

(1) The existing calculation methods

There are two existing calculation methods of bottom shear stress used to examine the bottom shear stress. At first, Method 1 is proportional to the square of $U(t)$ by considering the friction coefficient under a sinusoidal wave motion as proposed by Kabiling and Sato¹⁰⁾ given in Eq. (7).

$$t_o \left(t - \frac{j}{s} \right) = \frac{1}{2} r f_w U(t) |U(t)| \quad (7)$$

Here, $t_o(t)$ is the instantaneous bottom shear stress, t is time, s is the angular frequency, $U(t)$ is time variation of free stream velocity, j is phase difference between bottom shear stress and free stream velocity and f_w is friction velocity factor.

Second, Method 2 is proportional to the square of the instantaneous friction velocity, $U^*(t)$ incorporating the acceleration effect under a bit of

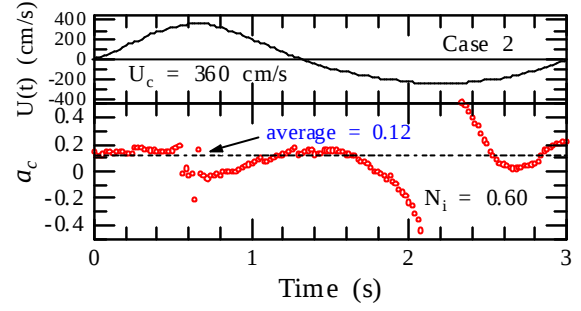


Fig. 2 Calculation example of acceleration coefficient, a_c

sawtooth asymmetric wave as proposed by Nielsen¹¹⁾ in Eqs. (8) and (9), as follows

$$U^*(t) = \sqrt{\frac{f_w}{2}} \left\{ \cos j U(t) + \frac{\sin j}{s} \frac{\partial U(t)}{\partial t} \right\} \quad (8)$$

$$t_o(t) = r U^*(t) |U^*(t)| \quad (9)$$

This method was obtained through an understanding that if the steady flow component is weak (e.g. in a surf zone) however, in the sense that its influence on the bed shear stress is small. It seems reasonable to derive the $t_o(t)$ from $u(t)$ by means of a simple transfer function based on knowledge from simple harmonic boundary layer flows as has done by Nielsen¹²⁾. The equations in terms of the free stream velocity and its derivative by considering the phase difference is hereafter given by Nielsen^{11),12)} as shown in Eq. (8).

(2) The new calculation method

The new calculation method of bottom shear stress under non-linear waves (Method 3) is based on incorporating velocity and acceleration terms all at once that is given through the instantaneous friction velocity, $U^*(t)$ as given in Eq. (10). Both velocity and acceleration terms are adopted from a calculation method proposed in Eq. (8) by Nielsen¹¹⁾, but that method could not give a good agreement with experimental data as shown by Suntoyo et al.⁴⁾, so in the new calculation method is proposed a new acceleration coefficient, a_c expressing the non-linearity effect on the bottom shear stress under cnoidal waves that is determined empirically from both experimental and baseline turbulent model results. The instantaneous bottom shear stress can be calculated proportional to the square of the proposed instantaneous friction velocity, as shown in Eq. (10),

$$U^*(t) = \sqrt{f_w / 2} \left\{ U \left(t + \frac{j}{s} \right) + \frac{a_c}{s} \frac{\partial U(t)}{\partial t} \right\} \quad (10)$$

$$t_o(t) = r U^*(t) |U^*(t)| \quad (11)$$

Here, a_c is the value of acceleration coefficient obtained from the average value of $a_c(t)$ calculated from experimental result as well as numerical model results, as expressed in Eq. (12). Fig. 2 showed a calculation example of the time - variation

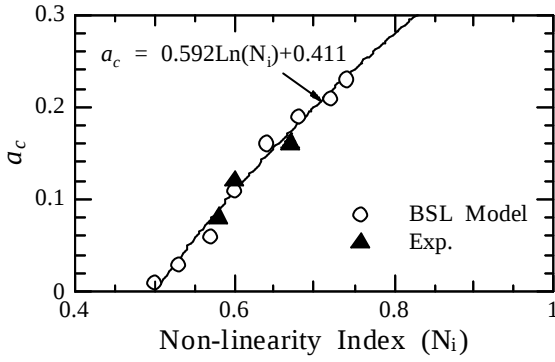


Fig. 3 Acceleration coefficient, a_c as function of N_i

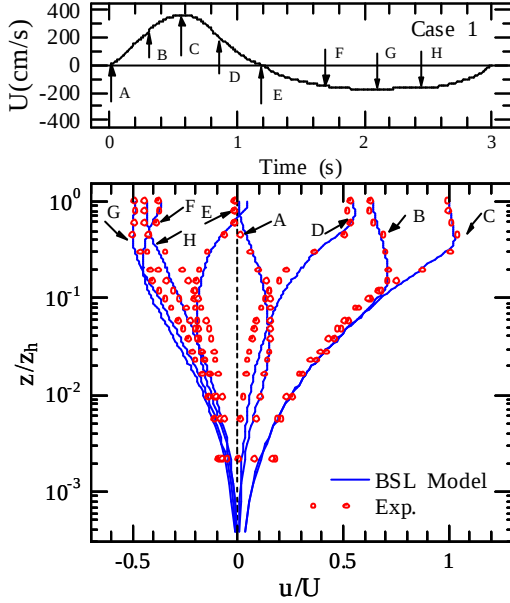


Fig. 4 Mean velocity profiles for Case 1 with $N_i = 0.67$

of the acceleration coefficient, $a_c(t)$.

$$a_c(t) = \frac{U^*(t) - \sqrt{f_w/2}U \left(t + \frac{j}{s} \right)}{\frac{\sqrt{f_w/2}}{s} \frac{\partial U(t)}{\partial t}} \quad (12)$$

The results of average value of acceleration coefficient, a_c from both experimental and numerical model results as function of non-linearity index, N_i were plotted in Fig. 3. Hereafter, an equation based on regression line to estimate the acceleration coefficient, a_c as function of N_i is proposed as given in Fig. 3. The increase in the non-linearity of wave brings out the increase of the value of acceleration coefficient, a_c . As seen that for the symmetric wave which has $N_i = 0.50$, the value of a_c is equal to zero, so the acceleration term is not the significant factor on calculation in Method 3, therefore Method 3 will be equal to Method 1.

Hereafter, friction velocity factor, f_w proposed by Tanaka and Thu¹³⁾ is used to examine the bottom shear stress for all methods, while the phase difference, j used it is an approximation obtained from the relation proposed by Suntoyo et al.⁴⁾.

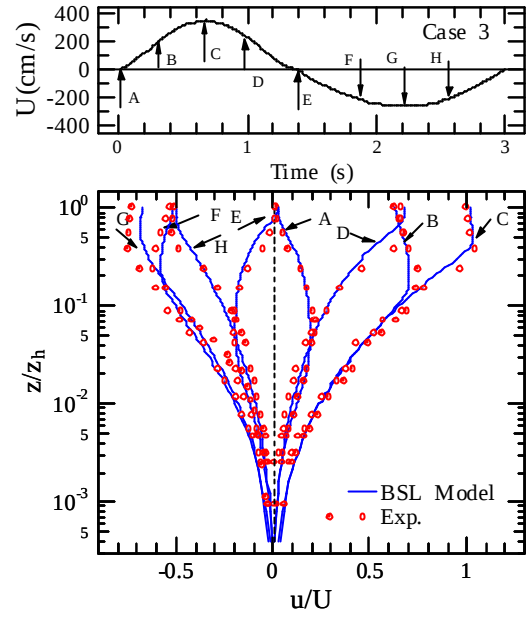


Fig. 5 Mean velocity profiles for Case 3 with $N_i = 0.58$

5. RESULTS AND DISCUSSIONS

(1) Mean velocity distribution

Mean velocity profiles in the rough turbulent boundary layer for cnoidal waves at selected phases were compared with the BSL numerical model as shown in Figs. 4 and 5. The solid line showed the BSL model while open and closed circles ($\circ$ and $\bullet$) showed the experimental results of mean velocity profile distribution. As seen that both for experimental and the BSL model results, the velocity overshoot is much influenced by the effect of acceleration and the velocity magnitude. The velocity overshoot at phases of B, C and D are higher than that of at phases of F, G and H. The mean velocity close to the bottom increase as increasing of the non-linearity index, N_i according to the increasing of acceleration effect at the crest part of wave flow.

The velocity profiles of experimental results showed a good agreement with the BSL model prediction at the phases of A, B, C, especially where the velocity overshooting occurs. During the deceleration phases where the pressure gradient is not so steep as in the present asymmetric wave cases, it seems that the BSL model slightly fails to cope with the flow situation.

(2) Turbulent intensity distribution

The turbulent intensity or the fluctuating velocity in x direction, u' can be computed using Eq. (13) as proposed by Nezu¹⁴⁾ from the turbulent kinetic energy provided in the BSL model.

$$u' = 1.052\sqrt{k} \quad (13)$$

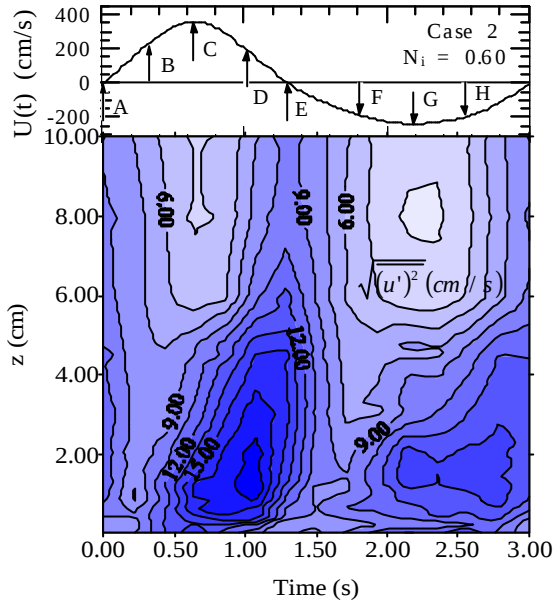


Fig. 6 Turbulent intensity distribution of experimental result for Case 2

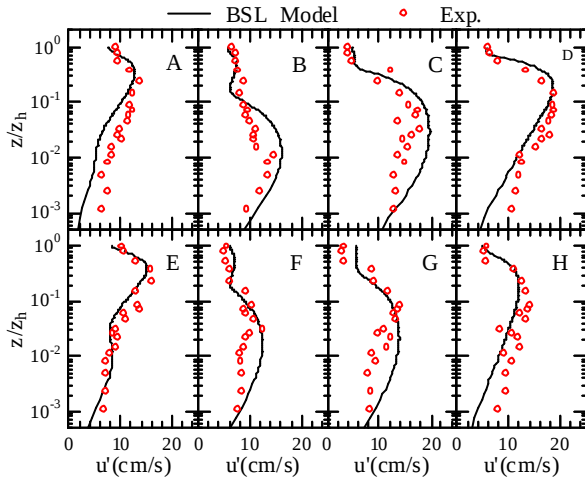


Fig. 7 Comparison between BSL model prediction and experimental result of turbulent intensity for Case 2

Fig. 6 shows the turbulent intensity distribution of experimental result for Case 2. This figure illustrates how the turbulence develops as the flow progresses in phase space. The turbulence builds up near the bed and constantly diffuses away from the bed across the boundary layer, as the boundary layer develops in time. The turbulent intensity almost uniformly distributed across the depth, where the free - stream velocity is zero, namely at phases of A and E. Moreover, higher turbulent intensity close to the bottom occurs at C and G phases due to the higher mean velocity at crest and trough of wave.

Comparison of BSL model prediction and experimental data of turbulent intensity at selected phases for Case 2 is shown in **Fig. 7**. An excellent agreement is shown across the depth at the phase of E. The model prediction far from the bed is

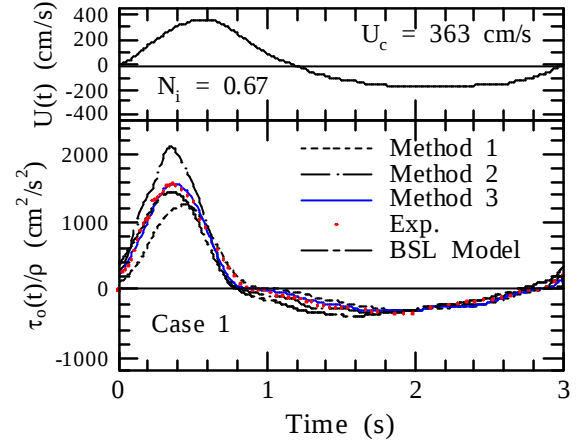


Fig. 8 Comparison among numerical model, calculation method and experimental results of bottom shear stress, for Case 1.

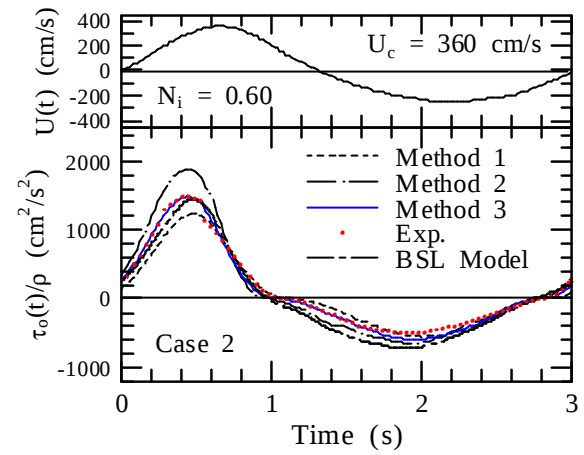


Fig. 9 Comparison among numerical model, calculation method and experimental results of bottom shear stress, for Case 2

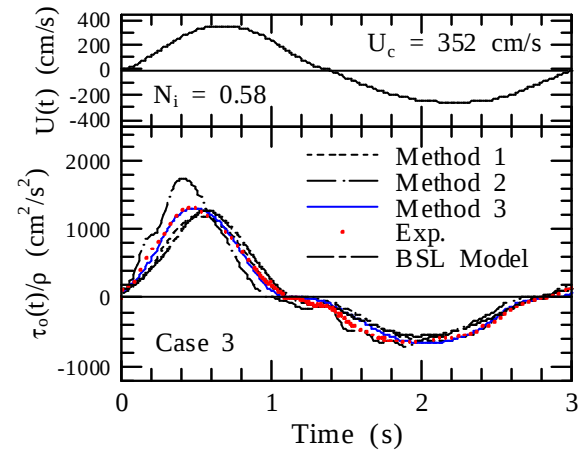


Fig. 10 Comparison among numerical model, calculation method and experimental results of bottom shear stress, for Case 3

generally good, while near the bed is not so much in good agreement. However, the prediction model qualitatively produces very good indication of the pattern of turbulence generation and it mixing.

(3) Bottom shear stress

Figs. 8, 9 and 10 show comparison among

numerical model, calculation methods and experimental results of bottom shear stress for Case 1, Case 2 and Case 3, respectively. Case 1 has the highest non-linearity index of wave, while Case 3 has the lowest non-linearity index of wave. Method 3 has given the best agreement with the experimental result along a wave cycle for all cases, while Method 1 gave under estimate value of bottom shear stress especially at crest part caused by incorporating the acceleration term was not done in Method 1. It is confirmed that the acceleration effect has significant role in the calculation of bottom shear stress under cnoidal waves. Although the acceleration term has been included in Method 2, however Method 2 gave over estimate value especially at positive wave cycle for all cases. It indicated that Method 2 was not a reliable method for calculating the bottom shear stress under cnoidal waves. Moreover, the BSL model prediction result showed more close to both the experimental result and Method 3 than Method 1 and Method 2 for all cases. Due to wave non-linearity, the wave-induced the bottom shear stress distribution is characterized by a large peak over a very short time interval preceding the wave crest. These characteristics are much more obvious for the higher non-linear wave case. As seen that Case 1 produced a largest peak over shortest time interval preceding the wave crest than others cases.

Hereafter, it can be concluded that the new method (Method 3) can be used to estimate the bottom shear stress under cnoidal waves for higher non-linearity up to the symmetric wave with $Ni = 0.5$ and the acceleration coefficient, a_c obtained from the regression line as shown in **Fig. 3** was sufficient for this calculation. Therefore, the new method of bottom shear stress under cnoidal wave that can be further used to an input sediment transport model under rapid acceleration in practical application.

6. CONCLUSIONS

In the present paper, the characteristics of the turbulent boundary layer under cnoidal waves according to the non-linearity index was examined both experimental results and the BSL numerical model. Moreover, the new calculation method of bottom shear stress for cnoidal waves has been proposed and the new method has given the best agreement with the experimental result than others calculation methods for all cases. It shows that the acceleration coefficient proposed in this method was sufficient for this calculation. Therefore this method

can be used to an input sediment transport model under rapid acceleration in practical application.

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