EFFECT OF CAPILLARY RISE ON SUBSURFACE RUNOFF IN THE REW MODELING FRAMEWORK

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This paper tried to clarify the effects of capillary rise on subsurface runoff through a recently suggested water balance model for representative elementary watersheds (REWs). We focused on saturated zone water balance of a REW model effected by the capillary rise, in comparison with to the Tank model. The results and discussions on the comparison led the followings: (1) Capillary rise is effective under arid climate than humid climate. (2) This tendency is enhanced under climates with clear difference between wet and dry seasons. (3) REW models have potential to describe and diagnose basin's processes based on the mass and momentum balance equations better than Tank models and other lumped water balance models. (4) REW models can be more suitable than Richards equation based models under arid climate if user's requirement for model structure is only unsaturated and saturated zones.

Key Words **:** *Capillary force, water balance, Tank model, runoff processes, diagnostic tool*

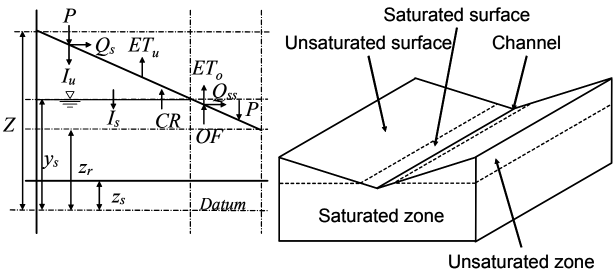
1. INTRODUCTION

Reggiani *et al.*[1] suggested conservation equations governing hillslope responses. The equations are from a unifying framework for watershed's balance equations for mass, momentum, energy, and entropy in Reggiani *et al.*[2),3),4)]. They introduced "representative elementary watershed (REW)" for the minimum spatial unit for the equations. The REW is sub-basin in other words and is like a lumped sub-model constituting a spatially distributed model. We call their balance equations as REW model for convenience in the followings.

A REW model is comparable to the water balance models with hydrological process descriptions. The bucket type model by Milly[5] is too simple to describe the processes, because the model neglects unsaturated zone processes and basin's topography. Eagleson's model[6] considered unsaturated and saturated zones, but water table depth is fixed all through the year and basin's topography is ignored. Woods' model[7] considered vegetation canopy, root zone and saturated zone, ignoring surface runoff. The REW model is in-between these existing water balance models and Richards[8] equation based models.

One of the characteristics of the REW model is the momentum balance equation for the unsaturated zone. This equation calculates water exchanged between unsaturated and saturated zones in the balance among gravitational force, capillary force and resistance force acting on the water in the unsaturated zone. The REW model is still under the testing phases, and no one has tested the effect of capillary force in applying the model to basins. In addition, no attention are paid to the differences between a REW model and other water balance models with focus on the capillary force in the characteristic momentum balance equation for the unsaturated zone.

This paper tried to evaluate the effect of the capillary rise on subsurface runoff in a REW model in contrast with well-known Tank model[9]. We adopted a REW model for a single and simple imaginary REW to avoid complex discussions in the comparison.



(a) Left side half of REW (b) Whole view of REW

Fig.1 Conceptual drawing of REW model. *P*: precipitation, ET_u and ET_o: evapotranspiration from unsaturated and saturated surface, Q_s and Q_{ss}: surface and subsurface runoffs, I_u and I_s are infiltration to unsaturated and saturated zones, *CR*: capillary rise, *OF*: Outflow from saturated zone, y_s: water table elevation, z_r: channel bed elevation, z_s: impermeable boundary elevation, *Z*: depth of soil layer.

2. MODEL DESCRIPTIONS

(1) Model structure

Fig. 1 shows the structure of the REW model for a single REW. This model consists of unsaturated and saturated zones separated vertically by a ground water table. The REW model shows saturated ground surface when ground water table is higher than the average channel bed elevation, and it disappears in the other case. The ground surface fraction other than the saturated surface is unsaturated surface, neglecting the ground surface fraction of the river.

In addition to the unsaturated and saturated zones, a REW model has three more sub-zones of concentrated overland flow, saturated overland flow and channel reach flow. The three sub-zones are neglected in this research to highlight the effect of capillary rise in REW model.

(2) Model behavior

a) Unsaturated zone

The behavior of a REW model is similar to other water balance models. Precipitation (P) to the unsaturated surface increases the saturation degree in the unsaturated zone, if it is less than the infiltration capacity of the soil. Otherwise, the residual precipitation instantaneously becomes surface runoff in this research. Precipitation to the saturated surface also becomes surface runoff instantaneously. The increased saturation degree causes expansion of saturated surface and increase in the gravitational force acting on the unsaturated zone water. If gravitational force is higher than capillary force, then the unsaturated zone water moves downward and raises the ground water table. If gravitational force is lower than capillary force in spite of the precipitation, the unsaturated water moves upward dropping the ground water table. The speed of water exchange between unsaturated and saturated zones depends on both the REW-scale hydraulic conductivity and the difference between gravitational force and capillary force.

After precipitation, the unsaturated zone starts to lose water through evapotranspiration (ET) at a speed proportional to potential evapotranspiration (PET). Decrease of saturation degree causes decrease of gravitational force and increase of capillary force, which results in the drop of ground water table and expansion of unsaturated surface.

b) Saturated zone

Saturated zone generates subsurface runoff, when ground water table is higher than the average channel bed elevation. The subsurface runoff is proportional to hydraulic conductivity and the difference between ground water table and the average channel bed elevation, hence this behavior is quite similar to that of Tank model[9]. This is why we compare REW model with Tank model.

(3) Governing equations

A single REW model in this research is governed by the following Eq.(1) to (3):

$$\rho\varepsilon\frac{d}{dt}(s_u y_u \omega_u) = \min\left\{\rho P\omega_u,\ \frac{\rho K_{sat}\omega_u}{\Lambda_u}\left[\frac{1}{2}y_u - \psi_u\right]\right\}\cdot\delta_P + \rho\varepsilon\omega_u v_u - \rho\omega_u\frac{1}{R}(\tanh\frac{5}{2}s_u)(1.0+R^{-5})^{-(1/5)}PET\cdot\delta_P \quad (1)$$

$$-\varepsilon\rho g s_u y_u \omega_u + \varepsilon\rho g\omega_u\left[\frac{1}{2}y_u - \psi_u\right] = K^{-1}\varepsilon\rho g y_u \omega_u v_u \quad (2)$$

$$\rho\varepsilon\frac{d}{dt}(y_s\omega_s) = -\rho\varepsilon\omega_u v_u - \frac{\rho K_{sat}\omega_o}{\cos(\gamma_o)\Lambda_s}\frac{1}{2}(y_s - z_r + z_s) \quad (3)$$

Eq.(1) is mass balance equation for the unsaturated zone that has storage change on the left hand side. The right hand side terms are P, capillary rise (CR) and ET. Eq.(2) is momentum balance equation for the unsaturated zone that consists of gravitational force and capillary force on the left hand side, and resistance force on the right hand side. Eq.(2) calculates v_u not from Darcy's law but from the balance of the three forces and deliver it to Eq.(1) and (3). Eq.(3) is mass balance equation. The left

Table 1 REW Model parameters and variables followed by parameter values and units in parentheses. Parameter values are listed if they are fixed.

Name	Description
P	Precipitation: Generated by Robinson & Sivapalan[10]. Only the intensity was changed for different climate.
PET	Potential evapotranspiration
R	Dryness index: annual P/annual PET
ε	Soil porosity: 0.35[10]
K_{sat}	Saturated hydraulic conductivity: 3.4×10^{-6} (m/s) [11]
K	Unsaturated hydraulic conductivity: $K_{sat}(s_u)^{\lambda}$ [12]
Λ_s	Typical length scale for seepage outflow: 10(m)
γ_o	Slope angle of the overland flow plane with respect to horizontal: 0.0
z_r	Average elevation of channel bed from datum: 7(m)
z_s	Average elevation of the bottom end of REW from datum: 0(m)
s_u	Saturation degree in unsaturated zone
Z	Depth of soil layer: 8.0(m)
y_u	Average thickness of unsaturated zone
y_s	Average thickness of saturated zone
ω_u	Unsaturated surface area fraction
ω_o	Saturated surface area fraction
ω_s	Area fraction of saturated zone: 1.0
ψ_u	Pressure head in the unsaturated zone
v_u	Vertical velocity in unsaturated zone
Λ_u	Typical length scale for infiltration: $s_u y_u$ (m)
δ_P	Function to be 1.0 if rained, otherwise 0.
t	Time
ρ	Water density: 1,000(kg/m^3)
g	Gravitational acceleration: 9.8(m/s^2)
ψ_c	Bubbling pressure: -0.20(m)
ψ_0	Capillary pressure at the inflection point: -0.30(m)
m	Dimensionless parameter related to the width of the pore radius distribution: 0.44
λ	Pore-disconnectedness index: 4.7

hand side term is storage change, and the right hand side terms are recharge from unsaturated zone and outflow (OF) to channel.

These governing equations need constitutive relationships for their solutions. We used the same relationships as Reggiani *et al.*[1], but we applied Kosugi's[13] soil characteristic function for the relationship between pressure head and saturation degree as shown in Eq.(4):

$$\psi_u = \begin{cases} \psi_c - (\psi_c - \psi_0)\cdot\left\{\dfrac{(s_u)^{-1/m} - 1.0}{m}\right\}^{1.0-m} & (s_u < 1.0) \\ 0.0 & (s_u = 1.0) \end{cases} \tag{4}$$

The meanings of the parameters and variables in Eq.(1) to (4) are listed in **Table 1**.

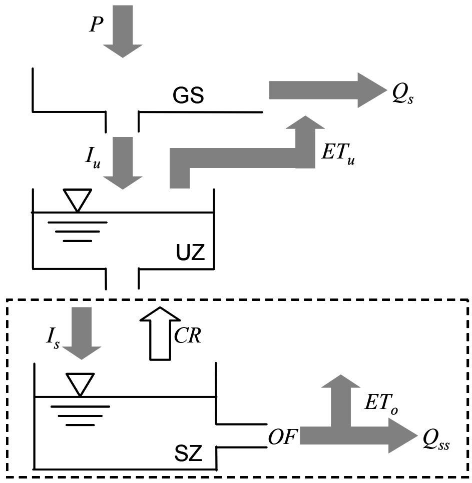


Fig.2 Difference in water balance calculation between REW and Tank model. Gray arrows indicate they are same in both models, whereas white arrow indicates it acts only in REW model. *CR* is white arrow, hence it works only in REW model. In this figure, GS, UZ, and SZ indicate ground surface, unsaturated zone and saturated zone, respectively. *OF* is the sum of ET_o and Q_{ss}. We focus on the processes in the dashed rectangular.

3. EFFECT OF CAPILLARY RISE ON SUBSURFACE RUNOFF

(1) Methodology

We explore the effect of momentum balance equation of REW model through the comparison between REW and well-known Tank models focusing on the saturated zone water balance of REW model in the dashed rectangular in **Fig.2**, because REW model becomes equivalent to Tank model if we neglect CR in Eq.(2). We calculated subsurface from the Tank model by Eq.(3) and (5), although the Tank model loses variable runoff coefficient and capillary rise.

$$dH/dt = \left(K_{sat}/2\cos(\gamma_o)\Lambda_s\right)\left(H - Z\right) \tag{5}$$

H , $K_{sat}/2\cos(\gamma_o)\Lambda_s$, and Z in Eq.(5) correspond to y_s , $K_{sat}\omega_o/2\cos(\gamma_o)\Lambda_s$, and $z_r - z_s$ in Eq.(3), respectively. Note that ω_o makes runoff coefficient variable in Eq.(3).

The Tank model receives I_s (negative CR) and neglects positive CR calculated in the REW model. The difference between two models appears in $Q_{ss} = OF - ET_o$ because ET_o is same in REW and the Tank models. ET_o is PET if rained, otherwise 0.

We carried out numerical experiments for two different climates in terms of annual P , annual PET , and phase difference between seasonally changing intensities of P and PET .

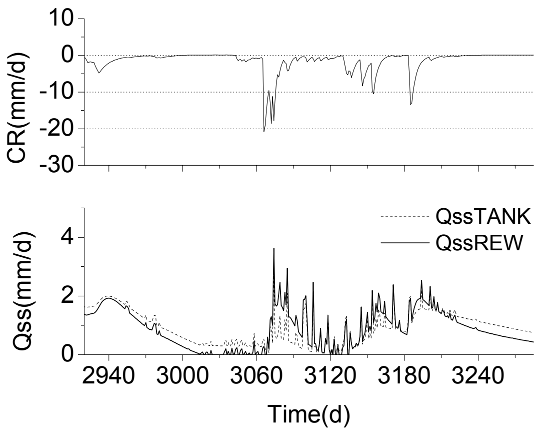


Fig.3 One year time series for capillary rise *CR* (upper panel) and subsurface runoff Q_{ss} from REW and Tank models (lower panel) for humid climate. Annual *P* and annual *PET* are 2,000mm and 1,000mm, respectively. Seasonal intensities of *P* and *PET* are in phase, causing mild climate seasonality. The last one year data is taken from a nine years long simulation.

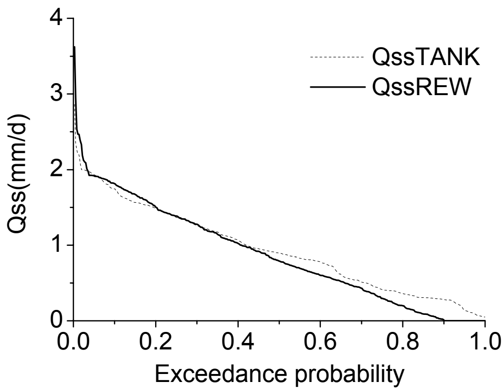


Fig.4 Flow duration curves for subsurface runoff Q_{ss} from REW and Tank models for humid climate. Annual *P* and annual *PET* are 2,000mm and 1,000mm, respectively. Seasonal intensities of *P* and *PET* are in phase. Q_{ss} data are from **Fig.3**.

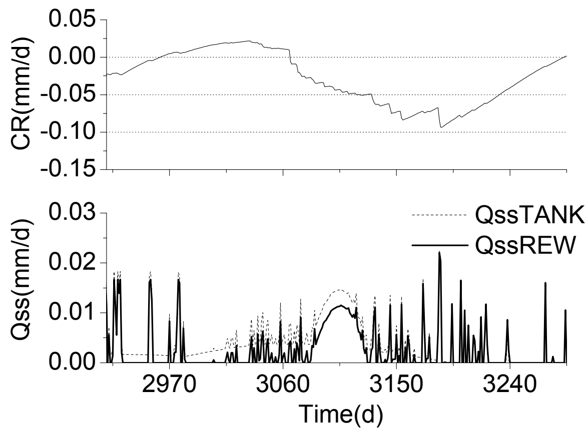


Fig.5 One year time series for capillary rise *CR* (upper panel) and subsurface runoff Q_{ss} from REW and Tank models (lower panel) for arid climate. Annual *P* and annual *PET* are 20mm and 2,000mm, respectively. Seasonal intensities of *P* and *PET* are of opposite phase, causing strong climate seasonality. The last one year data is taken from a nine years long simulation.

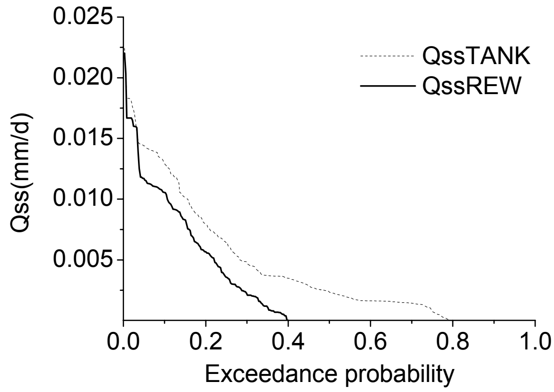


Fig.6 Flow duration curves for subsurface runoff Q_{ss} from REW and Tank models for arid climate. Annual *P* and annual *PET* are 20mm and 2,000mm, respectively. Seasonal intensities of *P* and *PET* are of opposite phase. Q_{ss} data are from **Fig.5**.

(2) Results

The effects of capillary rise and variable runoff coefficients appear as the difference in Q_{ss} as shown in **Fig.3** to **6**. **Fig.3** and **4** show daily time series of CR, Q_{ss}, and flow duration curve (FDC) for humid climate, and **Fig.5** and **6** for arid climate. These figures include the 9th year data of the totally 9 years to remove the effects of initial conditions.

Fig.3 and **4** are the results when annual P and annual PET are 2,000mm/y and 1,000mm/y, respectively. The seasonal intensities of P and PET are in phase. Positive CR looks almost negligible and the two models show small difference in daily Q_{ss}, but annual Q_{ss} from REW model is about 6.83% less than Tank model for moisturizing the unsaturated soil and enabling more intensive ET from the zone.

Fig.5 and **6** are the results when annual P and annual PET are 20mm and 2,000mm, respectively. The seasonal intensities of P and PET are of opposite phase making distinct wet and dry seasons, which brings intense capillary force, less subsurface storage, and less subsurface runoff. These figures show significant differences in Q_{ss} between two models. The REW model's Q_{ss} is 39.2% less than the Tank model on the annual basis, and the difference tries to satisfy the strong ET demand from the arid climate.

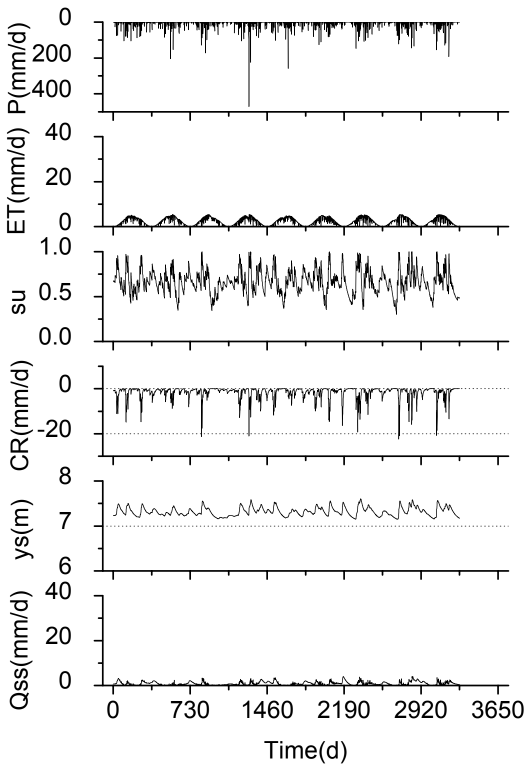


Fig.7 Time series of P, ET, s_u, CR, y_s, and Q_{ss} for nine years. Annual P and annual PET are 2,000mm and 1,000mm, respectively. Seasonal intensities of P and PET are in opposite phase, causing mild climate seasonality.

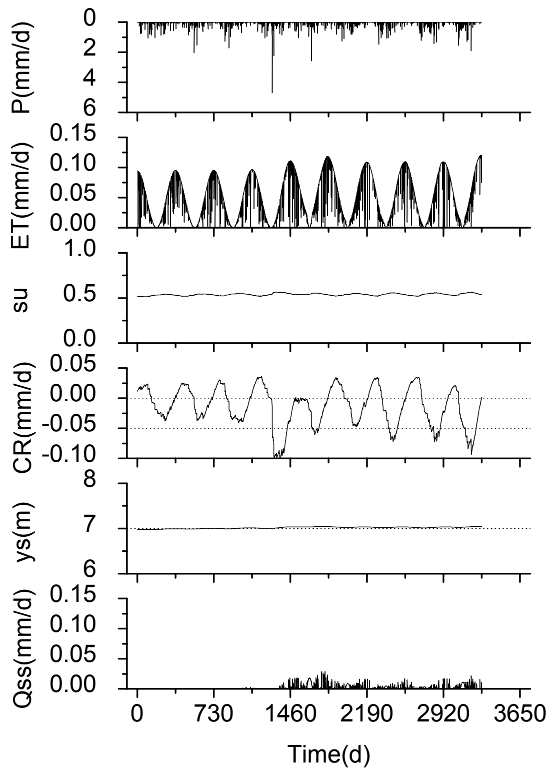


Fig.8 Time series of P, ET, s_u, CR, y_s, and Q_{ss} for nine years. Annual P and annual PET are 20mm and 2,000mm, respectively. Seasonal intensities of P and PET are of opposite phase, causing strong climate seasonality.

Fig.7 and **8** are daily time series of P, ET, s_u, CR (positive means upward), y_s, and Q_{ss}. As mentioned above, positive CR becomes more significance under arid climate than humid climate. This result clearly shows that REW model is a better tool in describing or diagnosing hydrological processes under severely arid climate with distinct wet and dry seasons than Tank models, although Tank model can simulate watershed water balance reasonably well through model parameter calibrations.

4. DISCUSSIONS

(1) Comparison with storage function model

A REW model is a lumped model if applied for a single REW, hence a REW is comparable with storage function model as well. The difference between Tank model and storage function model is just the way to calculate runoff from the models. This fact promises that quite similar results appear if we compare REW models with storage function models.

Capillary rise in REW models can be taken into Tank models, storage function models, and other storage type lumped models, but they need to evaluate capillary rise using lumped model parameters that has less physical meanings than those of REW models. Moreover, an advantage of REW models is that they calculate capillary rise or recharge into subsurface storage based on the balance among numerically evaluated gravitational force, capillary force, and resistance force. This force balance as well as the intensity of the capillary rise has potential to be a quite useful window in understanding, explaining, and diagnosing the hydrological processes of a basin.

(2) Comparison with Richards equation based models

This research did not compare REW model with distributed models based on Richards equation[8), hence we would like to note a major difference between these two models here.

The difference is about capillary rise, again. Richards equation based models operate to achieve hydrostatic pressure head distributions. Soil water always flows downward if the soil is in draining phases and it flows upward only when (1) the soil matrix has source points or (2) the matrix receives inflows from the matrix boundaries. These features are owing to the lack of the capillary rise components in Richards equation based models, and they do not allow describing capillary rise processes through such models. The capillary rise processes should be significant under arid climates than humid climates, as shown in this research. This result indicates Richards equation based models can work well under humid climate, but they can fail to do so without direct ET from the soil matrix under arid climates.

Unlike Richards equation based models, REW models describes capillary rise through the momentum balance equation. Even if the climate is arid, REW models can deliver water from saturated zone to unsaturated zone and try to satisfy the strong ET demand. This fact means REW models enable us to describe or diagnose the interactions between saturated and unsaturated zone better.

(3) Weak points and future of REW models

A REW model has weak points in the constitutive relationships such as REW scale relationships among hydraulic pressure head, saturation degree, and hydraulic conductivity as mentioned by Beven[14]. In addition, those relationships must be universal and transferable to a REW to another. Discoveries of such REW scale relationships and parameters from REW scale observations or satellite remote sensing data will help the usability of REW models, promising our scientific and technological innovations and advancing our current predictability of REW scale hydrological processes in ungauged basins.

5. SUMMARIES

This paper investigated the difference between REW models and Tank models to clarify the effect of capillary rise in REW models. The summaries of the results and discussions are as follows:

- Capillary rise is more effective under arid climates than humid climates. Strong seasonality with distinct climatic difference in wet and dry seasons enhances this tendency.
- REW models have potential to describe and diagnose basin's processes through the mass and momentum balance equations better than Tank models and other lumped water balance models.
- REW models can suit to arid climates more than Richards equation based models.
- Creating relationships and parameters for REW scale will broad the use of REW model and will advance our predictability in ungauged basins.

ACKNOWLEDGMENTS: The authors greatly thanks to the financial support by JSPS Research Fellowship for Young Scientists, Grand-Aided for Scientific Research (Grant-in-Aid for JSPS Fellows) and the research grant from the Maeda Engineering Foundation. We acknowledge versatile supports and thoughtful advices from Professor Katumi Musiake at Fukushima University and Professor Taikan Oki at University of Tokyo. We also acknowledge reviewer's insightful comments.

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(Received September 30, 2005)