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MULTIPLICATIVE RANDOM CASCADE HSA METHOD FOR HIGH RESOLUTION RAINFALL FIELD MODELING

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In order to downscale spatial rainfall field from a coarse scale into a finer one, the non-homogenous multiplicative random cascade method is often employed. Currently, this kind of downscaling method is less reliable even though it correctly preserves the statistical description and a long term average spatial pattern. It fails reproducing the spatial patterns in repeated trials; and there is a higher chance of having random outputs. These drawbacks are needed to overcome for applying the downscaled rainfall products in further analysis of hydrological and meteorological studies. In this study, a new method, named as Multiplicative Random Cascade Hierarchical and Statistical Adjustment (HSA) method, is introduced and tested to downscale 1.25 degree GAME Re-analysis data into 10-minute spatial resolution. The obtained results are highly improved, quite robust and reliable than the previous method.

Key Words : *spatial rainfall structure, downscaling, random cascade method, HSA method*

1. INTRODUCTION

A large scale difference that exists between the global scale (climate or atmospheric) models and the local hydrological models are offering great challenge in linking and viewing global to local scale phenomenon. These models are necessary to be coupled in order to understand and predict a clear scenario of local and regional impacts on hydrological cycle due to global environmental changes. Coarse scale products of global scale models are an inadequate basis for assessing local / regional scale impacts as it is hardly able to resolve many important sub-grid scale processes[1,2]. It is necessary to identify the sub-grid scale features for local or regional hydrological analysis, which is not seen in a coarser scale frame.

Accurate simulation of space time rainfall field is an important task in hydrology. It is an important binding forcing to understand the space time variability of hydrologic factors, and to drive small to large-scale, short to long-term simulations of runoff quantity and quality. There are numerous attempts to use products of global scale space time rainfall models, e.g. General Circulation Models (GCMs), in local scale hydrological analysis for a number of reasons. This demands a reliable disaggregation of a coarse GCM scale rainfall field to a smaller scale of local catchments[3].

There are several methods developed for the purpose of downscaling rainfall fields from a coarser to finer resolution. A scale invariance multifractal method, which assumes a cascade of process of development of intense eddies at finer resolution inside a weaker eddy based on turbulence theory of energy dissipation, has been repeatedly proven to be strongly relevant to the physical processes of rainfall generation mechanism[4,5,6,7]. This method has lead to the development of a non-homogenous multiplicative random cascade method, which is able to preserve the spatial statistics at a multiscale frame and also able to generate a long-term spatial pattern correctly. This method assumes the isotropic spatial statistics, which is not in line with the rainfall observation and fails to generate practically useful, especially short time scale rainfall field. As stated earlier, the need of spatial disaggregation is much crucial for the purpose of utilizing the disaggregated GCM scale spatial rainfall field; the focus in this paper is given on spatial structure of the rainfall field, leaving the part of study of temporal structure as a further

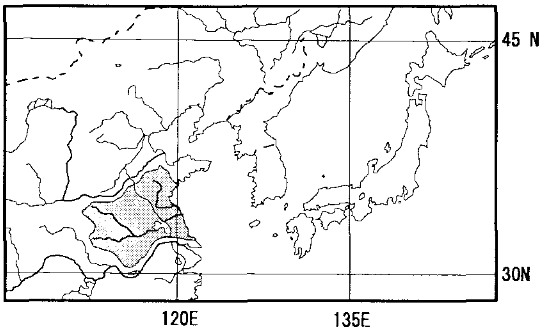


Fig. 1 : Location of the study region.

research topic in random cascade method. We have found that the inclusion of spatial correlation field in the multiplicative random cascade is successful to improve the disaggregation of the spatial rainfall field. This method is applied to a 560 km X 320 km region over eastern Chinese territory (Fig. 1) using hourly rainfall data of 1.25-degree resolution and disaggregated to 10-minute spatial resolution. The method is named as HSA method. This paper presents the improved method to disaggregate the spatial rainfall field using a non-homogenous multiplicative random cascade HSA method.

2. SPATIAL RAINFALL MODEL

Spatial disaggregation of rainfall field can be carried out using the extended discrete random cascade method[8,9,10]. In this process, a two-dimensional region with known volume of rainfall is successively divided into b equal parts (b = 2^d) at each step. During each subdivision the mass (or volume) obtained at the previous disaggregation step is distributed into the b subdivisions by multiplication by a set of "cascade generators" W, as shown schematically in Fig.2 (for d = 2 and b = 4).

For an area at level 0, denoted by Δ_0^0, has the outer length scale of L_0 and average rain intensity of R_0. The initial rain volume $\mu_0(\Delta_0^0)$ becomes $R_0 L_0^d$. At level 1, the rain volume $\mu_0(\Delta_0^0)$ divides into b = 4 sub-areas denoted as Δ_1^i, (i = 1, 2, 3, 4) and the sub-area rain volume $\mu_1(\Delta_1^i)$ is $R_0 L_0^d b^{-1} W_1^i$, (i = 1, 2, 3, 4). At level 2, each of the sub-area volume is further subdivided into b = 4, all together b^2 = 16 sub sub-area, denoting them as Δ_2^i, (i = 1, 2, ..., 16) and the corresponding volume $\mu_2(\Delta_2^i)$ is $R_0 L_0^d b^{-2} W_1^i W_2^i$, ($i$ = 1, 2, ..., 16). The process of sub-division is continued further until the n^{th} level up to b^n sub-areas, which are denoted as Δ_n^i, (i = 1, 2, ..., b^n). At the n^{th} level, the volume

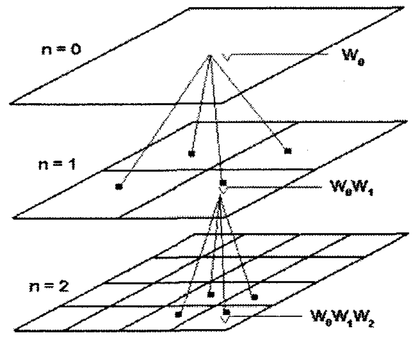


Fig. 2 Schematic of cascade branching.

in the sub-areas can be expressed as

$$\mu_n(\Delta_n^i) = R_0 L_0^d b^{-n} \prod_{j=1}^{n} W_j^i \quad ; (i = 1, 2, ..., b^n) \qquad (1)$$

The cascade generators W are non-negative random values with $E[W]$ = 1, which is imposed to ensure the mass conservation from one discretization level to the next. To get the cascade generator W values, Over and Gupta[9,10] have proposed a model called the beta-lognormal model as

$$W = BY \qquad (2)$$

where, B is a generator from the "beta model" that separates the rainy and non-rainy zone on the basis of discrete probability mass function and Y is obtained from lognormal distribution[11]. They are given by

$$B = \begin{cases} 0 & \text{for probability} = 1 - b^{-\beta} \\ b^{\beta} & \text{for probability} = b^{-\beta} \end{cases} \qquad (3)$$

$$Y = b^{-\frac{\sigma^2 \ln b}{2} + \sigma X} \qquad (4)$$

where, β is a parameter; X is normal random variate and σ^2 is a parameter equal to the variance of $\log_b Y$ with condition that $E[Y]$ = 1. The W is given by

$$W = \begin{cases} 0 & \text{when } P(B = 0) = 1 - b^{-\beta} \\ b^{\beta + \sigma X - \frac{\sigma^2 \ln b}{2}} & \text{when } P(B = b^{\beta}) = b^{-\beta} \end{cases} \qquad (5)$$

This model consists of only two parameters, β and σ^2. The parameter estimation method is proposed by Over and Gupta[9,10] by using the Mandelbrot-Kahane-Peyriere (MKP) function, named after Mandelbrot[6], Kahane and Pyeriere [12], which characterizes the fractal or scale-invariant behavior of the multiplicative cascade process. This method yields the following equations for the parameter β and σ^2,

$$\beta = 1 + \frac{\tau^{(1)}(q)}{d} - \frac{\sigma^2 \ln b}{2}(2q - 1) \qquad (6)$$

$$\sigma^2 = \frac{\tau^{(2)}(q)}{d} \ln b \qquad (7)$$

Here, $\tau^{(1)}(q)$ and $\tau^{(2)}(q)$ are the first and second derivative of the slope $\tau(q)$ with respect to q. The slope $\tau(q)$ represents the scaling relationship for different exponent q in the process of obtaining statistical moment across different level of subdivision. Choosing the value of q = 2 gives less variable estimates of β and σ^2 without affecting the simulation[13].

3. IMPROVED MODEL USING HSA METHOD

Repeated trials of rainfall generation have a rare possibility of yielding similar spatial rainfall field due to randomness of the cascade generator W. The spatial rainfall field generation process by that method is isotropic. However, the observed rainfall is non-homogenous and anisotropic. A generated spatial pattern without having a correlated spatial structure of rainfall scenario substantially different from the observed one are the known drawbacks of the method described in section 2, which has limited its popularity and practical application. We improve the method by including the spatial correlation effect in order to incorporate the non-homogenous anisotropy and increase reliability. The new method is named as HSA-method, which refers Hierarchical and Statistical Arrangement method. Fundamentally, this method learns from the neighboring region to understand the sub-grid scale local anisotropy by utilizing the multiple coarse grid scale information and their influence on small scale rainfall field in terms of the spatial correlation.

(1) Spatial correlation of the rainfall field

Generally rain-falling cloud clusters expand to few-hundred-kilometers but rainfall disaggregation is conducted for the detail of a smaller interval of few kilometers because the rainfall events of hydrologic interest are of smaller spatial scales than the scale of cloud clusters. When the disaggregation targets a smaller spatial scale, the neighboring cells of a rained cell possess higher chance of rainfall because they are likely to be encompassed within the same cloud cluster inside or nearby a rainy zone. Therefore, in smaller spatial scale, the rain events are observed to have strong spatial correlation. The spatial correlation in rainfall field corresponds to the rain-cell coverage and movements on its natural way along with the energy dissipation of the cloud mass. Therefore the rainfall disaggregation shall not be treated as a random event irrespective of their organization in space. Existence of spatial correlation in the spatial rainfall fields can be described by a spatial correlation function as the observations have shown the same shape and approximately the same scale sizes[14]. Fig. 3 shows the spatial correlation of the rainfall data over the study area.

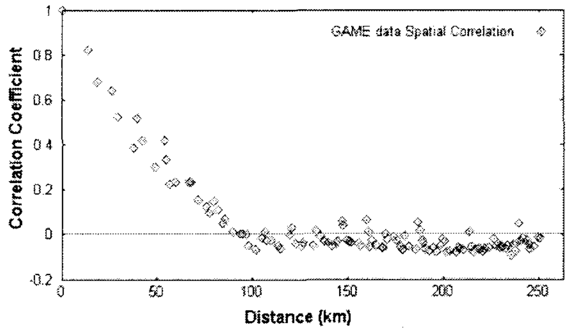


Fig. 3: Spatial Correlation of rainfall data

The spatial correlation gradually decreases upon increase of distance. The decaying shape of the spatial correlation function may be represented by a logarithmic function (equation 8). Its parameters are obtained simply by regression analysis of the observed spatial correlation data. A threshold distance may be set to omit spatial correlation at very far distance that may be out of interest in spatial rainfall disaggregation. This is given by

$$\rho_Z = \begin{cases} \alpha + \kappa \log_\lambda Z & \text{if } (\rho_Z > 0 \text{ and } Z < Z_0) \\ 0 & \text{if } (\rho_Z < 0 \text{ or } Z > Z_0) \end{cases} \qquad (8)$$

$$Z_0 = anti\log_\lambda \frac{\rho_Z - \alpha}{\kappa}, \text{ for } \rho_Z = 0 \qquad (9)$$

where, Z is distance in kilometers; ρ_Z is the spatial correlation value at Z; Z_0 is the threshold beyond which the spatial correlation remains zero by the use of the logarithmic spatial correlation function with α, κ and λ parameters.

(2) Incorporating the spatial correlation effect

In the process of cascading down of a two dimensional (d = 2) spatial field, the b^n numbers of sub-areas, named as Δ_n^i, (i = 1, 2, ..., b^n) are obtained at the n^{th} level with the grid dimension L_0/d^n. For each of these sub-areas, a spatial correlation reference index H is evaluated, which works as a spatial guide matrix. The reference index H is influenced by the average rain intensities R_m of the surrounding eight coarse scale grids (m = 1, 2, …, 8) and corresponding distances of the sub-area from the referred neighbor grid Z_n^m. For n^{th} level, the reference index H can be represented as,

$$H_n^{jk} = \sum_{m=1}^{8} R_m \rho_{Z_n^m} \quad ;\text{for} \quad \begin{matrix} j = 1,2,\ldots,d^n \\ k = 1,2,\ldots,d^n \end{matrix} \tag{10}$$

$$Z_n^m = \sqrt{(x_m - x_j)^2 + (y_m - y_k)^2} \tag{11}$$

where, H_n^{jk} is the non-negative reference index at n^{th} level for jk^{th} sub-area; R_m is the rainfall of m^{th} neighbor cell; $\rho_{Z_n^m}$ is the spatial correlation with the m^{th} neighbor viewed from jk^{th} location at n^{th} level, from where the distance up to the m^{th} neighbor becomes Z_n^m. For the sub-area, the x_j and y_j represents the central point of the sub-area; however, the x_m and y_m for coarse grid area are shifted to the nearest location of central grid area, as shown in Fig. 4, assuming that the average rainfall intensity of the coarse grid has no any specific rain centroid or dominant region inside it.

Sequential assignment of the W_n^i values following the calculation order of current model leads to non-correlated spatial arrangement due to dependence of the W_n^i values on the random variate X. A new form of model is proposed here to improve the haphazard arrangement condition such that,

$$W[\bullet] = BY \tag{12}$$

where, $W[\bullet]$ represents the W with its spatial address $[\bullet]$, which is missing in equation (2). The reference index H_n^{jk} is used to obtain the missing spatial address $[\bullet]$ since it has a clear two dimensional spatial reference j and k inside the sub-dividing region.

At every additional level (n+1), four more random cascade generators, $W_{n+1}^{i'}$ ($i' = b^{n+1}$-3, b^{n+1}-2, b^{n+1}-1, b^{n+1}) appear for newly disaggregated sub-area $\Delta_{n+1}^{i'}$ from Δ_n^i, ($i = b^n$). In the new model, their spatial address $[\bullet]$ is determined on the basis of comparison between the reference indexes $H_{n+1}^{j'k'}$ and the random cascade generators $W_{n+1}^{i'}$. The $W_{n+1}^{i'}$ may need to reshuffle its location within $\Delta_{n+1}^{i'}$ in order to attain same hierarchy of $H_{n+1}^{j'k'}$ locations. This process includes a spatial correlation structure into the cascade generators in successive progress of disaggregation. This process principally does not introduce arbitrary bias to the theoretical consideration of current random cascade method, because the generators $W_{n+1}^{i'}$ are mathematically independent to their spatial locations within the boundary sub-areas Δ_n^i at one level back until the

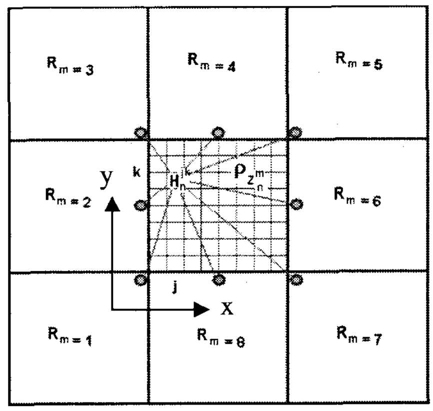
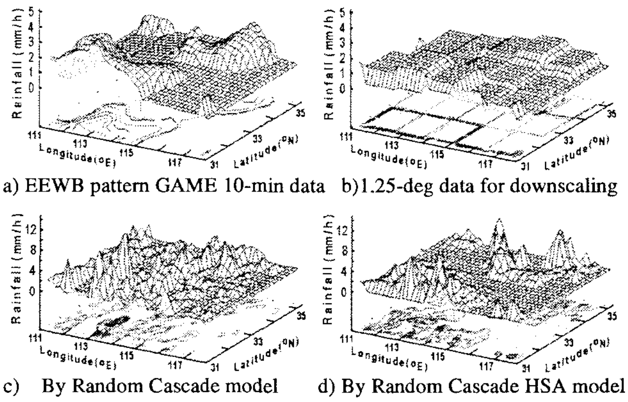


Fig. 4: Look up relation with eight surrounding cells to evaluate the reference matrix

$W_{n+1}^{i'}$ values are generated by equation (2). One relocation operation at this case involves only four W values. Re-allocation of the generators is not permitted in between the sub-grids from different grids to prevent overlapped assignment of $[\bullet]$.

(3) Statistical filter

The spatial H_n field is a smooth gradient surface and its shape is based on the surrounding coarse grid average rainfall. The lowest and highest zones of the H_n field present valuable information as these are most possibly non-rainy and rainy zones respectively. Most of cases, the peak rainy cells of the $\mu_n(\Delta_n)$ field may not be in accordance with the possible rainy zones of H_n field and / or non-rainy cells of the $\mu_n(\Delta_n)$ field may not be in accordance with the possible non-rainy zones of H_n field. Though these extreme high and low value cell numbers may not be significant to influence spatial statistics, they might have practical significance. Therefore at the n^{th} level, spatial locations of the extreme $\mu_n(\Delta_n)$ values are re-adjusted following hierarchical order of H_n field after statistical separation of extreme high and low values from both fields. This process is called as the statistical adjustment. This process might induce some bias, however, it may be considered as a compromise in theory to obtain improved result. The statistical adjustment is omitted if the correlation of H_n field and $\mu_n(\Delta_n)$ field is found higher than a target correlation, which is adapted 80% arbitrarily in this case. The extreme value separation, which takes part in statistical adjustment, is done by statistical measures the mean, standard deviation and a coefficient that widens or narrows the selection of number cells.



a) EEWB pattern GAME 10-min data b) 1.25-deg data for downscaling

c) By Random Cascade model d) By Random Cascade HSA model

Fig. 5 Comparison of downscaled results

4. SIMULATION RESULTS

An experimental data having resolutions of 10-minutes in space and one hour in time are used in testing the procedures described above. This dataset was made by fusing the spatial variability of HUBEX-IOP-EEWB data[15)] having resolutions of 10-minutes in space and one hour in time with GAME Re-analysis data[16)] having resolutions of 1.25-degree in space and six hours in time. Details of the data are given by Shrestha *et al.*[17)].

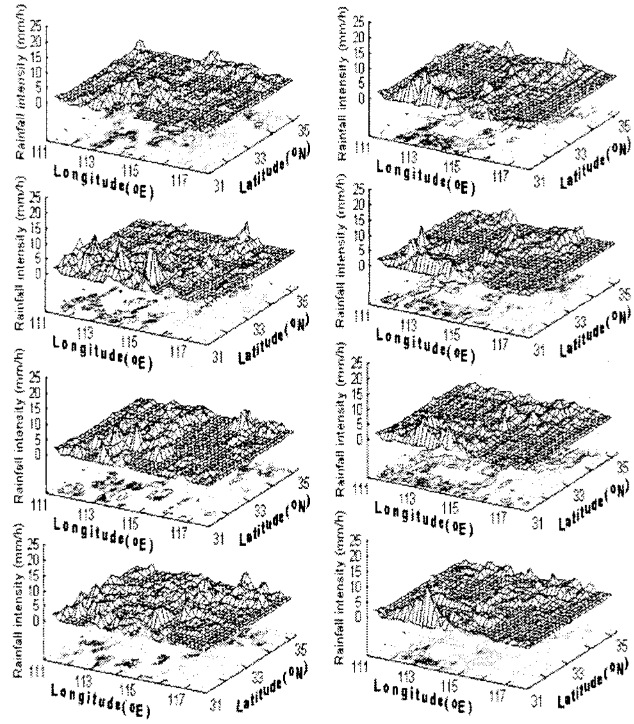
The experimental data (Fig. 5a) is aggregated up to make a coarse data of 1.25-degree resolution (Fig. 5b), which has to employ the downscaling process to obtain a finer resolution data, from which the coarser one is constructed. The degree of ability to reproduce the finer resolution data examines the effectiveness of downscaling method. The test simulation has used 15 grids of 1.25-degree resolution to generate 10-minutes spatial data for four months period at hourly time step.

Feeding Fig. 5b as the input data, the outputs obtained after disaggregation is Fig. 5c using the conventional method and Fig. 5d using the HSA method. Upon the downscaling trial of 2952 events, the average spatial correlation in between the downscaled data and the experimental data is found to be improved to 0.595 using HSA-method from 0.345 using the conventional one, which shows a good improvement in the downscaling performance.

5. DISCUSSIONS

(1) Separation of rainy and non-rainy zones

The Random cascade method including HSA method is found promisingly successful to produce the spatial pattern very similar to that of the source data. Separation of rainy and non-rainy zones was clear and almost exact in space, where it was expected to be appeared (see Fig. 5a and 5d). In the conventional random cascade method, the rainy and non-rainy zones are found mixing together (Fig. 5c), which disables it to produce a clustered region having rain and no-rain. The conventional method generates doughnut shaped rain structure quite often, which never occurs in real rainfall field. The doughnut shaped rain structure did not appear in the random cascade HSA method. The fractional coverage of rainy and non-rainy zones is almost same in both methods. Both methods yield the same result while checking the spatial statistics (e.g. mean and standard deviation) of the generated rainfall field. Thus, it clarifies our argument that the spatial re-organization of the generators, as described in section 3, is not inducing bias. Instead, the method is effective in getting the improved results.



(a) (b)

Fig. 6 Differences in results of repeated trials using (a) RC method and (b) RCHSA method

(2) Results in repeated trial of the experiment

There is no consistency in the spatial patterns those generated by the conventional method of downscaling (compare Fig. 5a and 5c), which forces a user either to depend on ensemble of the result or to get ultimately frustrated with the results. The random cascade HSA method is largely successful to reproduce the same spatial pattern, which is valid even in repeated trials unlike the conventional random cascade method (see Fig. 6a and Fig. 6b). The rainfall magnitudes at a particular cell, however, do not remain exactly same in the repeated trials in the random cascade HSA method too, as they depend upon the generators based on the cascade of multiplication of the random numbers inside the model despite their superb capability of preserving the same spatial patterns. We argue this as a beneficial point of the suggested method to infer the probability of uncertainty in rainfall phenomenon, which might help to understand the consequences of rainfall uncertainty in rainfall-runoff modeling.

6. CONCLUSION

The spatial rainfall disaggregation method based on the multiplicative random cascade theory is able to downscale the rainfall field from GCM output scale to finer scales. This method is able to describe the spatial statistics of disaggregated rainfall field properly but they are not sufficient property of the disaggregated rainfall field to become practically useful, especially in short time scale hydrologic analysis. This method is not able to model the existence of the spatial correlation in the rainfall field. Another drawback of the method is having the nature of generating significantly different spatial patterns of rainfall in repeated trials. This happens due to the cascades of multiplication of generators, which are associated with random numbers. The generators are highly affected by the random numbers.

With a modification in the cascade development method, those limitations are possible to remove effectively. We illustrated here the need of spatial information for the generators, which can be incorporated through a spatial correlation function in the method. The method effectively uses the coarse resolution rainfall field to derive a reference index matrix and the consecutively guide the spatial arrangement of the generators. This method is names as the HSA mèthod. The results are improved drastically by employing the proposed modification in the random cascade method, which demonstrates the inevitable consideration of spatial correlation of rainfall field that exists at finer scale. The test was conducted over eastern Chinese region covering 560 km X 320 km area.

The random cascade HSA method is found quite successful to separate rainy and non-rainy zones properly. This method reproduced similar spatial patterns of the rainfall field in every realization as it was expected. The average of the spatial correlation coefficient between the experimental data and simulated rainfall field is found to be 0.59, which was quite higher than that obtained by the conventional random cascade method. The ability of reproducibility and increased accuracy of the spatial rainfall modeling opens up chances to use the downscaled rainfall data in practical hydrological analyses of shorter time scale such as rainfall-runoff analyses.

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