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AN APPROACH TO UNCERTAINTY IDENTIFICATION IN HYDROLOGICAL MODELING

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We propose a methodology to identify prediction uncertainty through recognizing and quantifying the different uncertainty sources in a hydrologic model. Statistical second moment is used as a measure of uncertainty; also an index which originated from Nash coefficient of efficiency named Model Structure Indicating Index (MSII) is proposed to quantify model structure uncertainty. The results show that MSII can well reflect the goodness of model structure, while a larger value of MSII indicating a poorer structure of hydrologic model. The index can be used as a tool for implementing model quantitative comparison (selection).

Key Words: *Prediction Uncertainty, Monte Carlo Simulation, hydrological modeling*

1. INTRODUCTION

Hydrological modeling is the discovery of general laws and principles that govern the natural phenomenon under observation. A hydrologic model is an integration of mathematical descriptions of conceptualized hydrologic processes, which serves for a specific purpose. Consequently, the spatial scale, temporal scale, structure, architecture, and applicability of a model are restricted a lot by the hypothesis of the hydrologic model in most of the cases. As a result, there are numerous hydrologic models developed for various aspects, and the development of new hydrologic models or improvement of previously developed models continues in Japan and elsewhere. With rapid advances in computing technology, remote sensing, GIS and DBMS, the role of hydrologic models is enhanced as a tool in planning, decision making and tends to incorporating with other process models such like economic, social, political, administrative, and judicial models. Thus, the watershed hydrologic models will become a component in a larger management strategy. Furthermore, these models will become more global, not only in the sense of spatial scale but also in the sense of hydrologic details[1)]. Therefore, a methodology to assess the error, uncertainty and adequacy of adopting hydrologic models in specific purpose is needed. The present study provides a methodology for model comparison and selection through model uncertainty recognition and quantification.

For many years hydrologists have been interested in the effects of various uncertainties on the accuracy and reliability of the estimation of catchment hydrological variables such as peak flow and flood volume[2)]. Among early contributions, some of them focus on the rainfall uncertainty and its influence to the runoff (e.g., Storm *et al.*[3)]); some of them focus on the sensitivity of model structure due to the input error (e.g., Singh and Woolhiser[4)]). Recent researches relating to hydrologic model uncertainty most refer to parameter uncertainty identification, the procedure of parameter calibration, and their impact to simulation result[5)6)]. Among them, Generalized Likelihood Uncertainty Estimation (GLUE) Methodology[5)] offers a path of identifying parameter uncertainty. Nevertheless, parameter equifinality became the conclusion of GLUE; uncertainty related to input data and other factors are excluded. Even it is said that they could also be included in GLUE but this has not normally been done[7)].

The performance of hydrologic models is profoundly affected by the sources of uncertainty, briefly they are:

(a) Observed data,
(b) Data for model calibration,
(c) Parameter space, and
(d) Model structure.

Among those, data uncertainty occupies the most and contaminates other sources of uncertainty.

Underestimating or misunderstanding of these uncertainty sources and the interrelation among them may cause tremendous misleading on interpreting the result of hydrologic models. However, recognizing and quantifying these different sources of uncertainty in hydrologic models has received little attention in the research literature.

In this study, a methodology is proposed to recognize and quantify the different uncertainty sources. Firstly, Monte Carlo simulation method is applied to add bias item in model input data series (rainfall), then rainfall realizations, parameter space, and model outcomes (outflow discharge) under different bias level are acquired. Secondly, by examining the counter relationship between model simulation outcomes, calibration outcomes and observed watershed response series (discharge), an uncertainty structure is recognized. Finally, parameter uncertainty, calibration uncertainty, and model structure uncertainty caused by input data uncertainty are recognized, separated, and quantified through the methodology.

Statistical second moment is used as a measure of uncertainty, also an index which originated from Nash coefficient named Model Structure Indicating Index (MSII) is proposed to quantify model structure uncertainty which can be used as a tool for implementing model quantitative comparison (selection). For the demostration of the proposed method, a conceptual hydrologic model named Storage Function Method (SFM) is employed. Through fixing the value of one parameter of SFM, a poorer structure model is formed as a contradistinction in performing model comparison (selection). The results show that a larger value of MSII indicating a poorer structure of hydrologic model in a dynamic manner, that is, incorporating the MSII to the input uncertainty.

2. UNCERTAINTY RECOGNITION IN HYDROLOGICAL MODELING

In this study, prediction uncertainty which came from the four kinds of sources mentioned previously is classified into four categories: system uncertainty, entire uncertainty, inherent uncertainty, and structure uncertainty. The definition and the procedure to recognize them are described below.

(1) System uncertainty

Even it is well known that a hydrologic model is an approximation of the real phenomena based on the hydrologic cycle, still it is needed to stress that there always exiting discrepancy between model outcome and observed data, no matter how precise the model is and how perfect the model calibrated. That is the model predicting limitation underneath the hypothesis and architecture of the model, which is defined as the system uncertainty in this study.

The system uncertainty can be recognized by evaluating the discrepancy between observed watershed response series and the model outcome during the process of model parameter calibration. This can be clarified by the statement written below:

$$\tilde{y} = f(\tilde{x}, \theta) \tag{1}$$

where $\tilde{y}$ and $\tilde{x}$ denote outcome and input series of a hydrologic model $f(\cdot)$ respectively, and θ denotes a parameter set. Denoting X and Y are observed watershed input and response series data accordingly and assuming $\tilde{x} = X$, then θ can be determined by adjusting or tuning the value of parameters to make the difference between model outcome $\tilde{y}$ and observed watershed response data Y in an acceptable range according to some objective function. The procedure of adjusting the value of parameter set is well known as "model calibration". In this study, least sum of square error (LSE) method is selected as the objective function:

$$S_{LS}(\theta_m) = \sum_{n=1}^{N} (Y_n - f(X_n, \theta_m))^2 \tag{2}$$

where n denotes time step of the simulation time series; m denotes the parameter set to be examined.

The discrepancy between observed watershed response and model outcome during the process of calibration is supposed to be the minimum value by comparison to the possible coming events. This has often been observed that the goodness-of-fit between observed data and estimated data during calibration is better than what in validation, not to mention in implementation. Under this fact, we can say that the uncertainty occurring here denotes the predicting limitation of the model. Since this is the best that a model can achieve.

(2) Entire uncertainty

The discrepancy of the model outcome to the observed data can be shown as below:

$$Y = f(X, \theta) + \varepsilon \tag{3}$$

where ε denotes system uncertainty mentioned above, that is, the predicting limitation of the model under perfect calibration. In agreement with this definition, it is clear that the uncertainty came from the process of calibration. But the real uncertainty source is supposed to be the data which is used for calibration.

After calibrating the model parameter, the calibrated parameter space will reflects its uncertainty through the model structure and propagates to the model outcome. This uncertainty can be recognized by examining the discrepancy between observed watershed response data and model outcome by using input data and parameter sets. Also this is the way of most of the researches dealing with parameter sensitivity. This uncertainty is defined as the entire uncertainty in this study, since actually this is the utmost uncertainty a model could has under existing input uncertainty.

(3) Inherent uncertainty

Another categorized uncertainty what we called "inherent uncertainty" is very difficult to be recognized since it always contaminated by the uncertainty of model structure and input data. Inherent uncertainty is different to what so called parameter uncertainty since the propagation effect of model structure is excluded here intentionally. Inherent uncertainty represents the sensitivity of parameter space which determined according to the input uncertainty and reflects to model outcomes. This can be examined by the discrepancy among model outcomes derived from different best fit parameter sets. Watershed response data is not used here, which indicates the model structure uncertainty is eliminated as much as possible.

(4) Structure uncertainty

The distance between entire uncertainty and inherently uncertainty is structure uncertainty. Since the parameter set used for extracting both of the uncertainty is the same. The different is the data used for comparison: observed data and outcome during calibration. The propagation effect plays an important role here, which is why the difference between entire uncertainty and inherent uncertainty is called structure uncertainty in this study.

The schematic diagram of uncertainty structure proposed here is depicted in **Fig.1**, which reveals a static view of model uncertainty structure, that is, the input uncertainty (data uncertainty) is neglected or fixed to a certain magnitude. In order to know the behavior of model structure uncertainty under different input uncertainty level, a dynamic view of model uncertainty structure is introduced by increasing the input uncertainty during the processes of uncertainty recognition. The behavior of model structure uncertainty under

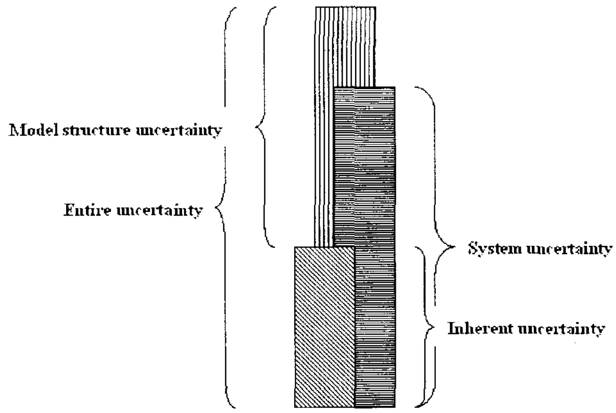

Fig.1 Schematic diagram of the uncertainty structure.

different input uncertainty can be examined by observing the discrepancy among different model outcomes accordingly.

3. UNCERTAINTY QUANTIFICATION

Several indexes are selected to perform uncertainty quantification in this study; the basic form of the index is statistical second moment and Nash coefficient. The categorized prediction uncertainties defined in previous section are mathematically described as below:

(1) Index for quantifying system uncertainty

$$SD_s = \sqrt{\frac{\sum((Q_o - Q_b) - \overline{(Q_o - Q_b)})^2}{n}} \tag{4}$$

$$Nash_s = 1 - \frac{\sqrt{\sum(Q_o - Q_b)^2}}{\sqrt{\sum(Q_o - \overline{Q}_o)^2}} \tag{5}$$

where Q_o and Q_b indicate the observed watershed response series and model outcome by using the best fit parameter set, n is the time step of the time series. SD_s quantifies the error variance, the residuals, between observed watershed response and the best fitted simulated outcomes; while $Nash_s$ elucidates the performance of the best fitted simulated outcomes. SD always larger than 0, while the range of $Nash$ coefficient is: $-\infty < Nash \leq 1$.

(2) Index for quantifying entire uncertainty

$$SD_e = \sqrt{\frac{\sum((Q_o - Q_e) - \overline{(Q_o - Q_e)})^2}{n}} \tag{6}$$

$$Nash_e = 1 - \frac{\sqrt{\sum (Q_o - Q_e)^2}}{\sqrt{\sum (Q_o - \overline{Q}_o)^2}} \quad (7)$$

where Q_e is the model outcomes acquired by using a parameter set within the whole parameter space and entire rainfall realizations. Consequently, SD_e explains the total prediction error.

(3) Index for quantifying inherent uncertainty

$$SD_i = \sqrt{\frac{\sum ((Q_b - Q_e) - \overline{(Q_b - Q_e)})^2}{n}} \quad (8)$$

$$Nash_i = 1 - \frac{\sqrt{\sum (Q_b - Q_e)^2}}{\sqrt{\sum (Q_b - \overline{Q}_b)^2}} \quad (9)$$

SD_i and $Nash_i$ explain the inherent uncertainty because Q_e is calculated with a possible parameter set identified by other model outcomes, thus, the discrepancy between Q_b and Q_e indicates the error inevitable in a model formulation.

(4) Index for structure uncertainty

In order to implement dynamic view of the relationship among system uncertainty, structure uncertainty and parameter uncertainty caused by input data uncertainty, the Nash coefficient is used to formulate a Model Structure Indicating Index (MSII) and is defined as:

$$MSII = \frac{Nash_i - Nash_e}{Nash_s} \quad (10)$$

The difference between entire and inherent uncertainty denotes the structure uncertainty. This is used as the numerator in the equation, while system uncertainty is used as a denominator to form a criterion to evaluate model structure. The index expresses the total variance unexplained by the standard deviation in a dimensionless form. The range of MSII is: $0 \le MSII < \infty$.

4. ALGORITHM FOR UNCERTAINTY RECOGNITION & QUANTIFICATION

Instead of sampling the parameter space directly like what GLUE did, the study here generates the parameter set space by introducing noise item into input data with specified probability distribution. Here Normal distribution with mean equals to zero and standard deviation from 1.0 to 9.0 (mm/hr) is used to acquire model parameter space and outcomes under different input uncertainty. For each iteration, 10000 model outcomes for each specified input uncertainty were derived from the combination of rainfall series and parameter set generate output series through the model. The system uncertainty and the prediction ability were identified and recognized by corresponding parameter set. Detail algorithm for uncertainty recognition and quantification is listed in **Fig.2.**

For input uncertainty σ_x = 1.0, 2.0,..., 8.0, 9.0 (unit: mm/hr)

1. Use Monte Carlo simulation to sample 100 rainfall realizations according to a real recorded event by adding noise item to it.
2. Use LSE to determine 100 parameter sets regarding to its corresponding rainfall realization.
   $$S_{LS}(\theta_j) = \min(\sum_{n=1}^{N} (Y_n - f_n(\bar{x} + \xi_j, \theta_j))^2), j = 1,...,100$$
3. System uncertainty ε_s is acquired by examining the discrepancy between 100 best fit model outcomes during calibration and observed watershed response. Mean value of SD_s and $Nash_s$ for 100 cases are used for uncertainty quantification.
   $$Y = f(\bar{x} + \xi_j, \theta_j) + \varepsilon_s, j = 1,...,100$$
4. Entire uncertainty ε_e is acquired by examining the discrepancy between observed watershed response and 10000 model outcomes. Mean value of SD_e and $Nash_e$ for 10000 cases are used for uncertainty quantification.
   $$Y = f(\bar{x} + \xi_j, \theta_j) + \varepsilon_e;$$
   $$j = 1,...,100; k = 1,...,100$$
5. Inherent uncertainty ε_i is acquired by examining the discrepancy between 100 best fit model outcome during calibration and the rest 9900 model outcome. Mean value of SD_i and $Nash_i$ for 10000 cases are used for uncertainty quantification.
   $$y_l = f(\bar{x} + \xi_l, \theta_k) + \varepsilon_i, l \ne k;$$
   $$l = 1,...100; k = 1,...,100$$
   $$y_l = f_l(\bar{x} + \xi_l, \theta_l), l = 1,...,100$$
6. Apply Eq. (10) to acquire MSII of the Model.

End

where $\xi \sim N(0, \sigma_x^{\,2})$, n denotes time step of the simulation time series, $\bar{x}$ denotes the observed input data series.

Fig.2 Algorithm for uncertainty recognition and quantification.

5. MODEL DESCRIPTION AND PROCESS OF OBSERVED DATA

To demonstrate uncertainty identification, Storage Function Model (SFM) proposed by Kimura[8] is applied at the Yasu River basin (355km^2) in this study. The form of SFM is as:

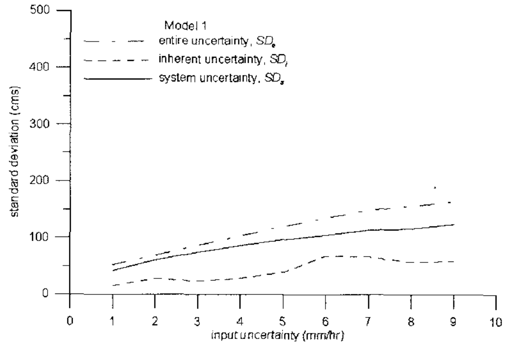

Fig.3 Standard deviation of model 1.

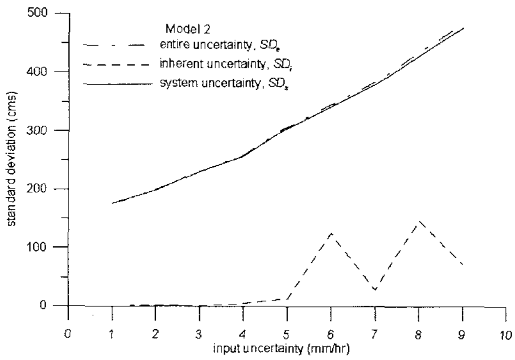

Fig.4 Standard deviation of model 2.

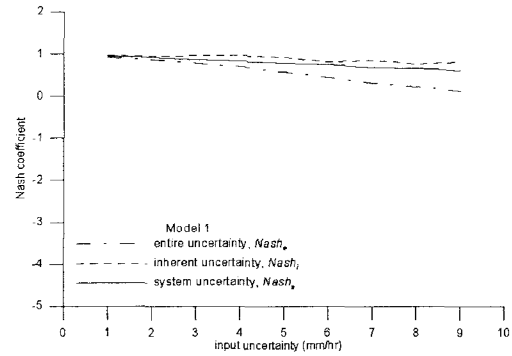

Fig.5 Nash coefficient of Model 1.

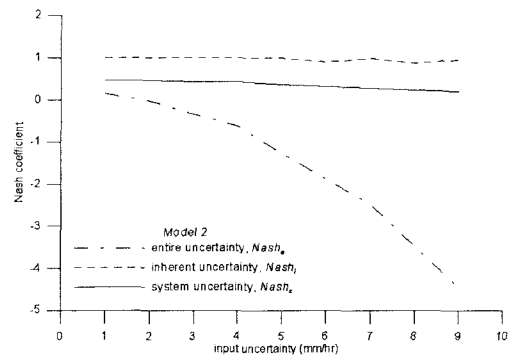

Fig.6 Nash coefficient of Model 2.

$$\frac{dS}{dt} = r_e(t - T_l) - q, \quad S = kq^p$$

$$r_e = \begin{cases} f \times r, & if \ \sum r \le R_{SA} \\ r, & if \ \sum r > R_{SA} \end{cases} \qquad (11)$$

where S = water storage height; r = rainfall intensity; q = runoff height; t = time step; and T_l = the lag-time. This model is often used for the flood runoff calculation in a basin with an area of less than five hundred square kilometers in Japan[9].

Parameter p is a constant, commonly the value is 0.6, f is the ratio of contribution area of the watershed which generates outflow, R_{SA} is accumulating saturated rainfall. Parameter k is solved by the equation proposed by Nagai *et al.*[10].

A fully functional SFM with parameter T_l, f, and R_{SA} is called Model 1 in this study. By fixing the value of R_{SA} to 0.0, a poorer-structured model with comparison to the original one called Model 2 is manipulated as a contradistinction in this study.

Rainfall data was collected from four rainfall gauging stations inside the Yasu River basin (355km^2); they are Yasu, Minakuchi, Kouka and Oogawara. The average precipitation was obtained by using Thiessen polygon method.

6. RESULTS

In **Fig.3** and **Fig.4**, the standard deviation of entire, inherent and system uncertainty of model 1 and 2 are shown. The larger magnitude of entire uncertainty indicates a broader range of model outcomes. Which also indicates a poorer model performance; also this can be examined in **Fig.5** and **Fig.6**: the Nash coefficient of entire, inherent and system uncertainty of Model 1 and Model 2. It is always expected that entire uncertainty increases while input uncertainty increasing; where the discrepancy between entire and inherent uncertainty is in the same tendency. System uncertainty is supposed located in the middle between entire and inherent uncertainty. The range between entire and inherent uncertainty indicates the goodness of a model structure; that is, the ability of a model simulates the behavior of the watershed through observed data. The broader the distance is the worse structure the model has. In **Fig.5** and **Fig.6**, the range between inherent and entire uncertainty of Model 1 is far smaller than Model 2. This can be interpreted that the parameter sensitivity of Model 2 is worse than Model 1.

The reason can be explained by **Fig.7** and **Fig.8**, which show the upper and lower bound of entire model outcomes and model outcomes during calibration of Model 1 and Model 2 under input uncertainty 4.0 mm/hr. It shows that Model 2 is incapable to simulate the watershed behavior as Model 1 does. Even the range inside upper and lower bound of Model 2 is far larger than Model 1, the true watershed response is out side the range. This indicates no matter how perfect the calibration is, predicting limitation of Model 2 is worse than model 1. MSII of Model 1 and Model 2 are

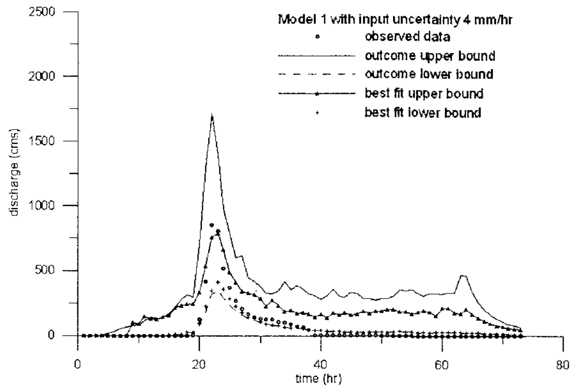

Fig.7 Upper and lower bound of Model 1 outcome with input uncertainty 4 mm/hr.

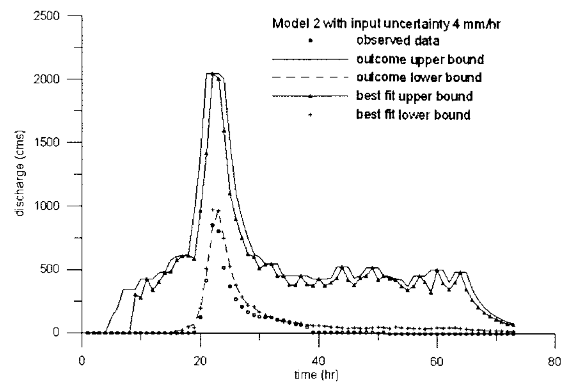

Fig.8 Upper and lower bound of Model 2 outcome with input uncertainty 4 mm/hr.

illustrated in **Fig.9**. It shows that the gradient of MSII of Model 2 increasing more rapidly than Model 1. Which not only reveal the goodness of hydrologic model in a static view but also reflects the input uncertainty propagates itself through model structure in a dynamic manner.

7. CONCLUSIONS

A methodology for uncertainty recognition and quantification is proposed which can be used as a tool for hydrologic models quantitative comparison or evaluation. Instead of sampling the parameter space directly like what GLUE did, the study here generates the parameter set space by introducing noise item into input data with specified probability distribution. This reflects the truth that parameter uncertainty came from uncertainty of data to hand and the way the model structure responses it. The results show that within increasing input uncertainty, the distance between entire and inherent uncertainty is also increased. A smaller magnitude of the ratio of inherent uncertainty to entire uncertainty indicates the system uncertainty (i.e. the prediction limitation) is larger, which means lacking of the ability to simulate the true watershed behavior. MSII is proposed to evaluate the goodness of model structure. The results show that

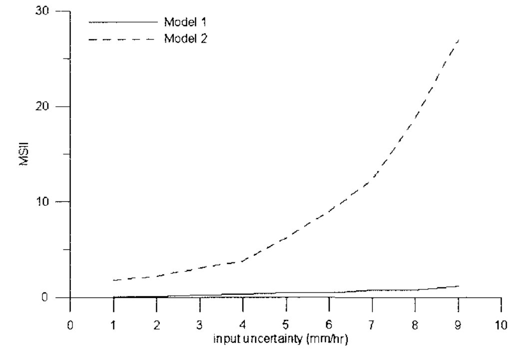

Fig. 9 MSII of Model 1 and Model 2.

a larger value of MSII indicates a poorer structure of hydrologic model, within increasing input uncertainty the tendency becomes more apparently. The index can be used as a tool for implementing model quantitative comparison (selection).

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