

# MODELING OF TOTAL SEDIMENT LOAD TRANSPORT IN ALLUVIAL RIVERS

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Study of total sediment load transport is an important consideration of the fluvial process in many nowadays rivers. In order to improve the prediction of sediment transport and morphological behavior of the channel bed, a mathematical model taking bed load, suspended load and wash load into account has been developed. In contrast to previous researches, wash load has been especially accentuated from the perspective of cohesive sediment transport. For a better simulation of the interrelationship between different sediment loads and the river bed, cohesive and non-cohesive regimes were distinguished for the bed depending on the critical clay content. Two cases of with and without wash load have been performed using a 1D numerical modeling approach. The comparison between the computational results and field data exhibited its advantages over the conventional models.

**Key Words :** wash load, cohesive sediment, bed regime, numerical modeling

## 1. INTRODUCTION

An important aspect of fluvial process is the movement of sediment in alluvial rivers, to which river morphology and river channel changes are closely related<sup>1)</sup>. The past decades have witnessed the great development in coarse sediment transport which is considered to be the dominated factor of the channel formation. Although all of the published sediment transport formulae are not accomplished following purely theoretical pursuit, it is usually able to find a formula meeting specific engineering requirements.

Nevertheless, with a large amount of construction of hydraulic structures during the campaign of river training and river restoration, fine sediment transport is attracting more and more attention. Besides its highly environmental relevance due to collecting nutrients and pollutants, fine sediment is also observed to play an important role in the bed evolution. As such sediment is generally considered as wash load and is neglected by most forecast models in the state-of-the-art of river engineering practices, it in many cases leads to questionable results. In order to improve the

prediction of fluvial process in nowadays rivers facing more and more prominent problems of coarse as well as fine sediment, it is both necessary and important to take wash load into account in the modeling of total sediment load transport.

Preliminary study of wash load transport has been carried out by some researchers, e.g. Zhong et al<sup>2)</sup> and Umeda et al<sup>3)</sup>. Most of the theories to explain the motion mechanism of wash load have been put forward based on a common presumption that wash load moves in dispersed particles. They seemed to be successful in explaining the phenomena of wash load being carried out by water and having nearly no exchange with the local bed in some natural rivers. However, when employed to predict sedimentation problems due to more complex flow conditions caused by hydraulic structures, they were usually thrown into trouble. As is well-known that cohesiveness increases with the decreasing of the particle size, more physically rational explanation seems to consider the cohesiveness between fine particles, which has been touched by some researchers<sup>1), 4)</sup>. But unfortunately, little investigation has been furthered either because of its sophistication or in their mind wash load is

independent of the flow and seldom participates bed shaping processes.

In this paper, the overall sediment is classified into bed load, suspended load and wash load. Hence the flow domain is subdivided into two layers. Bed load is confined to move in a thin layer in the proximity of the river bed, above which is the region occupied by suspended load and wash load. The exchange of sediment between the two layers is through the upward and downward fluxes at the interface. But the result of sediment deposition and erosion occurs on the bed directly. Suspended load and wash load are obviously distinguished due to their different motion mechanism. The non-cohesive sediment, dislodged mainly by gravity and moving individually, is considered as suspended load, which coincides with the traditional definition. While cohesive sediment, tending to aggregate owing to cohesiveness and forming larger units of floc, is categorized into wash load.

## 2. MODEL DESCRIPTION

Models for sediment transport in alluvial rivers generally consist of three modules: hydrodynamics, sediment transport and bed variation. In order to simplify the calculation due to the wide spectrum of sediment size distribution, the bed materials are divided into a series of fractions.

### (1) Flow field

For long-term prediction of river bed evolution, such simplification is generally made that the flow is assumed to be steady and non-uniform. The continuity and momentum equations governing the 1D open channel flow are presented as below.

$$\frac{\partial Q}{\partial x} = q \quad (1)$$

$$\frac{1}{gA} \frac{\partial}{\partial x} \left( \frac{Q^2}{A} \right) + \frac{\partial}{\partial x} (z_b + h) + S_f = 0 \quad (2)$$

where  $Q$  = discharge;  $x$  = longitudinal coordinate;  $q$  = lateral inflow;  $g$  = gravitational acceleration;  $A$  = cross-sectional area;  $z_b$  = elevation of the channel bed;  $h$  = water depth;  $S_f$  = energy slope.

### (2) Bed regime

Erosion experiments related to bed cohesiveness have been carried out by several groups<sup>5), 6)</sup>. Adding cohesive sediment to a coarse bed, Torfs observed that the non-cohesive behavior of the bed was increasingly suppressed. Suggested by van Ladden<sup>5)</sup>, cohesive and non-cohesive bed regimes could be distinguished according to the bed composition.

And the clay content in the bed surface was considered to be the governing parameter for the transition of both regimes.

In order to evaluate the influence of different bed regimes on sediment transport quantitatively, a new erosion mechanism is introduced here. We assume that any river bed is made from a non-cohesive bed and a cohesive matrix. The cohesive matrix has some structural strength that can prevent the sediment from removing. Then the erosion rate is determined by both the erodibility from the non-cohesive bed and the possibility of the sediment passing through the cohesive matrix. The classic formulae are employed to estimate the erosion amount from the non-cohesive bed, but it is not explicitly linked to the erosion rate. And the aforementioned possibility is related to the strength of the cohesive matrix. As a preliminary study, we introduce a bed regime function in this paper to evaluate this possibility as follows:

$$f_b = \begin{cases} 1 & \text{if } p_{\text{clay}} < p_c \\ \frac{1 - p_{\text{clay}}}{1 - p_c} & \text{if } p_{\text{clay}} \geq p_c \end{cases} \quad (3)$$

where  $f_b$  = bed regime function;  $p_{\text{clay}}$  = clay content in the bed surface;  $p_c$  = critical clay content. For a non-cohesive bed, the strength of the cohesive matrix is so small that the sediment can pass it freely, i.e.  $f_b = 1$ , the classical formulae are still valid. While for a cohesive bed, some of the sediment is not fortunate enough to pass the matrix, then  $f_b < 1$ .

### (3) Bed load transport

Among numerous formulae, the approach for bed load transport rate proposed by Ashida and Michiue<sup>7)</sup> has achieved a broad application to the rivers in Japan. Considering different bed regimes, the Ashida-Michiue Formula for each size fraction of bed load is modified as below.

$$\frac{q_{bk}}{\sqrt{sgd_k^3}} = 17 f_b p_{bk} \tau_{*ek}^{3/2} \left( 1 - \frac{u_{*ck}}{u_*} \right) \left( 1 - \frac{\tau_{*ck}}{\tau_{*k}} \right) \quad (4)$$

where  $k$  = sediment size fraction (here only bed load fraction);  $q_{bk}$  = bed load discharge for fraction  $k$ ;  $s$  = specific gravity of sediment;  $d_k$  = diameter of fraction  $k$ ;  $p_{bk}$  = percentage of fraction  $k$  in the bed composition;  $\tau_{*k}$ ,  $\tau_{*ck}$ ,  $\tau_{*ek}$  = dimensionless shear stress, critical shear stress and effective shear stress for size fraction  $k$ , respectively;  $u_*$ ,  $u_{*ck}$  = friction velocity and critical friction velocity for size fraction  $k$ , respectively. To calculate the critical friction velocity for each size fraction, the following equations firstly contributed by Egiazafoff and later extended by Ashida and Michiue are adopted.

$$\frac{u_{*ck}^2}{u_{*cm}^2} = \begin{cases} 0.85 & \text{if } d_k / d_m < 0.4 \\ \frac{1.64}{[\lg(19d_k / d_m)]^2} & \text{if } d_k / d_m \geq 0.4 \end{cases} \quad (5)$$

where  $m$  = subscript expressing mean size. The famous Iwagaki formula is used to calculate the critical friction velocity  $u_{*cm}$  for the mean size  $d_m$ .

#### (4) Suspended load transport

The similarity between suspended load and wash load is that they are both kept aloft in the water column by the flow during transportation. In relatively low sediment-concentrated flow, suspended load and wash load are assumed to behave individually in the water column, which can be described by the advection-diffusion equation. A 1D form yields

$$\frac{\partial}{\partial t}(AC) = \frac{\partial}{\partial x} \left[ A \left( \varepsilon_{sx} \frac{\partial C}{\partial x} - uC \right) \right] + B_s(E - D) \quad (6)$$

where  $C$  = mean volumetric sediment concentration;  $t$  = time;  $\varepsilon_{sx}$  = diffusion coefficient in longitudinal direction;  $u$  = water velocity;  $B_s$  = sediment deposition width;  $E$  = near-bed erosion flux;  $D$  = near-bed deposition flux.

Eq. (6) is assumed to be valid for each size fraction. As is readily seen, the near-bed fluxes appear as source terms and need to be evaluated. There is evidence that near-bed fluxes are generated by the simultaneous erosion and deposition, but from the perspective of mathematical modeling, it is preferred to consider the two processes discontinuously.

The deposition flux is calculated from

$$D_s = w_s C_a \quad (7)$$

where  $w_s$  is the sediment settling velocity and  $C_a$  is the ambient concentration near the bed.

While for the erosion rate, it is generally assumed to be the one under equilibrium condition, that is

$$E_s = w_s C_e \quad (8)$$

where  $C_e$  is the equilibrium concentration near the bed. It is noticed that the cohesive matrix has some effect on the equilibrium concentration. As a result the equilibrium concentration derived from the classic formulae (e.g. the formulae proposed by Ashida and Michiue) should be multiplied by the bed regime function.

The boundary conditions for the equation set are specified as follows. An equilibrium sediment concentration profile is given at the upstream inflow boundary. The well-known Lane-Kalinske formula

is used to determine this profile.

$$\frac{C}{C_e} = \exp \left[ -15 \left( \frac{z - z_a}{h} \right) \left( \frac{w_s}{u_*} \right) \right] \quad (9)$$

where  $z_a$  = reference level (in this model  $z_a = 0.05h$ , is the same as the interface of the bed load layer and the suspended load layer);  $C_e$  = near-bed equilibrium concentration;  $z$  = depth in vertical direction.

For the downstream outflow boundary, the gradient of the sediment concentration is assumed to be zero.

#### (5) Wash load transport

Comparing with the movement of suspended load dislodged under gravity, the inter-particle cohesiveness becomes crucially important for the wash load transport. During transportation, particles of wash load tend to collect together and form larger units of flocs. Thus an individual floc contains appreciable sub-population of grain sizes. Particles contained by a floc move as a whole at the same velocity which might be much larger than individual particles. When a floc deposits, it breaks into pieces.

As has mentioned before, the movement of wash load in the water column still satisfies the advection-diffusion equation, i.e. Eq. (6), which can be adopted to determine the mean concentration of all fractions of wash load.

When solving the advection-diffusion equation, the main difference between suspended load and wash load lies in the setting of the upstream inflow boundary and near-bed fluxes. On the inflow boundary, wash load is considered to have a uniform distribution in the vertical direction and is attributed a specific value resulting from experimental input values or regressive results of the field data. The near bed deposition flux and erosion flux are discussed below more in detail.

##### a) Deposition flux

Deposition of wash load is considered as a stochastic process following Krone's law depending on the settling velocity of the floc and whether or not it is capable of surviving the near bed shear stress<sup>8)</sup>. The mathematical form of the deposition flux of wash load is

$$D_w = \begin{cases} C_w w_f \left( 1 - \frac{\tau_b}{\tau_{cd}} \right) & \text{for } \tau_b < \tau_{cd} \\ 0 & \text{for } \tau_b \geq \tau_{cd} \end{cases} \quad (10)$$

where  $D_w$  = deposition flux of wash load;  $C_w$  = volumetric concentration of wash load;  $w_f$  = settling velocity of the floc;  $\tau_b$  = near-bed shear stress;  $\tau_{cd}$  = critical shear stress for deposition.

In the above formula, the settling velocity of the floc  $w_f$  is an important factor that needs to be determined. Experimental methods are generally employed. The behavior of flocs is found to have strongly dependence on the sediment concentration<sup>8)</sup>. At low concentrations flocs exist as individual units but joint together at higher concentration to form a network. As a result, the settling velocity increases with concentration at low concentrations, then attains a maximum value and thereafter decreases at large concentrations. A general form of the settling velocity of a floc dominated by the sediment concentration is

$$w_f = \begin{cases} K_1 c^m & \text{for } c < c_h \\ K_1 c_h^m [1.0 - K_2 (c - c_h)]^{m_1} & \text{for } c \geq c_h \end{cases} \quad (11)$$

where  $c$  = massive concentration of wash load (i.e.  $c = \sigma C_w$ , and  $\sigma$  is the sediment density,  $2.65 \text{ g/cm}^3$  in this paper);  $c_h$  = onset concentration of hindered settling;  $K_1, K_2$  = coefficients depending on particle mineralogy;  $m, m_1$  = coefficients depending on particle size and shape.

After deposition, the floc turns into dispersed particles. In order to calculate the deposition flux of each size fraction, it is therefore necessary to estimate the size distribution of a broken floc. Assuming the percentage of a specific size fraction in the floc is governed by its concentration and proportional to the concentration, the following relation could be obtained

$$D_{wk} = \frac{C_{wk}}{C_w} D_w \quad (12)$$

where  $D_{wk}$  = deposition flux for size fraction  $k$ ;  $C_{wk}$  = volumetric concentration for size fraction  $k$ .

#### b) Erosion flux

The basic idea to evaluate the erosive amount follows Partheniades' algorithm<sup>8), 9)</sup>. According to his experimental result, the erosion rate is proportional to the excess of the shear stress over the critical erosive shear stress normalized with respect to the latter. Considering the effect of bed regime and the ratio of each fraction of wash load available from the bed, we have

$$E_{wk} = \begin{cases} 0 & \text{for } \tau_b < \tau_{cek} \\ m_{ek} f_b p_{bk} \left( \frac{\tau_b}{\tau_{cek}} - 1 \right) & \text{for } \tau_b \geq \tau_{cek} \end{cases} \quad (13)$$

where  $E_{wk}$  = erosion flux for size fraction  $k$ ;  $m_{ek}$  = erosion parameter for size fraction  $k$ ;  $\tau_{cek}$  = critical erosive shear stress for size fraction  $k$ .

#### (6) Bed variation

Since the bed morphology is molded by total sediment load transport, the mass conservation of

sediment is adapted to the following form

$$(1 - \lambda) \frac{\partial z_b}{\partial t} + \frac{B_s}{B} \left[ \frac{\partial q_b}{\partial x} + (E_s - D_s) + (E_w - D_w) \right] = 0 \quad (14)$$

where  $B$  = river bed width;  $\lambda$  = porosity of sediment;  $q_b$  = bed load transport rate.

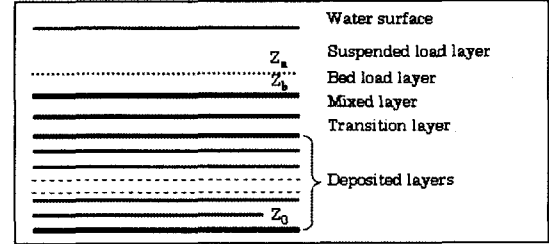


Fig.1 Vertical profile of a cross-section.

Simulation of the bed sorting process has followed the method proposed by Liu<sup>10)</sup>. The natural bed is discretized vertically into a series of layers above a datum level  $Z_0$  (Fig. 1). And the upper layer of the bed is named mixed layer, followed by a transition layer and a sequence of deposited layers. The deposited layers function as a reserve for sediment loads. The mixed layer exchanges with the sediment transport but is kept a constant thickness. The variation of the bed morphology is expressed by the changing of the thickness of the transition layer and the total number of the deposited layers. Based on the above principle, formulae for bed sorting processes can be designed.

### 3. MODEL VERIFICATION

As a simple application, the Yodo River system in Japan has been selected as a reference (Fig. 2). Originating from Lake Biwa, joined by three main upstream rivers, i.e. the Uji River, the Kizu River and the Katsura River, the Yodo River system forms a total catchment area of about  $8,240 \text{ km}^2$ , and finally flows into Osaka Bay. Due to a great number of groynes along the river, wash load transport is a crucial concern during the river recovery.

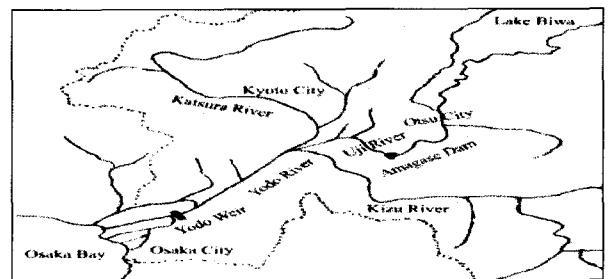


Fig.2 Description of study domain.

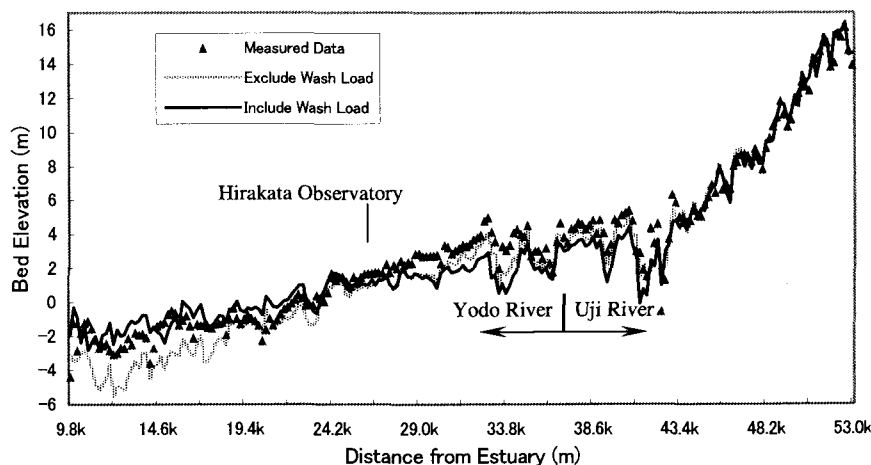
The river reach in the study domain is modeled

longitudinally by 339 cross-sections. And each cross-section is made up of a main channel and a flood plain on either side, which have been attributed different roughness. The mean bed elevation in 1975 is selected as the initial bed (Shortage of field data, river bed in 1976 has been used for the Kizu River). Dividing the initial bed into 40 layers in vertical direction with the thickness of each layer 0.5m, long-range series of daily discharge (hourly discharge during flood periods) and yearly navigation dredging data from 1976 to 1998 are employed for the verification. Cases of with and without wash load are performed, respectively. And 75 $\mu$ m is considered as the cutoff size for wash load.

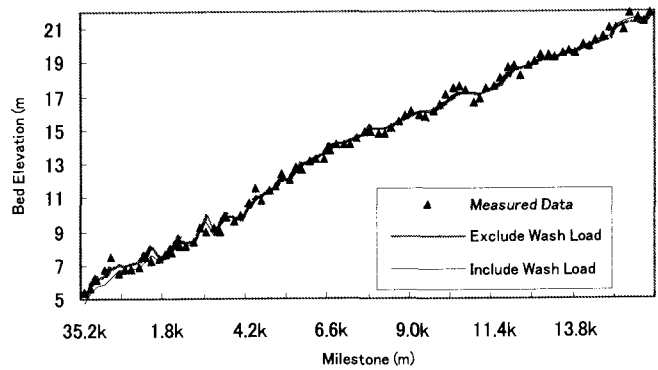
Limited to the available data at present, the comparison of the variation of the total sediment amount and the wash load concentration are not directly fulfilled in this paper. Instead the computational result has been compared with the mean bed elevation data for the river system and water stage data at Hirakata Observatory measured in 1998. Main parameters adopted in the computation are listed in Tab.1.

**Table 1** Coefficients adopted in the computation

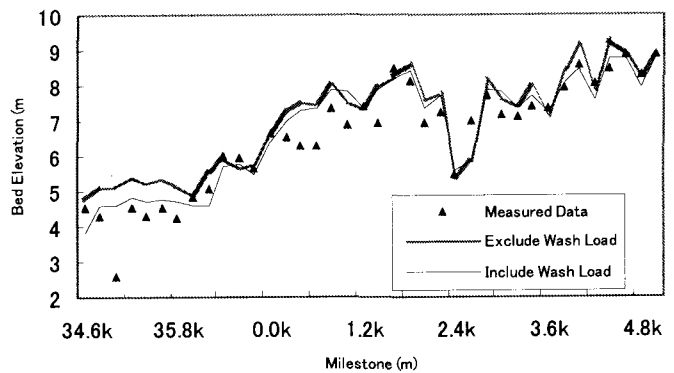
Coefficient	Description	Value
$p_c$	critical clay content	10.0%
$\tau_{cd}$	critical deposition shear stress	0.25 N/m <sup>2</sup>
$\tau_{cek}$	critical erosive shear stress	0.8-0.65 N/m <sup>2</sup>
$m_{ek}$	erosion parameter (same for all wash load fractions)	$2.0 \times 10^{-10}$ m/s
$c_h$	onset concentration for hindered settling	10.0 kg/m <sup>3</sup>
$K_1$	coefficients used in the formula for the settling velocity of the floc	0.006 m <sup>4</sup> /kg/s
$K_2$		0.01 m <sup>3</sup> /kg
$m$		4/3
$m_1$		4.65
$\lambda$	porosity of sediment load	0.4



**Fig.3** Bed elevation of the Yodo River in 1998.



**Fig.4** Bed elevation of the Kizu River in 1998.



**Fig.5** Bed elevation of the Katsura River in 1998.

For the two main tributaries, the Kizu River and the Katsura River (Fig.4 and Fig.5), there is no much difference between including and excluding cases. Both cases could give reasonable results similar to the field data. From which such conclusions could be drawn that in these two rivers, wash load is mostly carried out by water and has nearly no exchange with the local river bed, which coincides with the fact that almost no appreciable fine sediment amount could be found in either of the river bed. Due to a large dam (Amagase Dam) upstream, sediment flowing into the Uji River has been effectively controlled. Little morphological variation could be observed in the river bed, and this phenomenon has been adequately reproduced by the calculation of either case (see Fig. 3). Omitting wash load, the excluding case gives a much lower elevation than the field data in the downstream area of the Yodo River (especially in the downstream of the Hirakata Observatory) where fine sediment plays an important role. It means there is a significant overestimation of the fine sediment erosion, which is caused by the inherent deficient of non-cohesive based forecast models. The current model manifests its

advantages over the classical ones by more reasonable explanation of the mechanism of deposition and erosion due to cohesiveness. Although slightly larger than measured data in some cross-sections, pronounced improvements have been achieved after consideration of wash load transport.

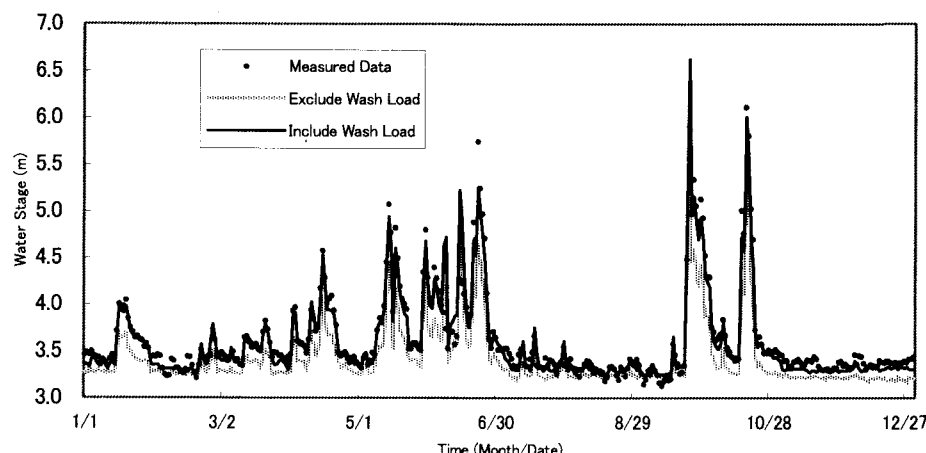


Fig.6 Water stage at Hirakata Observatory, 1998.

The comparison of the water stage at Hirakata Observatory is given in Fig. 6 above, which covers the water level variation during both dry seasons and flood periods in 1998. One can find that the case including wash load reproduces the water level variation quite well, which significantly humbles its counterpart neglecting effect of wash load.

During the computation, it was found that the wash load transport rate is very sensitive to the critical erosive shear stress and erosion parameter. While changing of the critical deposition shear stress in a specific range has almost no effect on the result. It might result from the limitation of the 1D flow model. Generally, wash load tends to deposit on the flood plains where the flow velocity is very slow but be washed out in the main channel due to the high velocity. While the 1D model is not able to resolve the great difference of the velocity distribution. As a result, it has a potential to overestimate the shear stress in flood plains.

#### 4. CONCLUSIONS

Owing to the necessity and importance of considering wash load transport in many rivers nowadays, traditional sediment transport models have been extended. Wash load was re-defined according to its specific motion mechanism and analyzed from the viewpoint of cohesive sediment transport. Taking into account the effect of river bed on sediment movement, a new erosion mechanism has been put forward and the clay content in the bed surface has been introduced as a simple indicator to evaluate the transport-related bed cohesiveness.

Computational results show that the new model is able to overcome some unreasonableness stemming from the imperfection of previous models and soundly improve the accuracy of simulation. Nevertheless, due to the complexity of cohesive sediment transport, some of the coefficients are empirically tuned based on limited laboratorial experiment results found in literatures, further investigation and detailed physical explanation are required in the future.

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