# SPECTRAL ANALYSIS OF SPATIAL RAINFALL FIELD TO INVESTIGATE UNCERTAINTY IN HYDROLOGICAL MODELING

Roshan SHRESTHA[1], Yasuto TACHIKAWA[2] and Kaoru TAKARA[3]

[1]Student Member, M. Eng., Dept. of Urban and Environmental Engineering, Kyoto University (Kyoto 606-8501, Japan)
[2]Member of JSCE, Dr. Eng., Associate Professor, DPRI, Kyoto University (Gokasho, Uji 611-0011, Japan)
[3]Fellow of JSCE, Dr. Eng., Professor, DPRI, Kyoto University (Gokasho, Uji 611-0011, Japan)

The uncertainty in hydrologic model outputs is the accumulated effects of input data, model structure and process description, which all are also scale dependent features. It is hard to pinpoint the basic reason of uncertainty; however, it is understandable that the dominant reason remains changing on the basis of data type, model structure and parameterization over a range of scale. In this paper, the spatial rainfall structure has been analyzed by spectral observation over a wide range of spatial scale, which presents a good insight to understand the spatial rainfall structure dynamics. The scale dependency of spatial rainfall characteristic is detected as a dominant reason to impart uncertainty in larger scale relative to catchment scale. A significant sensitivity of reference grid position in multi scale frame is also noticed, which displays the extent of anisotropic rainfall structure and its effect on different scale. The Huaihe River basin (132,350 $km^2$) and its sub-basins are taken as a case study.

***Key Words*** : *spatial rainfall structure, multi resolution, uncertainty, wavelet response*

## 1. INTRODUCTION

There are many unanswered questions regarding the uncertainty such as a) how to quantify the uncertainty components; b) how does uncertainty inherit from preceding processes; c) how to identify the generated uncertainty within ongoing processes; or d) how the uncertainty is interacting with the scale issues. The different dominant hydrological processes at different scale of analysis obviously make a different interaction with forcing data. Since the rainfall is one of the major forcing data, its interaction can be one of the dominant reasons to cause uncertainty in hydrologic modeling. There is a strong need to understand the limitation of rainfall data to impart the uncertainty in a wide range of scales such that it may help to set up the direction of uncertainty investigation. A new paradigm is felt necessary in this regard to penetrate these issues and make a more clear understanding.

A spectral analysis of the spatial rainfall structure presented here displays how the space-time rainfall field changes its characteristic in a changed scale frame. This change affects the hydrological analysis, probably both the distributed and lumped cases, since the lumped models often use the up-scaled point data to obtain the catchment average input rainfall and distributed models are certainly sensitive to spatial scales as well.

Basin and sub-basins of the Huaihe River (Table 1) in China are taken as the case to study (Fig. 1). Grid rainfall data is referred from the EXPT data[1)] with ten-minute spatial and one-hour temporal resolutions. This data is a merged experimental data from HUBEX IOP EEWB data[2)] and GAME Re-analysis 1.25-degree data (Version 1.1)[3)] for the period of May 1, 1998 to August 31, 1998.

## 2. SPECTRAL ANALYSIS OVERVIEW

The spectral analysis is conventionally based on Fourier analysis, in which the signal is compared with a number of basis functions composed of sines and cosines of different frequencies. This kind of analysis is unable to analyze multi resolution data for scale effect investigation[4)]. The scale based wavelet analysis is another way of spectral analysis that is introduced first by Grosmann and Morlet[5)] to apply in geophysical seismic signal processing,

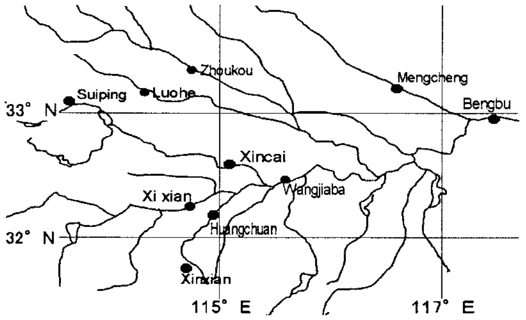



**Fig. 1** Location of basin confluences.

**Table 1** Catchment area of test basins.

| No. | Catchment Name | Area (km$^2$) |
|---|---|---|
| 1 | Suiping | 2,093 |
| 2 | Wangjiba | 29,844 |
| 3 | Bengbu | 132,350 |

which now is widely used in analyzing the structure of time series by using variable size windows[6)]. The wavelet analysis is believed to offer a different approach to looking at signals such as the use of longer time windows to detect low frequency (large scale) information and vice versa[7)]. Consequently, the signal is decomposed into components of different time scales and this offers a method for a local examination of the signal, a multi-scale outlook, and a time-scale analysis[8)]. Similar approaches are tested in the investigation of the stationarity of hydrologic and climatic time series[9)]; to identify dominant time scale[10)]; and in multi resolution analysis as well[11)].

Interpretation of signal frequency into the time scale phenomenon is straightforward to understand. In addition, this method may also be suitable to analyze the spatial information, where the spatial data structures have significant influence, for example, in the distributed hydrologic analysis. There are strong research interests to know how the spatial data changes in a multi resolution transfer. However, a little is known about the methods and consequences of spatial signal processing by means of wavelet analysis. Investigating the spectral analysis in this direction is a new topic of the research.

In hydrology, the spatial signals and temporal signals are equally important. The static and dynamic heterogeneity of hydrologic variables in space and time have significant influence in overall response. The topography, channel configuration, land-use classes etc. are some examples, which impart the static heterogeneity in space. The rainfall, soil moisture, evapo-transpiration etc. are some examples, which impart the dynamic heterogeneity in space. Influence of the heterogeneity on the hydrologic response is dependent on the spatial scale, which may be evaluated by a wavelet response function. The obtained plot of wavelet response function with multi resolution scale in this case can be interpreted as a 'scale spectrum', which provides a mean to investigate the consequences of the heterogeneity of spatial signal in the range of large to small scale. Studying the relative amplitude of the wavelet response function at different scales gives an idea about the dominant scales where the heterogeneity have strong influence in the response function.

## 3. FUNDAMENTALS OF THE ANALYSIS

### (1) Signal processing in space

The signal in space is considered as non-uniform continuous signal that may or may not have periodicity; however, it can be separated into artificial intervals for analysis. The segments of signal in a defined interval may be interpreted as resolution in one-dimensional space. This spatial signal is intended to be subjected to the spectral analysis. The wavelet response coefficient (WRC) transformed from the signal is given by curvilinear integration of the signal and the wavelet response function given by

$$W(\alpha,\beta)=\int S(x)\overline{\xi_{\alpha,\beta}(x)}dx \qquad (1)$$

where $x$ is the space; $S(x)$ is the spatial signal; $\overline{\xi_{\alpha,\beta}(x)}$ is the representative wavelet response function to transform the signal into $W$. The wavelet function $\xi_{\alpha,\beta}(x)$ plays the role of convolution-kernel and can be defined as

$$\xi_{\alpha,\beta}(x)=\alpha^m\xi\left(\frac{x-\beta}{\alpha}\right);\quad \alpha>0 \qquad (2)$$

where $\alpha$ may be interpreted as a dilation $(\alpha>1)$ or contraction $(\alpha<1)$ factor of $\xi(x)$, corresponding to different scales of observation. $m$ is an exponent. The parameter $\beta$ is a spatial translation factor or shift of the function that allow the study of signal $S(x)$ locally to go around the space $x$.

For a discrete system, the discrete-space signal can be used instead of continuous form of Equation (2). This takes a form of

$$\xi_{i,\alpha,\beta,n}(x)=\alpha^{mi}\xi\left(\alpha^i x-n\beta\right) \qquad (3)$$

where $n$ is a positive integer, which defines the translation of multiple of $\beta$ from a reference grid position that forms the discrete steps; $\alpha$, $\beta$ and $m$ are constants; $i$ is a scale factor, which together with $\alpha$ works analogous to a contraction or dilation factor.

In hydrological analysis, the spatial variables are two or three-dimensional. For example, the case of a spatial rainfall field, which is intended to be subjected into signal processing analysis, is two-dimensional. As the dimension changes from one to two, the basic wavelet function changes from $\xi(x)$ to $\xi(\vec{x})$ form where the $\vec{x}$ denotes the vector spatial field. This arises some practical difficulty to apply the $\xi(\vec{x})$ function because how the scalar values $m$, $\alpha$ and $\beta$ is to be changed is not known. Also there is confusion on the response function itself to describe the vector field of spatial rainfall. An easier and realistic alternate way is desired in this regard.

**(2) Alternate way of wavelet response**

In order to simplify the complexities of the classical form of wavelet response function to obtain the WRC, a different way is examined here. This method utilizes the statistical moments of the data distribution. Thus the WRC becomes a function of the statistical moments of the data distribution, which may be written as

$$W_{\vec{x}}(\ )=f[\mu_1(t),\mu_2(t),...;t] \qquad (4)$$

where $W_{\vec{x}}(\ )$ is the WRC of vector spatial field; $t$ stands for time; $\mu_1(t)$ and $\mu_2(t)$ are first and second moments of the data distribution at particular time $t$ such that

$$\mu_1(t)=\int S(x)p_x(x)dx \quad \text{; at time } t \qquad (5)$$

$$\mu_2(t)=\int[S(x)-\mu_1(t)]^2 p_x(x)dx \quad \text{; at time } t \qquad (6)$$

It is well known that the statistical moments are the descriptors of the data property. Therefore, the transformed function using the moments have strong basis of reflecting the basic data property in evaluation of the WRC, which ultimately may appear in the spectrum to display the proper significance of the data structure.

The WRC has been supposed to indicate analogous catchment response such as to produce runoff from the rainfall. This has been attempted to be related by an exponent model[12], as it is one of the simplest non-parametric models. An experiment with an alternate model may be future research work. Thus an instantaneous WRC (IWRC) is given by

$$W_l(t)=e^{\mu_1(t)}-1 \qquad (7)$$

$$W_d(t)=e^{\mu_1(t)}e^{\sqrt{\mu_2(t)}}-1 \qquad (8)$$

where $W_l(t)$ represents the IWRC for a lumped spatial data as if $\mu_2(t)=0$; $W_d(t)$ represents the IWRC for a spatially distributed data.

**(3) Wavelet response of time series**

A series of the spatially integrated IWRC forms a time series over a longer period of observation series, which if integrated in time domain, gives the temporally integrated WRC of the spectrum for that particular space-time scale. This is given by

$$W_{\vec{x},t}(\ )=\int_0^t W_d(\tau)\ \overline{\psi_{\vec{x}}(\tau)}d\tau \qquad (9)$$

At constant time scale $\tau=t$, the temporal response function $\overline{\psi_{\vec{x}}(t)}$ turns to a constant and $W_{\vec{x},t}(\ )$ refers to the WRC of $\vec{x}$ spatial scale. Changing the data scale $\vec{x}$ by upscaling or downscaling process and then re-calculating the WRC for the range of spatial scales yield a set of WRC for multiple scales from the same mother data. The differences between the multi scale WRC and the reference scale WRC produce a scale spectrum of wavelet responses, which visualize the data property deviation within the multi scale frame relative to the reference scale. This is given by

$$W_{\vec{x},t}(\vec{x}_R)=\int_0^t [W_{\vec{x},\tau}(\ )-W_{\vec{x}_R,\tau}(\ )]d\tau \qquad (10)$$

When the differences between the multi scale WRC are observed at a fixed $\Delta$ scale interval, it produces a delta scale spectrum, which is given by

$$\Delta W_{\vec{x}_i,t}(\ )=\int_0^t [W_{\vec{x}_{i+\Delta},\tau}(\ )-W_{\vec{x}_i,\tau}(\ )]d\tau \qquad (11)$$

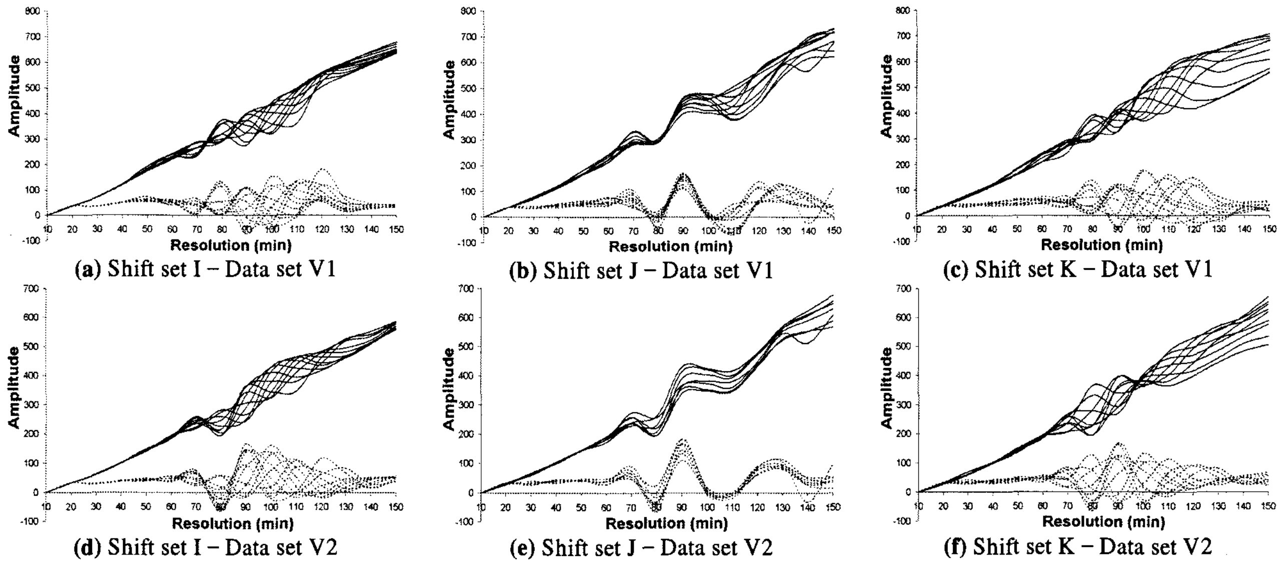
The $\Delta W_{\vec{x}_i,t}(\ )$ at a series of $i^{th}$ position of $x+\Delta$ scale gives the delta scale spectrum with reference to $x$ scale at the $\Delta$ scale interval.

**(4) Reference grid system**

The reference grid needs to be investigated whether it imparts any response to change the data structure or not, especially in multi resolution analysis. Shifting the reference grid to either horizontal direction or vertical direction or diagonal direction may give non-identical spectrums, which form particular scale spectra and delta scale spectra for the particular shift direction. This is useful to investigate the significance of the reference grid system by observing the changes in spectral plot due to the reference grid shift.

## 4. SIMULATION RESULTS

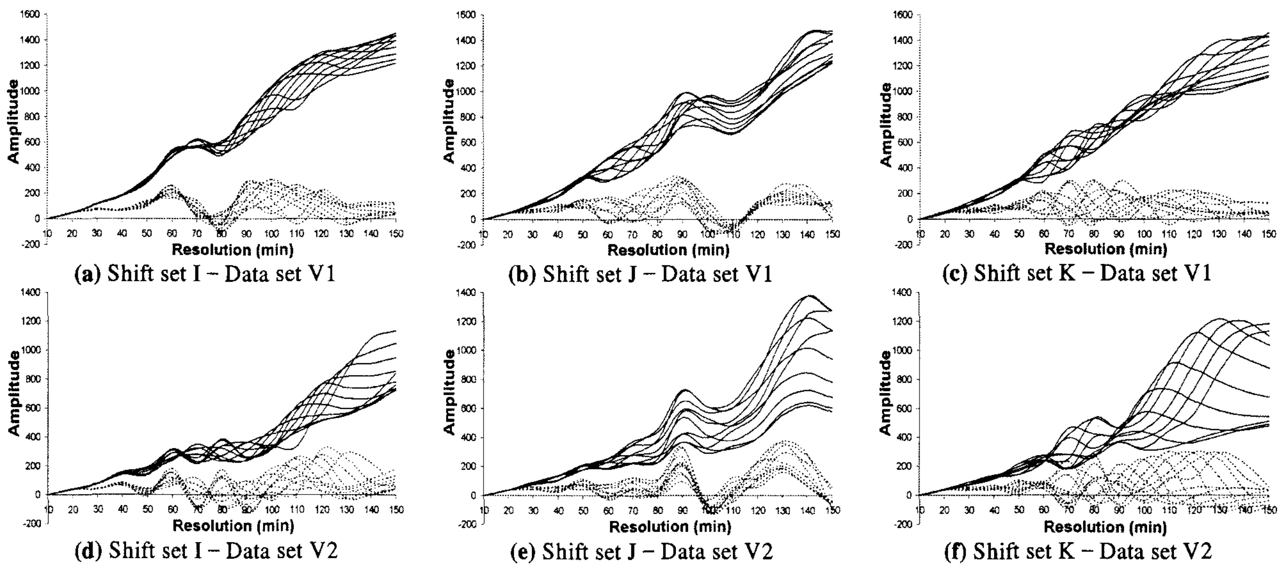
The simulation is conducted by taking the spatial rain data of fixed boundaries of different sized hydrologic catchments to evaluate the WRC at 10 minute reference scale and their scale spectrum for the study catchments. Any solid line of Fig. 2 through Fig. 4 represents the scale spectrum and any broken line of those figures represents the delta scale spectrum at $\Delta=10$ minutes.



(a) Shift set I – Data set V1 (b) Shift set J – Data set V1 (c) Shift set K – Data set V1

(d) Shift set I – Data set V2 (e) Shift set J – Data set V2 (f) Shift set K – Data set V2

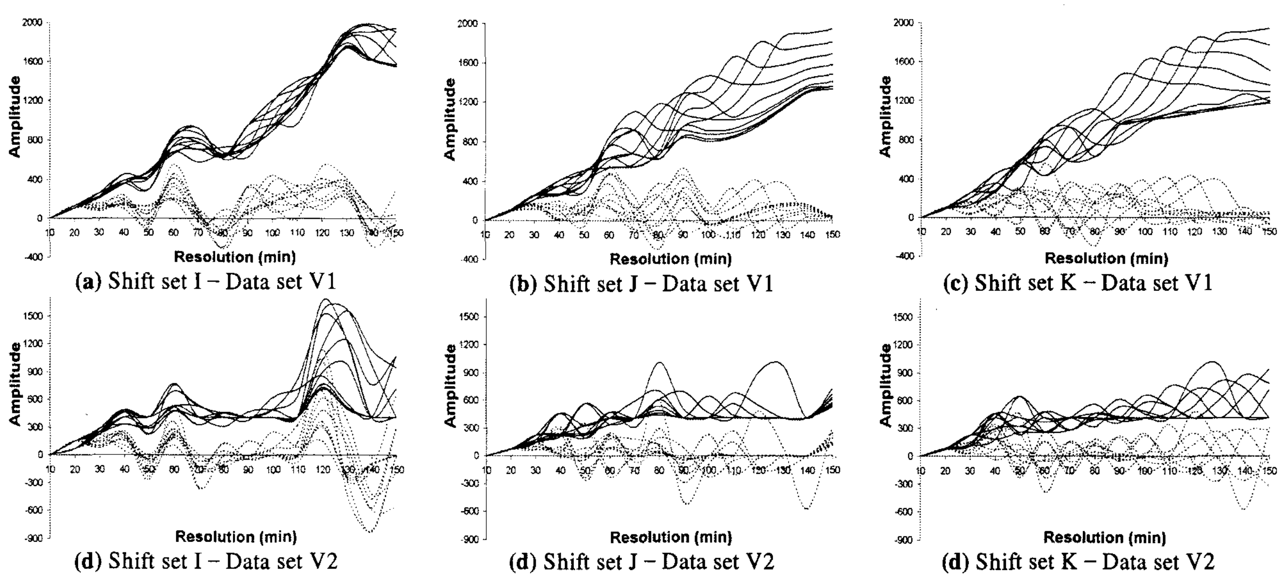
**Fig.2** Scale spectrum (solid line) and delta scale spectrum (broken line) for Bengbu (132,340 km$^2$)



(a) Shift set I – Data set V1 (b) Shift set J – Data set V1 (c) Shift set K – Data set V1

(d) Shift set I – Data set V2 (e) Shift set J – Data set V2 (f) Shift set K – Data set V2

**Fig.3** Scale spectrum (solid line) and delta scale spectrum (broken line) for Wangjiaba (29,844 km$^2$)



(a) Shift set I – Data set V1 (b) Shift set J – Data set V1 (c) Shift set K – Data set V1

(d) Shift set I – Data set V2 (d) Shift set J – Data set V2 (d) Shift set K – Data set V2

**Fig.4** Scale spectrum (solid line) and delta scale spectrum (broken line) for Suiping (2,093 km$^2$)

The vertical axis of Fig. 2 through Fig. 4 is amplitude of WRC. With change in resolution along horizontal axis, an amplitude change pattern can be seen forming a spectrum that gives an idea how the data features are deviating in multi resolution frame of the spatial domain. It may be interpreted as an indicator of particular change in two-dimensional spatial variability of the data.

The massive fluctuations of the response amplitude appear at different scales in different catchments but they are surely at relatively coarser resolutions. The effects of reference grid shift are plotted in I, J and K sets (each includes 9 set of simulations) of spectral plots referring the shift in horizontal, vertical and diagonal directions respectively, which reveals non-identical behavior of spectral change for shift in separate directions. The spectral fluctuation looks similar in both scale spectra and delta scale spectra, though one to one comparisons might have differences.

In the process of changing the data scale, there is high chance of change in accumulated value of time series within a fixed boundary of a catchment due to influence of differences in surrounding cell data. This is not favored in hydrologic analysis and hence the accumulated data is usually maintained by using some arbitrary corrector[1]. In this experiment, two sets of data V1 and V2 are examined, where V1 refers to unmodified multi scale data and V2 refers to modified multi scale data by arbitrary corrector to maintain the same accumulated value. Surprisingly, the spectral plots of V1 and V2 set data produced under the same condition, the same catchment area and the same direction of reference grid translation are found quite different (see V1 and V2 sets of spectral plots), unlike the conventional expectation.

## 5. DISCUSSIONS

### (1) Interpretation of the spectral plots

The differences in scale spectra at different catchment boundaries indicate that the size of catchment perhaps has a non-linear effect on them. The birth of massive spectral dispersion corresponds to the initiation of transitional resolution range[13], which may appear when grid cell size grows much coarser relative to catchments area; or when the grid boundary divides the catchment of much smaller area than the grid cell size itself; or when the grid boundary position shifts gradually in accordance of the changed spatial resolution. The rise and fall in spectral line occurs due to response of multi resolution data with catchment area.

The significant difference between V1 and V2 sets of plots shows that the forcing process to maintain the same accumulated value is not a non-responsive process, but it may impart uncertainty. The concept of preserving water balance is in fact modifying the data scenario in multi scale frame. There is complex interaction of the modification in space-time variability due to the intruded bias in the data, which has significant influence though accumulated value remains the same; and because of that there are differences in the spectral plot.

The spectral plot can identify the upper limit data resolution for distributed modeling. Even in lumped modeling it may bring important information to investigate the catchment average value, which is the upscaled point measured data. Normally the basin average value is not much investigated, though it is well known that it may differ significantly from the truth[14]. The biases in lumped basin average value are treated by adjusting parameters of subsequent models like runoff models, which ultimately may affect the estimation of suitable parameter for the particular soil type, geomorphology and catchment characteristics. There is an importance to understand how the data is upscaled to get the lumped input for the analysis, because the lumped data itself may have uncertainty based on scale difference between the source and target domain and the reference grid position as seen in the spectral plots here.

The differences in spectral plots by shifting reference grid toward different directions indicate the rainfall is not homogenous. The detected spectral changes are due to either the directional bias or the orderly arrangement that the rainfall structure holds. Perhaps the layered spectral lines (Fig. 3(b) or 3(e)) and twisting lines (Fig. 3(a) or 3(d)) are representing the parallel and crossing shift of reference grids with such rain arrangements or vice versa. The overall spectral form may be the function of catchment characteristic and scale domain of multi resolution system, but how the two spectrum lines differ to each other may be the function of the anisotropic rain structure.

### (2) Spectral analysis and uncertainty

One of the big challenges in hydrologic process understanding is to find out the basic phenomena that plays significant role in producing uncertainty[15]. The uncertainty in hydrologic model output is in fact accumulated effect of uncertainties in input data, model structure and process description, which all are also scale dependent features. It is hard to pinpoint a single reason of uncertainty in the modeling procedure; however, it is understandable that the dominant reason of uncertainty remains changing based on type of data, model structure and parameterization over a wide range of scale. The understanding of uncertainty in hydrology is still limited although there are many approaches and models available for analysis. The presented methodology may assist to figure out the basic phenomenon of uncertainty.

**Table 2** Marginal resolutions (minutes) in the spectrum.

| Resolution (Minute) | Bengbu | Wangjiba | Suiping |
|---|---|---|---|
| Scale Spectra | 60 | 50 | 20 |
| Delta Scale Spectra | 60 | 40 | 20 |

Rainfall, being a major forcing data, certainly imparts uncertainty. If the rain structure is fluctuating, it causes massive uncertainty in the hydrologic analysis. The induced uncertainty from rainfall might be due to uncertainty in rainfall measurement or due to effect of scale difference between the rainfall field and model structure. There is significant importance to understand the scale relevant uncertainty in order to identify the scale where uncertainty level is tolerable and where it is not so or the induced uncertainty is terribly high. The fluctuations of spectral lines may be interpreted analogous to uncertainty imparted at different scale, which may stand as one prominent understanding of uncertainty. This helps to identify a marginal point (Table 2) that separates the critical and non-critical part. In the non-critical part, the detected uncertainty may be due to the model structure and/or parameters. In the critical part, the uncertainty is more dominated by the rainfall structure than the effect of model structure or parameters.

## 6. CONCLUSION

A spectral analysis of the spatial rain structure is presented here to display the characteristic changes of space-time rainfall field in a multi scale frame.

The use of space-time integrated wavelet coefficients at different scales is able to display the fluctuation in spatial heterogeneity of the rainfall structure and able to identify the dominant range of scale where the uncertain component may evolve unexpectedly haphazard, which conventionally was an unseen problem.

The effect of bias introduced in an attempt to maintain the accumulated value of rainfall in a catchment is detected in the form of changes in the spectral plot, which has shown that the intruded bias may serve as a source of uncertainty.

A significant sensitivity of the reference grid system is detected when it is moved to different direction. The non-identical spectral plots obtained from the shift of reference grids to different directions have revealed the anisotropic response of spatial rainfall fields in multi scale frame, which indicates the anisotropic uncertainty phenomenon due to joint effect of scale and spatial rain structure.

## REFERENCES


1) Shrestha, R., Tachikawa, Y., Takara, K.: Model behavior of distributed hydrological modeling with different forcing data resolution, *In: Weather Radar Information and Distributed Hydrological Modeling, IAHS Publication no. 282, pp169~176, 2003.*
2) Tanaka, K.: Activities in Huaihe River Basin (HUBEX) study area, *Asia-Pacific Network for Global Change Research, Report of 1997-1998, pp. 69-91.*
3) ftp://hydro.iis.u-tokyo.ac.jp/GAME/GAIN/GAME-REANL/
4) Mallat, S.: A theory for multiresolution signal decomposition: the wavelet representation, *IEEE Trans. Pattern Analysis Machine Intell., **11**, pp. 674~693, 1989.*
5) Grossmann, A., Morlet, J.: Decomposition of Hardy function into square integrable wavelets of constant shape, *SIAM J. Math. Anal., pp 723~736, 1984.*
6) Roques, S., Meyer, Y., (Eds.): Progress in Wavelet Analysis and Applications, *Editions Frontieres, Derux (785 pp), 1993.*
7) Daubechies, I.: Ten lectures on Wavelets. *CSBM-NSF Series Application Mathematics, vol. 61, SIAM Publication, Philadelphia, PA (357pp)*
8) Liu, P. C.: Wavelet spectrum analysis and ocean wind waves, *In: Foufoula-Georgiou, E., Kumap, P. (Eds.). Wavelets in Geophysics, Academic Press, New York, pp151~166, 1995.*
9) Foufoula-Georgiou, E., Kumar, P. (Eds.): Wavelets in Geophysics, *Academic Press, New York, (373pp), 1995.*
10) Venugopal, V., Foufoula-Georgiou, E.: Energy decomposition of rainfall in the time-frequency-scale domain using wavelet packets, *J. of Hydrology, **187**, pp3~27, 1996.*
11) Labat, D., Ababou, R., Mangin, A.: Rainfall-runoff relations for karstic springs. Part II: continuous wavelet and descrete orthogonal multiresolution analysis, *J. of Hydrology, **238**, pp149~178, 2000.*
12) Eagleson, P. S.: Dynamics of flood frequency, *Water Resources Research, 8(4), pp 878~898, 1972.*
13) Shrestha, R., Tachikawa, Y., Takara, K.: Catchment response of up scaling procedure in distributed hydrologic modeling, *Proc. of the Fifth Int. Summer Symposium, JSCE, pp 193~196, 2003.*
14) Sivapalan, M., Bloschl, G.: Transformation of point rainfall to areal rainfall: Intensity-duration frequency curves, *J. of Hydrology, **204**, pp150~167, 1998.*
15) Kundzewicz, Z. W. (Eds.): New uncertainty concepts in hydrology and water resources, *IAHS Series, Cambridge university press (322pp), 1995.*


**(Received September 30, 2003)**