ACCELERATION EFFECT ON SHEAR STRESS IN TURBULENT BOTTOM BOUNDARY LAYER UNDER SAW-TOOTH WAVES

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Bottom shear stress estimation is the crucial step, which is required as an input to most of sediment transport model. Acceleration effect in waves with a bit of saw-tooth asymmetry causes the higher bottom shear stress occurs at higher acceleration and hence a higher probability for sediment transport. The acceleration effect on turbulent bottom boundary layer is investigated with the experiments in oscillating wind tunnel over rough bed under saw-tooth wave by Laser Doppler Velocimeter (LDV) to measure velocity distribution. Bottom shear stress will be estimated by fitting the logarithmic velocity profile to the measured velocity. Furthermore, the experimental results will be examined with the bottom shear stress calculation method incorporating acceleration effects.

Key Words : *Turbulent bottom boundary layer, bottom shear stress, acceleration effect, saw-tooth wave*

1. INTRODUCTION

In shallow water, a bottom boundary layer will be formed along the sea bottom where molecular viscosity and Reynolds stress play an important role in fluid motion for laminar flow and for turbulent flow, respectively. The bottom shear stress is an important factor in coastal processes, causing both cross-shore and longshore sediment transport. In nearshore area where the waves are close to breaking, they become non-linear, hence the simple harmonic variation as sinusoidal wave can not describe the boundary layer behavior occurring in the surf zone in which major part of nearshore sediment are transported. In addition, the free stream velocity variation, the water particle velocity and bottom shear stress significantly deviate from those under sinusoidal wave. Schäffer and Svedsen[1)] had shown that a simplified approach of skewed wave or saw-tooth wave could be used to represent the asymmetric shape of breaking wave. Samad and Tanaka[2)] had investigated the flow behavior in bottom boundary layer under saw-tooth wave for both laminar and turbulent flow condition by numerical model for smooth bed.

The bottom shear stress in the rough turbulent boundary layer under saw-tooth wave has been observed by Suntoyo et al.[3)], which pointed out that the bottom shear stress variation was dependent on high acceleration and deceleration. Saw-tooth wave also can be used to cover sediment transport under rapid acceleration and the relative contribution of pre-suspended sediment to the onshore sediment transport in swash zone[4)]. Fredsøe and Deigaard[5)] examined a saw-tooth non-linear wave over a rough bed and found a good agreement between the logarithmic velocity profile approach and turbulent model. However, the acceleration effect investigation in the bottom shear stress calculation has not deeply examined yet.

In this paper, we investigate the incorporation importance of the acceleration effect in the calculation of the instantaneous bottom shear stress under saw-tooth wave for the rough bed turbulent boundary layers.

2. LABORATORY DATA

(1) Experimental Set-up

The experiments have been carried out in an

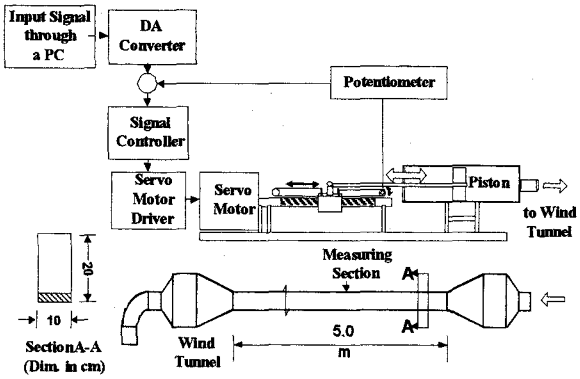

Fig.1 Schematic diagram of experimental set-up.

oscillating wind tunnel connected with the piston system with air and smoke as the working fluid and the working particle, respectively. The experimental system consists of two major components, namely an oscillatory flow generation unit and a flow-measuring unit.

The oscillatory flow generation unit was made up of signal control and processing components along with piston mechanism. The piston displacement signal has been fed into the instrument through a PC. Input digital signal has been converted to corresponding analog data through a digital-analog (DA) converter. A servomotor, connected through a servomotor driver, was driven by the analog signal. The piston mechanism has been mounted on a screw bar, which was connected to the servomotor. The feed-back on piston displacement, from one instant to the next, has been obtained through a potentiometer that compared the position of the piston at every instant to that of the input signal, and subsequently adjusted the servomotor driver for position at the next instant. A schematic diagram of experimental set-up is shown in **Fig.1**.

The flow measurement unit comprised of a wind tunnel and one component laser Doppler velocimeter (LDV) for flow measurement. The wind tunnel is connected to the piston system that has a dimension of 5 m length, 20 cm and 10 cm in height and width, respectively. The triangular shape elements were used as the roughness element having a dimension of 5 mm height and 10 mm width were pasted over the bottom surface of the wind tunnel with distance of 12 mm along the wind tunnel. Near the measuring section the sidewalls of the wind tunnel were made of transparent fiberglass sheets to facilitate LDV measurements.

(2) Experimental Conditions

Experiments have been carried out for three cases; namely Case 1 and Case 2 are for saw-tooth wave cases and Case 3 is for sinusoidal case.

Table 1 Experimental conditions.

Exp.	Case 1	Case 2	Case 3
Uc(cm/s)	199	477	400
T(s)	3	3	3
ks(cm)	7.1	2.6	10.0
a_m/ks	13.5	89.3	19.1
Re	1.35x10^5	7.71x10^5	5.42 x10^5
α	0.351	0.314	0.554

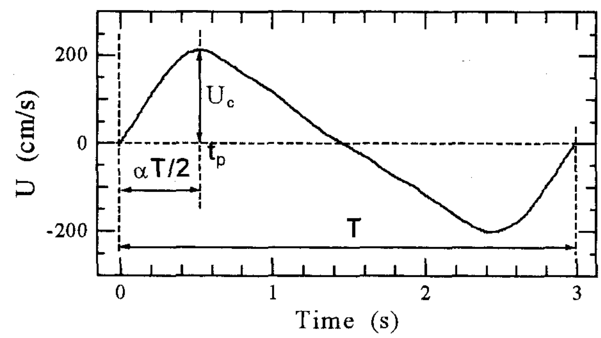

Fig.2 Saw-tooth wave definition.

Furthermore the conditions are presented in **Table 1**.

Here, a_m/k_s: the roughness parameter, k_s: the Nikuradse's roughness equivalent and a_m: U_c/ω, where U_c: the velocity at wave crest, T: the wave period, R_e: the Reynolds number ($R_e=U^2_c/\nu\omega$), ω: the angular frequency, ν: the kinematics viscosity (ν=0.141 cm^2/s). And α: the wave skewness parameter, where an increase in α indicates a decrease in wave skewness ($\alpha= 2t_p/T$), t_p: the time interval measured from the zero-up cross point to wave crest in the time variation of free stream velocity, as shown in **Fig. 2**.

The Reynolds number magnitude defined for each case has sufficed to locate cases in the rough turbulent regime.

3. VELOCITY PROFILES

Velocity profiles in the boundary layer both of the saw-tooth wave cases at selected phases have been compared with those from laminar solution as shown in **Figs. 3** and **4**, respectively. It can be seen that for laminar flow, the velocity overshoot is much influenced by degree of wave skewness and magnitude of Reynolds number. The larger velocity overshoot is occurred due to the higher acceleration under wave crest, but under trough the velocity overshoot is much smaller due to the smaller acceleration, as shown in **Fig. 3**.

The vertical velocity distribution for turbulent flow significantly deviates from laminar flow. Due

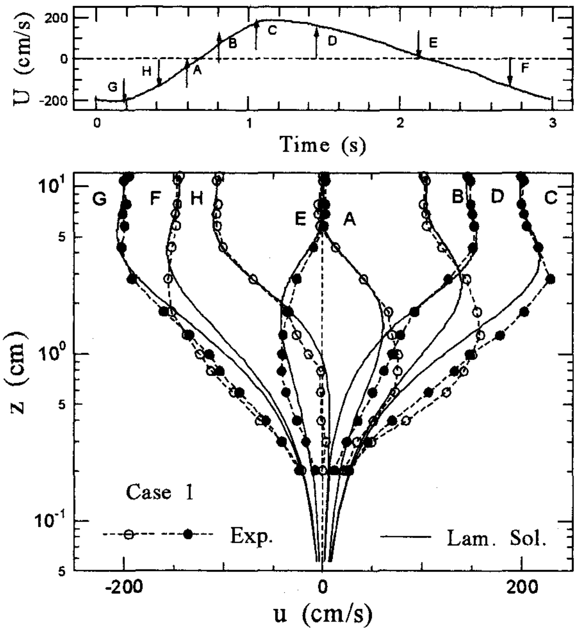

Fig. 3 Vertical velocity distribution, Case 1.

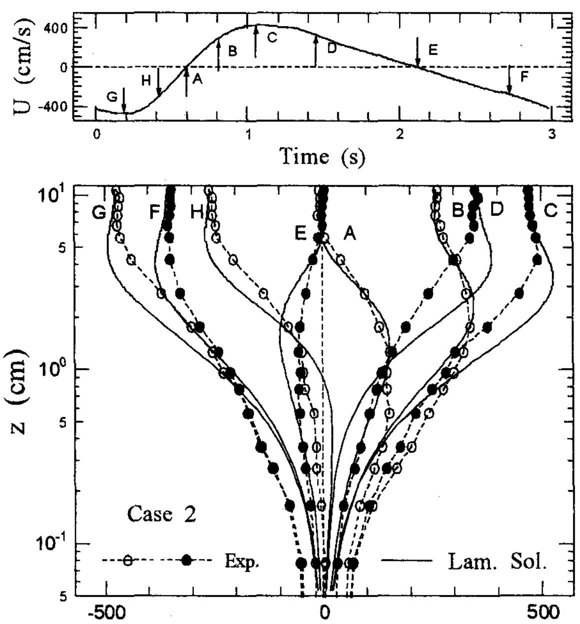

Fig. 4 Vertical velocity distribution, Case 2.

to turbulence production near the bottom, the velocity overshoot significantly reduced.

4. ESTIMATION OF BOTTOM SHEAR STRESS

Bottom shear stress is estimated by fitting the logarithmic velocity distribution to the measured data, which is given in Eq.(1),

$$u = \frac{U_*}{\kappa} \ln\left(\frac{z}{z_0}\right) \quad (1)$$

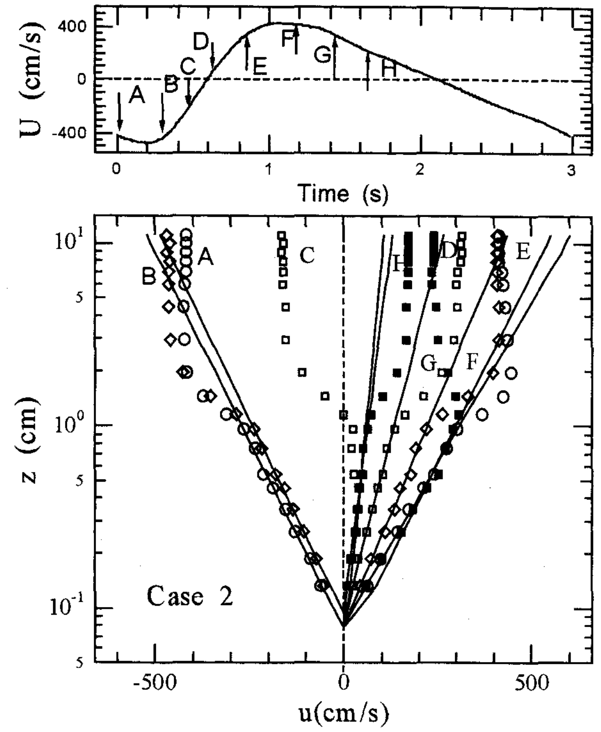

Fig. 5 Log-fitting to measured velocity profile, Case 2.

where, u: the flow velocity in the boundary layer, κ : the von Karman's constant (=0.4), U_*: the shear velocity, z: the cross-stream distance from theoretical bed level and z_o: roughness height. Eq.(1) is the logarithmic velocity profile expressed in term of z_o denoting the value of z at which the logarithmic velocity profile predicts a velocity of zero. For a smooth bottom z_o=0, but for rough bottom, the elevation of theoretical bed level is not a single value above the actual bed surface.

Furthermore, the value of z_o for fully rough turbulent flow is obtained as

$$z_o = k_s / 30 \quad (2)$$

The value of z_o for the fully rough turbulent flow is obtained by extrapolation of the logarithmic velocity distribution above the bed, to the value $z = z_o$ where u vanishes. In this present study the value of z_o is taken from the average value of the extrapolation results of z_o at high velocity. The values of z_o are 0.238 cm, 0.085 cm and 0.334 cm for Case 1, Case 2 and Case 3, respectively.

By plotting u against $ln(z/z_0)$, a straight line is drawn through the experimental data, and the value of U_* can be obtained from the slope of this line.

Fig. 5 show the logarithmic law has been approved within the wide range in the near bottom region for Case 2 at the selected phases of velocity profile.

5. CALCULATION METHODS OF BOTTOM SHEAR STRESS

(1) Consideration of the friction coefficient under sinusoidal wave motion.

First consideration for the bottom shear stress calculation based on an expression for friction coefficient under sinusoidal wave motion. The time-variation of bottom friction can be expressed by an equation, as proposed by Samad and Tanaka[6] for an arbitrary variation of $U(t)$, as follows:

$$\tau_o\left(t-\frac{\varphi}{\omega}\right)=\frac{\rho}{2}f_w U(t)|U(t)| \quad (3)$$

where τ_o: the bottom shear stress, φ: the phase difference, f_w: the wave friction coefficient, which has been extensively studied by many researchers for sinusoidal wave induced flow. Furthermore, the friction coefficient formula that will be used in this calculation are both of its for rough bed turbulent as proposed by Tanaka and Thu[7] and Nielsen[8] that is given in Eqs. (4) and (5) respectively, as follows:

$$f_w=\exp\left\{-7.53+8.07\left(\frac{a_m}{z_o}\right)^{-0.100}\right\} \quad (4)$$

and

$$f_w=\exp\left\{5.5\left(\frac{a_m}{k_s}\right)^{-0.2}-6.3\right\} \quad (5)$$

(2) Consideration of the acceleration effect under a bit of saw-tooth asymmetry wave

The instantaneous bottom shear stress is calculated from the variation of the free stream velocity as proposed by Nielsen[9], the acceleration effect can be obtained through with the instantaneous friction velocity, as expressed by

$$U_*(t)=\sqrt{f_w/2}\left[\cos\varphi\, U(t)+\sin\varphi\left\{\frac{U(t+\delta t)-U(t-\delta t)}{2\omega\delta t}\right\}\right] \quad (6)$$

$$\tau_o(t)=\rho|U_*(t)|U_*(t) \quad (7)$$

where δt: the time step of the velocity measurement.

Furthermore, it is based on the above equations, namely, from Eq.(3) to Eq.(7) will be examined several methods as given in **Table 2.** Method 1 denotes the proposed method with consideration of the friction coefficient under sinusoidal by apply the friction coefficient formula in Eq.(4) as proposed by Tanaka and Thu[7]. Method 2 includes the acceleration effect due to the existence of rapid acceleration on the saw-tooth asymmetry wave, whereas Method 3 excludes the acceleration effect for Nielsen's method.

Table 2 Summary of proposed methods to compute the instantaneous bottom shear stress.

Method	$\tau_o(t)$	f_w	$U_*(t)$
1	Eq. (3)	Eq. (4)	-----
2	Eq. (7)	Eq. (5)	Eq. (6)
3	Eq. (7)	Eq. (5)	$=\sqrt{\frac{f_w}{2}}\cos\varphi\, U(t)$

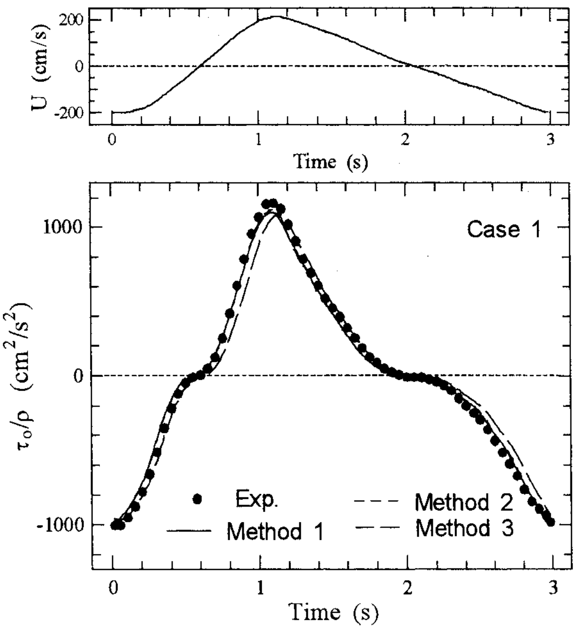

Fig. 6 Comparison of experimental result bottom shear stress and calculation methods, Case 1.

All of the instantaneous bottom shear stress calculation methods as given in **Table 2** apply the phase difference that is obtained from each experimental cases, namely φ = 6.0°, φ = 18.1° and φ = 42.7°, for Case 1, Case 2 and Case 3, respectively.

Afterwards, the three proposed methods are applied to calculate the instantaneous bottom shear stress from the time-variation free stream velocity from the three experimental cases.

From observing in **Fig. 6** shows that Method 2 has given a good agreement with the experimental result of Case 1, while Method 3 gave the underestimated value due to exclude the acceleration effect in the calculation, and also the phase difference still remain appear though the phase difference φ value has been included in the calculation in Method 3. Method 1 also gave a good agreement with the experimental result of Case 1.

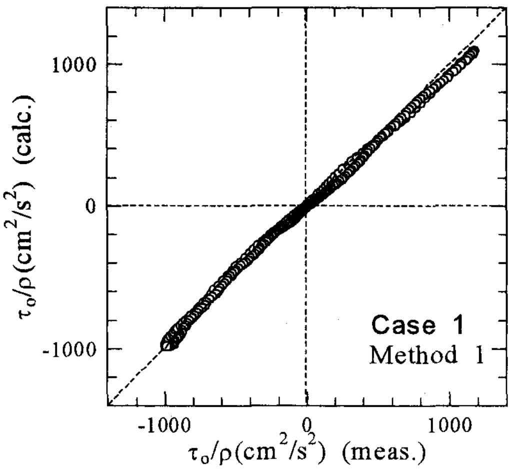

Fig. 7 Correlation between bottom shear stress of measurement and calculation for Case 1, Method 1.

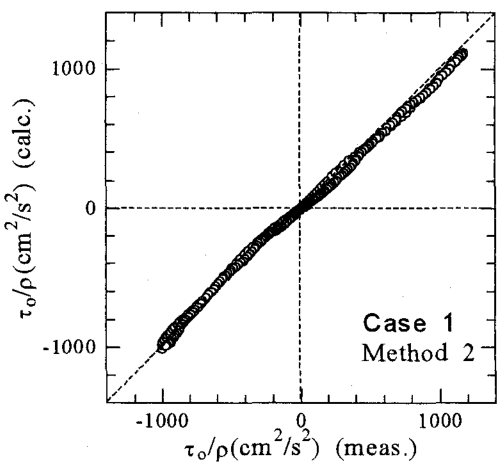

Fig. 8 Correlation between bottom shear stress of measurement and calculation for Case1, Method 2.

It means that the wave friction coefficient given by Tanaka and Thu[7] fits for the instantaneous bottom shear stress calculation at Re = 1.35 x 10^5 under saw-tooth wave with the roughness parameter, a_m/k_s = 13.5 and the wave skewness parameter, α = 0.351.

Furthermore, the correlation between the measurement and the calculation for Case 1 is plotted in **Figs. 7**, **8** and **9**. As previously mentioned that Method 1 and 2 have a good agreement between the measurement and the calculation, while Method 3 has not so good agreement with the measurement result.

For the higher Reynolds number value and the smaller the wave skewness in Case 2, the acceleration effect is significantly influential to the instantaneous bottom shear stress calculation, as shown in **Fig. 10**. Because of the bottom shear stress under saw-tooth wave has a characteristic in which at the peak part is larger than the trough part, so a good agreement between the experimental result and the calculation method can more be obtained at

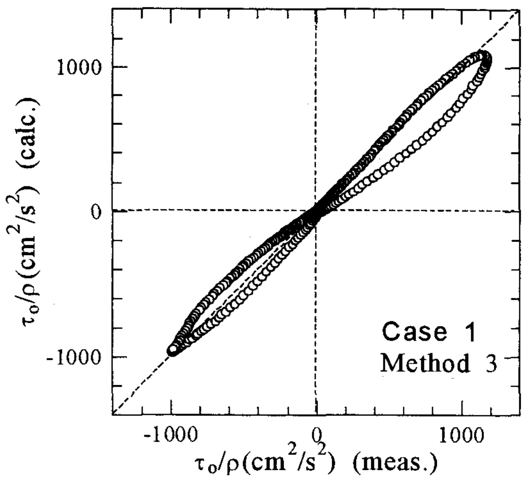

Fig. 9 Correlation between bottom shear stress of measurement and calculation for Case 1, Method 3.

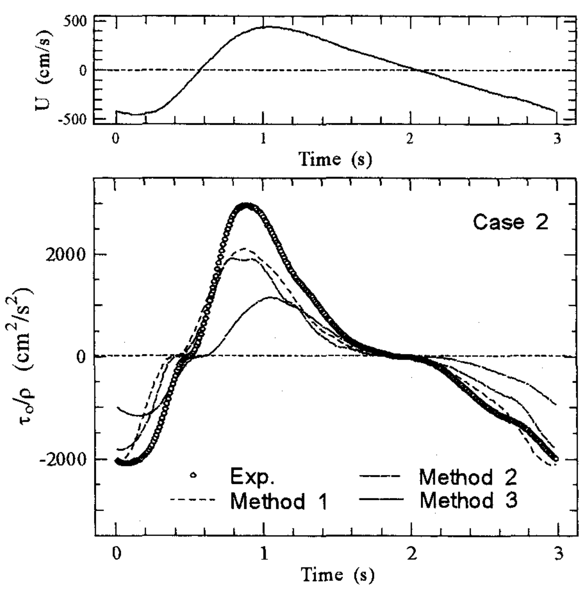

Fig. 10 Comparison of experimental result bottom shear stress and calculation methods, Case 2.

the trough part than at the peak part. Method 1 and 2 are more close to the experimental result than Method 3 along the time-variation bottom shear stress, nevertheless a good agreement between the experimental result and the calculation method could not obtained yet.

Fig. 11 shows that the acceleration effect is not so significantly influential in the instantaneous bottom shear stress calculation at the simple harmonic wave, as shown in figure that Method 2 gave a not so good agreement, while Method 1 is more adequate to calculate the instantaneous bottom shear stress under sinusoidal wave.

6. CONCLUSIONS

The acceleration effect on shear stress in turbulent

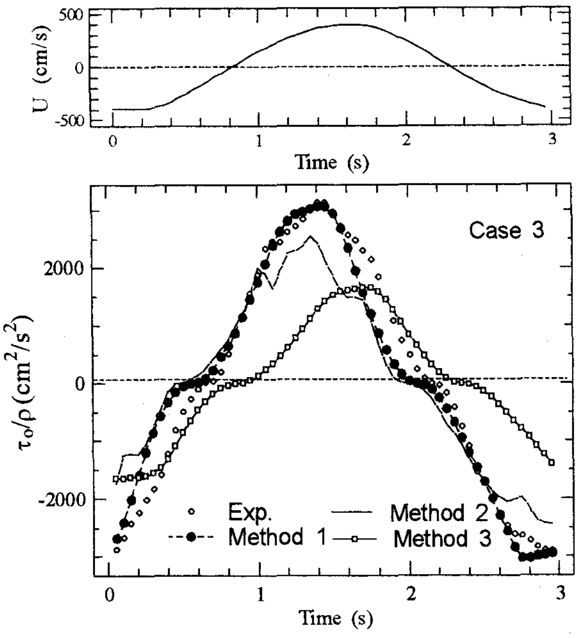

Fig. 11 Comparison of experimental result bottom shear stress and calculation methods, Case 3.

bottom boundary layer has been investigated and a number of conclusions can be drawn, as follow

1. The instantaneous bottom shear stress calculation method incorporating the acceleration effect as proposed by Nielsen[9] under saw-tooth wave has given a good agreement with the experimental result for the small Reynolds number, at Re = 1.35 x 10^5 and the underestimated value occurs due to exclude the acceleration effect in the calculation, and also the phase difference still remain appear though the phase difference φ value has been included in the calculation in Method 3.
2. For the higher Reynolds number value and the smaller the wave skewness in Case 2, the acceleration effect is significantly influential to the instantaneous bottom shear stress calculation. Because of the bottom shear stress under saw-tooth wave has a characteristic in which at the peak part is larger than the trough part, so a good agreement between the experimental result and the calculation method can more be obtained at the trough part than at the peak part.

To overcome this phenomenon need to include whole aspect that influence the acceleration effect in the instantaneous bottom shear stress, so it can be obtained a more reliable calculation method to calculate the instantaneous bottom shear stress which is required as input to sediment transport model.

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(Received September 30, 2002)