ON SPATIAL DISCRETIZATION SCALE TO AVOID SPURIOUS OSCILLATIONS IN FEM SOLUTIONS OF 2D AND QAUSI-3D GROUNDWATER FLOW MODELS

Xiangwei ZHANG[1], Kuniyoshi TAKEUCHI[2] and Hiroshi ISHIDAIRA[3]

[1]Student member of JSCE, M. Eng., Graduated School of Civil & Environmental Engineering, Yamanashi University (400-8511, Kofu 4-11-3, Yamanashi, Japan)
[2]Member of JSCE, Ph. D, Professor, Department of Civil & Environmental Engineering, Yamanashi University (400-8511, Kofu 4-11-3, Yamanashi, Japan)
[3]Member of JSCE, Dr. of Eng., Associate Professor, Department of Civil & Environmental Engineering, Yamanashi University (400-8511, Kofu 4-11-3, Yamanashi, Japan)

For the large-scale 2D and quasi-3D groundwater flow numerical simulation based on FEM, spatial discretization scale has significant influence on the result of simulation. A large element size may result in inappropriate local water balance, and then lead to spurious oscillation. Although Zhang et al. (2001) showed the criterion on "time step" to avoid the spurious oscillation for the fully implicit quasi-3D groundwater flow finite element scheme, the criterion of "spatial scale" against spurious oscillation in quasi-3D groundwater flow finite element solution has not been given. In this paper, the criteria to select spatial scale L for the 2D and quasi-3D groundwater finite element schemes are derived based on Zhang et al. (2001), and the effects of spatial scale on spurious oscillation solutions in the finite element solutions of 2D and quasi-3D groundwater flow models are discussed using two examples.

Key Words: *quasi-3D groundwater flow model, spatial discretization scale, spurious oscillation*

1. INTRODUCTION

A number of quasi-3D groundwater flow numerical models using FEM have been applied to multi-aquifer groundwater resources management at different scale. Medium- and small-scale models are used for local groundwater management, and large-scale models are used mostly to address issues related to regional or basin scale management. However, for the large-scale quasi-3D groundwater flow simulation based on FEM, spatial scale has significant influence on the result of simulation. A large element size may perhaps result in inappropriate local water balance[1), 2), 3)], and then lead to spurious oscillation.

In the 1970's, the spurious oscillation problem had received attention by Neuman et al.[1), 2)] and Fujii[3)]. It was shown that when storage mass matrix is non-diagonal, if the maximum principle is to be preserved, the time step Δt must not be too small. Wood[4)] discussed spurious oscillation in the 4-node finite element scheme of the unsaturated flow equation, and gave temporal scale criterion to avoid spurious oscillation.

More recently, Chen and Ewing[5)] carried out an analysis on the stability and convergence of finite element method for 2D reactive transport in groundwater on basis of the maximum principle, and proposed a mixed finite element method without spurious oscillation. Lal[6)] used Fourier analysis to evaluate numerical errors of 2D groundwater flow model, and discussed impact of temporal and spatial scale on numerical errors. However, in all above papers, spurious oscillation analyses are defined by the 2D saturated and unsaturated groundwater flow problems. For quasi-3D groundwater flow model, although Zhang et al.[7)] showed the criterion of "time step" to avoid the spurious oscillation for the fully implicit quasi-3D groundwater flow finite element scheme, the criterion on "spatial scale" against the spurious oscillation has not been given.

In this paper, the mixed finite element discretizations of quasi-3D groundwater flow model are proposed, and the stability analyses of the discretizations in the case of time weighting factor θ being equal to 1 and 0.5 are carried out based on the maximum principle.

Secondly, the criteria on selection of spatial scale L to avoid spurious oscillation for the 2D and quasi-3D

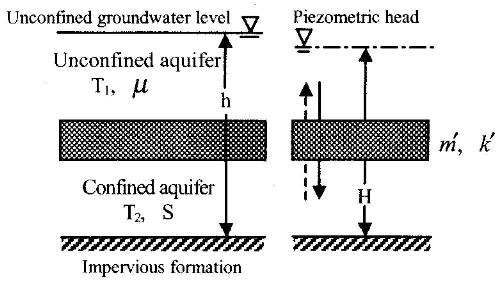


Fig. 1 Schematic diagram of two-layered aquifer system

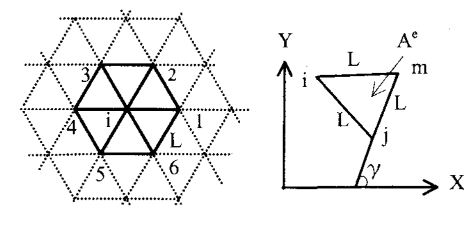


Fig.2 Subdomain divided into equilateral triangular elements

groundwater finite element schemes are derived when time weighting factor θ is equal to 1 and 0.5. Especially, in the case of time weighting factor is equal to 0.5, Neuman's result is corrected.

In the last section, two examples of 2D and quasi-3D groundwater flows are presented to verify the criteria of spatial scale, and effects of spatial scale on oscillation solutions in the finite element solutions of 2D and quasi-3D groundwater flow models are discussed.

2. QUASI-3D GROUNDWATER FLOW MODEL AND DISCRETIZATION

(1) Quasi-3D groundwater flow model

In general, the continuity equation of unconfined groundwater flow in a multi-aquifer system is nonlinear. However, if ratio of drawdown to saturated thickness is less than 20%, then for a nonlinear free-surface model the linear contribution is anywhere between 75 and 100% of the drawdown due to pumping. In other words, the linear approximation to some nonlinear models is sufficiently accurate[8)]. Therefore, a two-aquifer system as shown in **Fig.**1, may be described by following linearized equations.

a) For unconfined aquifer the flow is described by:

$$\frac{\partial}{\partial x}(T_1\frac{\partial h}{\partial x})+\frac{\partial}{\partial y}(T_1\frac{\partial h}{\partial y})+\frac{k'}{m'}(H-h)+w_1=\mu\frac{\partial h}{\partial t} \quad (1)$$

where h, H are hydraulic heads in the phreactic and confined aquifers; μ, T_1 are storativity and transmissivity in the unconfined aquifer; m', k' are thickness and hydraulic conductivity of the semi-pervious layer; w_1 is sink and source per area in unconfined aquifer.

b) For confined aquifer the flow is described by:

$$\frac{\partial}{\partial x}(T_2\frac{\partial H}{\partial x})+\frac{\partial}{\partial y}(T_2\frac{\partial H}{\partial y})+\frac{k'}{m'}(h-H)+w_2=S\frac{\partial H}{\partial t} \quad (2)$$

where S is storativity in the confined aquifer; T_2 is transmissivity in the confined aquifer; w_2 is sink and source per area in confined aquifer.

(2) The finite element scheme with time weighting factor

In order to discretize Eq.(1) and Eq.(2) in space, triangular elements for each aquifer are adopted. The finite element meshes for all aquifers are identical to each other. Application of Galerkin method together with finite differences in time leads to the following matrix equations.

a) For the unconfined aquifer

$$\mathbf{P}_1[\theta\mathbf{h}^{k+1}+(1-\theta)\mathbf{h}^k]-\mathbf{R}_1[\theta(\mathbf{H}^{k+1}-\mathbf{h}^{k+1})+(1-\theta)(\mathbf{H}^k-\mathbf{h}^k)]+\mathbf{F}_1\frac{\mathbf{h}^{k+1}-\mathbf{h}^k}{\Delta t}=\mathbf{W}_1 \quad (3)$$

b) For the confined aquifer

$$\mathbf{P}_2[\theta\mathbf{H}^{k+1}+(1-\theta)\mathbf{H}^k]-\mathbf{R}_2[\theta(\mathbf{h}^{k+1}-\mathbf{H}^{k+1})+(1-\theta)(\mathbf{h}^k-\mathbf{H}^k)]+\mathbf{F}_2\frac{\mathbf{H}^{k+1}-\mathbf{H}^k}{\Delta t}=\mathbf{W}_2 \quad (4)$$

where θ is a time weighting factor satisfying $0\le\theta\le 1$, Δt is time step, and k indicates the number of time step. If the total number of nodes in each individual aquifer is N, then $\mathbf{h}^k$, $\mathbf{H}^k$ are the N-dimensional vectors of nodal head values at time t_k in the unconfined and confined aquifers, and $\mathbf{h}^{k+1}$, $\mathbf{H}^{k+1}$ are the same vectors at time $t_{k+1}=t_k+\Delta t$. In addition, $\mathbf{P}_1$, $\mathbf{P}_2$ are $N\times N$ symmetric positive semi-definite conductance matrices in the unconfined and confined aquifers; $\mathbf{R}_1$, $\mathbf{R}_2$ are $N\times N$ diagonal leakage matrices; $\mathbf{F}_1$, $\mathbf{F}_2$ are $N\times N$ symmetric storage mass matrices in the unconfined and confined aquifers. For any element *e*, $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{F}_1$, $\mathbf{F}_2$ may be written as:

$$\begin{cases}\mathbf{P}_1^e=\dfrac{T_1^e}{4A^e}\mathbf{C}_1^e,\mathbf{P}_2^e=\dfrac{T_2^e}{4A^e}\mathbf{C}_1^e\\ \mathbf{R}_1^e=\mathbf{R}_2^e=\dfrac{k'A^e}{12m'}\mathbf{C}_2^e\\ \mathbf{F}_1^e=\dfrac{A^e\mu^e}{12}\mathbf{C}_2,\mathbf{F}_2^e=\dfrac{A^eS^e}{12}\mathbf{C}_2\end{cases} \quad (5)$$

where A^e represents the area of the triangle *e*, and the total number of elements *e* is M. T_1^e, T_2^e represent transmissivities of element e in the unconfined and confined aquifers. μ^e, S^e are storativities of element e in the unconfined and confined aquifers. In which

$$\mathbf{C}_1^e=\begin{bmatrix}b_ib_i+c_ic_i & b_ib_j+c_ic_j & b_ib_m+c_ic_m\\ b_ib_j+c_ic_j & b_ib_i+c_ic_i & b_jb_m+c_jc_m\\ b_ib_m+c_ic_m & b_jb_m+c_jc_m & b_ib_i+c_ic_i\end{bmatrix},\ \mathbf{C}_2^e=\begin{bmatrix}2&1&1\\1&2&1\\1&1&2\end{bmatrix} \quad (6)$$

As shown in **Fig.**2, if the coordinates of node i, j, m are employed, then the terms in Eq.(6) associated with element e are given by

$$\begin{cases} b_i = y_j - y_m, & c_i = x_j - x_m \\ b_j = y_m - y_i, & c_j = x_m - x_i \\ b_m = y_i - y_j, & c_m = x_i - x_j \end{cases} \quad (7)$$

3. THE CRITERION ON THE SELECTION OF SPATIAL SCALE

(1) Spatial scale criterion for quasi-3D finite element scheme

For simplicity, the study domain is divided into equilateral triangle elements with side length L, as shown in **Fig.**2. For confined aquifer, the discretization of Eq.(4) becomes:

$$(\theta-\alpha)(\sum_{j=1}^{6} H_j^{k+1} - 6H_i^{k+1}) + (1-\theta+\alpha)(\sum_{j=1}^{6} H_j^{k} - 6H_i^{k}) - \beta\{\theta[(\sum_{j=1}^{6} H_j^{k+1} + 6H_i^{k+1})$$
$$-(\sum_{j=1}^{6} h_j^{k+1} + 6h_i^{k+1})] + (1-\theta)[(\sum_{j=1}^{6} H_j^{k} + 6H_i^{k}) - (\sum_{j=1}^{6} h_j^{k} + 6h_i^{k})]\}$$
$$= 12\alpha(H_i^{k+1} - H_i^k) \quad (8)$$

where $\alpha = \dfrac{L^2 S}{8T_2 \Delta t}$, $\beta = \dfrac{L^2 k'}{8T_2 m'}$ (9)

According to Eq.(8), head H_i^{k+1} of node i at time t_{k+1} can be written as

$$H_i^{k+1} = \frac{1}{A_i^{K+1}}(A_j^{k+1} \sum_{j=1}^{6} H_j^{k+1} + A_j^k \sum_{j=1}^{6} H_j^k$$
$$+ A_i^k H_i^k + B_j^{k+1} \sum_{j=1}^{6} h_j^{k+1} + B_i^{k+1} h_i^k + B_j^k \sum_{j=1}^{6} h_j^k + B_i^k h_i^k) \quad (10)$$

where

$$\begin{cases} A_i^{k+1} = 6(\theta+\beta\theta+\alpha), & A_j^{k+1} = \theta-\alpha-\beta\theta \\ A_j^k = 1-\theta-\beta+\beta\theta+\alpha, & A_i^k = 6(\theta+\beta\theta+\alpha-1-\beta) \end{cases} \quad (11a)$$

and

$$\begin{cases} B_j^{k+1} = \beta\theta, & B_i^{k+1} = 6\beta\theta \\ B_j^k = \beta(1-\theta), & B_i^k = 6\beta(1-\theta) \end{cases} \quad (11b)$$

The Maximum Principle for the parabolic Eq.(1) and Eq.(2) states that if there are no sources or sinks in the interior of the study region, the value of the hydraulic head H in the interior of the region must lie between the maximum and minimum values of H on the boundary of the region at t=0[3), 4)]. This principle is then used for the discretized equation.

Quasi-3D groundwater flow is composed of horizontal and vertical flows in two directions. Hence, on basis of the above Principle, H_i^{k+1} is a weighted mean with positive weight factors of water heads ($H_j^{k+1}, H_j^k, H_i^k, h_j^{k+1}, h_i^{k+1}$) at its neighboring nodes in time-space, and hence there cannot be a local maximum in the interior of the time-space region, i.e. no spurious oscillation[4), 5)].

The weighting factors of $h_j^{k+1}, h_i^{k+1}, h_j^k, h_i^k$ in Eq.(10) are positive, and the sum of the weighting factors on the right-hand side of Eq.(10) is 1, that is:

$$\frac{1}{A_i^{k+1}}(6A_j^{k+1} + 6A_j^k + A_i^k + 6B_j^{k+1} + B_i^{k+1} + 6B_j^k + B_i^k) = 1 \quad (12)$$

Hence the weighting factors of H_j^{k+1}, H_j^k, H_i^k have to satisfy the following criteria:

$$\begin{cases} A_j^{k+1} = \theta - \alpha - \beta\theta > 0 \\ A_j^k = 1 - \theta - \beta + \beta\theta + \alpha > 0 \\ A_i^k = 6(\theta + \beta\theta + \alpha - \beta - 1) > 0 \end{cases} \quad (13)$$

Neuman[2)] indicated that Eq.(4) is unconditionally stable when $\theta \geq 0.5$. Especially, when $\theta = 1$, Eq. (4) is called fully implicit form, which corresponds to a forward difference scheme in time; whereas $\theta = 0.5$ corresponds to Crank-Nicholson scheme. However, according to Eq.(13), we found that Eq.(4) is not unconditionally stable as what Neuman has claimed in the cases of $\theta = 0.5$ and $\theta = 1$. The fact can be shown as follows.

a) When $\theta = 0.5$, from Eq.(13), the following conditions can be obtained,

$$\begin{cases} \alpha < \frac{1}{2}(1-\beta) \\ \alpha > \frac{1}{2}(\beta-1) \\ \alpha > \frac{1}{2}(1+\beta) \end{cases} \quad (14)$$

Eq.(14) is impossible to be satisfied with any value of β, that is to say, spurious oscillation solution may perhaps appear in Eq.(4). Wood[4)] also obtained similar conclusion, i.e. oscillation solution may perhaps occur in the 2D unsaturated groundwater finite element scheme when $\theta = 0.5$.

The reason is that since storage matrix $\mathbf{F}_2$ is non-diagonal and the third term in right-hand side of Eq.(4) includes both implicit and explicit parts, the coefficients of implicit and explicit parts in Eq.(8) are not equal to 0.5 when $\theta = 0.5$. That is to say, Eq.(8) does not correspond to Crank-Nicholson scheme. It implies that Neuman[2)] neglected the effect of non-diagonal matrix $\mathbf{F}_2$ on the weighting factors of implicit and explicit parts of Eq. (4)[7)].

b) When $\theta = 1$, according to Eq.(13), the following conditions should be satisfied,

$$\begin{cases} \alpha < 1 - \beta \\ \alpha > 0 \end{cases} \quad (15)$$

Substituting Eq.(9) into Eq.(15), the criterion on the selection of element size L, to avoid the spurious oscillation solution in the finite element scheme of confined aquifer flow model, can be expressed as,

$$L_{CA} < \sqrt{\frac{8T_1 m' \Delta t}{\Delta t k' + \mu m'}} \quad (16)$$

where the subscript 'CA' represents 'confined aquifer'.

Similarly, for unconfined aquifer, the criterion of L for finite element scheme can be obtained,

$$L_{UA} < \sqrt{\frac{8T_2 m' \Delta t}{\Delta t k' + S m'}} \quad (17)$$

where the subscript 'UA' represents 'unconfined aquifer'.

Considering Eq.(16) and Eq.(17), we can obtain the criterion of spatial scale L to avoid the spurious oscillation for the quasi-3D groundwater flow finite element scheme. That is

$$L_{Q3D\max} < \min\left(\sqrt{\frac{8T_1 m'\Delta t}{\Delta t k' + \mu m'}}, \sqrt{\frac{8T_2 m'\Delta t}{\Delta t k' + S m'}}\right) \quad (18)$$

where the subscript 'Q3D' represents 'Quasi-3D groundwater model'.

Eq.(18) includes two criteria of unconfined and confined aquifers at spatial scale, and it states that a value of spatial scale L which is larger than that is prescribed by the criteria would imply that spurious oscillation occurs.

(2) Spatial scale criterion for 2D finite element scheme

When hydraulic conductivity k' of semi-pervious layer reduces to zero, i.e. $\beta = 0$, the quasi-3D groundwater flow model is converted to 2D groundwater model. From Eq.(8), H_i^{k+1} of node i at time t_{k+1} also can be calculated by

$$H_i^{k+1} = \frac{1}{6+6\alpha}\{(1-\alpha)\sum_{j=1}^{6} H_j^{k+1} + \alpha\sum_{j=1}^{6} H_j^k + 6\alpha H_i^k\} \quad (19)$$

Based on the maximum principle, the weighting factor of H_j^{k+1} satisfies the following condition,

$$(1-\alpha) > 0 \quad (20)$$

Substituting of Eq.(9) into Eq.(19), the criterion on the selection of spatial scale L to avoid spurious oscillation for 2D groundwater flow finite element scheme is obtained as

$$L_{2D} < \sqrt{\frac{8T_2\Delta t}{S}} \quad (21)$$

where the subscript '2D' represents 'two dimensional groundwater model'.

Eq.(18) and Eq.(21) indicate that when Δt is given, relatively small numerical scheme width L to avoid spurious oscillation, which satisfy the criterion (18) or (21), should be adopted.

Eq.(16)~(21) were derived using equilateral triangle finite element grids. However, for irregular triangle finite element grids, spurious oscillation also can be approximately judged by the criteria[9].

4. VERIFICATION OF SPATIAL SCALE CRITERION AND DISCUSSION

Usually, the calculation area is firstly divided considering hydrogeological information, and then Δt is chosen. However, Δt of groundwater model is often given in the integrated surface-subsurface water modeling. The reason is that Δt of flood routing is small but not in groundwater modeling. Querner[10] indicated that in order to combine the stream routing model and groundwater model, groundwater model use often time steps of 1 day to as much as 10days. In this case, therefore, the selection of element size is important to avoid oscillation.

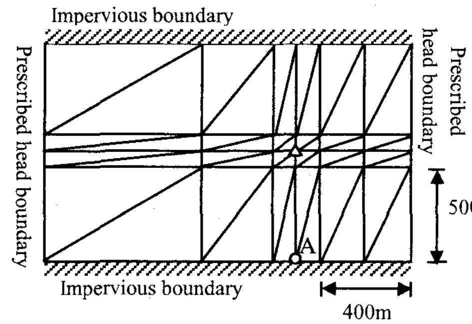


Fig.3 Domain and meshes for simulation study (from Kinzelbach, 1990)

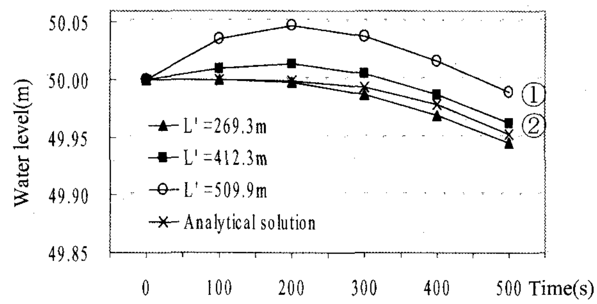


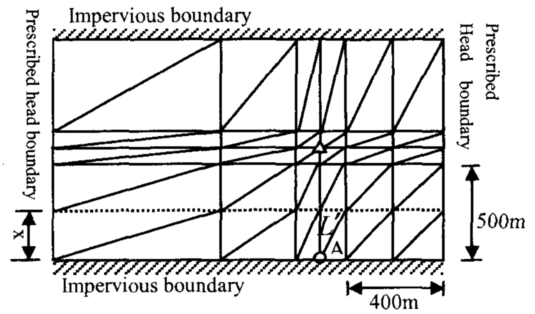
Fig. 4 Results with different L' when Δt =100s. Lines ① and ② are two examples of spurious oscillation when L' =509.9m, 412.3m, which exceed the theoretical limit

The following examples are presented to verify the criteria (18) and (21), as well as to analyze effect of spatial scale on stability of the schemes. The first example is used to check the spurious oscillation criterion (21) for 2D groundwater problem, and exhibits a comparison between result of FEM and the analytical solution. The second example is a quasi-3D groundwater flow simulation of Kofu basin to verify criteria (16), (17) and (18), and to analyze impact of spatial scale on oscillation solution of quasi-3D groundwater flow finite element scheme.

(1) Effect of element size on the oscillation for 2D groundwater flow finite element scheme

A 2D numerical test from Kinzelbach[11] is used to verify the criterion (21), and to analyze the influence of spatial scale on oscillation solution. In the test a pumping well is located at the center of upper- and down-boundary, and the node A for illustration is located at down-boundary. **Fig.**3 shows the location of well and the illustration node A.

The unconfined aquifer has a constant transmissivity of $0.1m^2/s$ and a storativity of 0.001. A uniform initial water level of 50m at each node and constant pumping rate of $0.5m^3/s$ are assumed. In addition, it is also assumed that the water level remains (50m) along the right- and left-hand boundary. The domain was divided into 35 nodes and 48 triangular elements.



X : Distance from down impervious boundary.
L' : The maximum side length of elements around point A.

Fig. 5 Subdivided meshes using dotted line at x

For example shown in **Fig.**3, on the basis of Eq.(21), if Δt is equal to 100s, then theoretical limit of element size L is 282.8m around node A, i.e. in order to avoid spurious oscillation, element size should be smaller than 282.8m. However, the maximum side length L' of elements around node A is 509.9m, it is larger than the theoretical limit. As a result, the solutions are unstable (line ① in **Fig.**4).

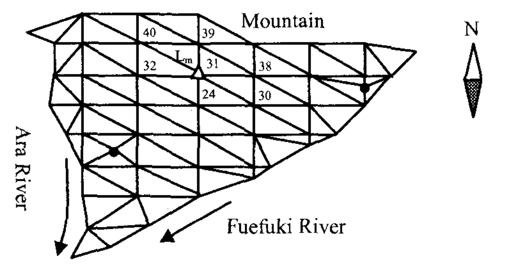
To avoid oscillation solution, we subdivide element around node A using dot line, as shown in **Fig.**5. When x=500m, the length L' is 509.9m. In this case, mesh in **Fig.**5 is same as that in **Fig.**3 L' is greater than theoretical limit of 282.8m, spurious oscillation occurs (**Fig.**4). If the mesh around node A is contracted along with dot line x, the spurious oscillation will decrease with the reduction of x. For examples, if x=400m, then L'=412.3m, oscillation solution is smaller than the solution of L'=509.9m, once x=250m i.e. L'=269.3m (it is smaller than the theoretical limit), spurious oscillation will disappear. The results for L'=509.9m, 412.3m and 269.3m are shown in **Fig.**4.

(2) Effect of element size on the oscillation for quasi-3D groundwater flow finite element scheme

In order to verify equations (16), (17) and (18), a quasi-3D groundwater flow model based on FEM is applied to Kofu basin with area of 63.8km^2. The thickness of the unconfined, confined aquifers and semi-pervious layers are 40, 38 and 20 m respectively. The study area was divided into triangular finite element network with 74 elements and 51 nodes shown in **Fig.**6. Element sizes were chosen based on research report of Nippon Koei CO., LTD. and CTI engineering CO., LTD[11), 12)]. Two rivers flow from the northeast to the southwest (Fuefuki river), and from the north to the south (Ara river) with the lengths of 10.8 km (Fuefuki river), 9.7 km (Ara river), respectively.

a) Calculation conditions

(a) 25m/d and 20m/d of hydraulic conductivities for the unconfined and confined aquifers are assumed, respectively. The conductivity of semi-pervious layer is 1.0×10^{-4} m/d, and the storativities of the unconfined



Δ: Well; 24~40: Nodal numbers; ●: Observed well.
L_m : The maximum side length of elements around node 40

Fig. 6 Study area subdivided into triangular grids in Kofu Basin

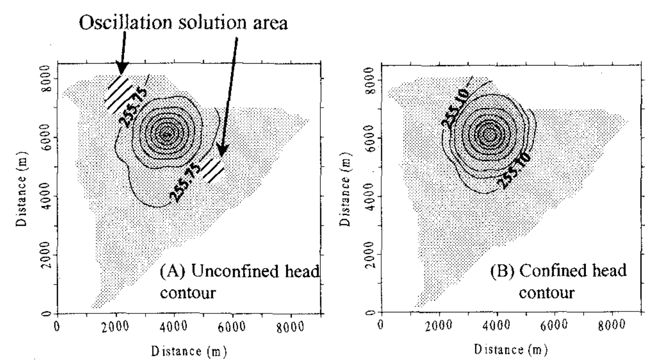


Fig. 7 A spurious oscillation example at t=15d with L_m=1463m (i.e. L_m exceeds theoretical element size limit) in unconfined aquifer and head contour of confined aquifer.

and confined aquifers are 0.05 and 0.025, respectively.

(b) The initial head of 256.0m for the unconfined aquifer at each node is assumed. The piezometric head of the confined aquifer at each node is 255.6m, and the precipitation and pumping water are set to zero.

(c) Fuefuki river and Ara river are assumed with constant head boundaries. The upper boundary closing to mountain is assumed to be impervious boundary.

(d) Constant pumping rates of 0.5m^3/s, 0.1m^3/s from the unconfined and confined aquifers are assumed at node 31, respectively.

b) Effect of element size

For above quasi-3D problem, if time step Δt of 15days is given, according to Eq.(16) and (17), the spatial scale limits of unconfined and confined aquifers are 1448.2m and 1958.3m, respectively, that is to say , the allowed element size (L_{Q3Dmax}) to avoid oscillation is 1448.2m. However, for mesh in **Fig.**6, L_m is equal to 1463m at nodes 40 and 30, and it exceeds L_{Q3Dmax}. As a result, the oscillation appears in the unconfined aquifer in the area of symmetrical nodes 40 and 30 adjacent to well 31, but no oscillation occurs in the confined aquifer (**Fig.**7).

The approach to overcome above numerical difficulty is to subdivide the elements around node 31 by the dotted line shown in **Fig.**8, and taking $L_m \leq$ 1448.2m.

Fig.9 gives different results at node 40 using the meshes in **Fig.**6 (L_m=1463m) and **Fig.**8 (L_m=1130m) with Δt=15days. It is evident that when large element

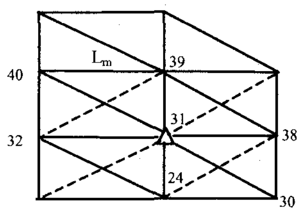


Fig. 8 Subdivided meshes around node 31

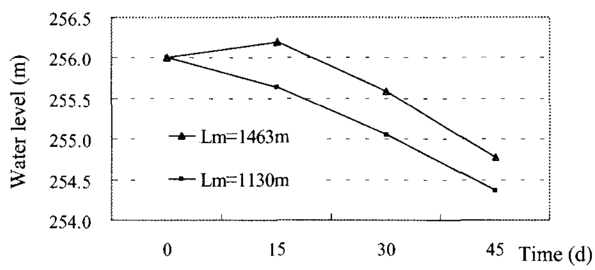


Fig. 9 Unconfined water levels at node 40 using different L_m with $\Delta t = 15 days$

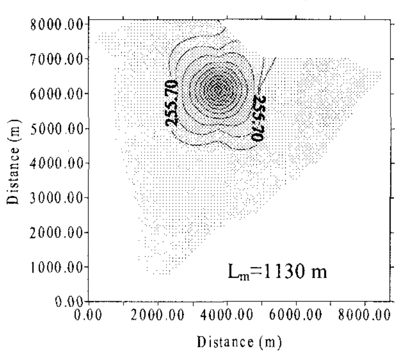


Fig.10 Unconfined water level contour at t=15d with L_m=1130m

size L_m=1463m (it is greater than L_{Q3Dmax}) is applied, the oscillation occurs in the numerical simulation process. If the element size decreases, and taking $L_m \leq L_{Q3Dmax}$, for example, L_m=1130m, the numerical solution is stable at node 40 (**Fig.**9). **Fig.**10 also shows that no oscillation solution appears on the whole of study area when L_m=1130m.

5. CONCLUSIONS

On the basis of above analyses, this study leads to the following conclusions:

1) The spatial scale problem to avoid spurious oscillation for 2D and quasi-3D groundwater flow finite element schemes were mathematically analyzed, and then the spatial scale criteria to avoid oscillation solution for 2D and quasi-3D groundwater finite element schemes were discussed and proposed in the case of $\theta = 0.5$ and $\theta = 1$.

Especially, in this study Neuman's result was corrected, i.e. when $\theta = 0.5$, quasi-3D groundwater flow finite element scheme does not correspond to Crank-Nicholson scheme, the scheme may perhaps lead to oscillation solution.

2) From Eq.(18) and Eq.(21), it is shown that the element size have significant influences on the calculated results of 2D and quasi-3D groundwater flow models, which may perhaps result in inappropriate local water balance, and lead to oscillations. In practice, if Δt is given, relatively small element size L should be selected for quasi-3D and 2D groundwater numerical simulations.

3) As demonstrated in two examples for 2D and quasi-3D models, the spatial scale criteria of (16), (17), (18) and (21) are valid for controlling spurious oscillation problems.

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