AN INVESTIGATION OF APPROPRIATE LES METHOD FOR WIND FLOW OVER TWO-DIMENSIONAL TOPOGRAPHY

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Turbulent flow past a complex topography, represented by an idealized two-dimensional hill with gentle slope is simulated by LES technique. Flow Reynolds number is moderately high to exhibit difficulties in the mostly sought boundary conditions in LES of practical flows. Simulations are performed to illustrate inadequacies of the commonly used boundary conditions and a modification is suggested that include pressure gradient effects. It is found from this study that the log-law modified to account for the local and instantaneous pressure gradient effects is a better alternative to the conventional log-law and non-slip conditions.

Key Words: LES, complex topography, high *Re*, pressure gradient

1.INTRODUCTION

Recent advancements of numerical methods in fluid-flow simulations have made it possible to calculate turbulent flows of various kinds. Large-Eddy Simulation (LES) method, for example, has proved very successful in simulating simple flows over smooth boundaries and is considered to be a very promising tool in engineering and environmental applications. Its application to flows in natural environment, however, is not quite straightforward. Even if a flow of a single phase without interfaces of multiple fluids is to be considered in such applications as atmospheric environment, boundaries are generally rough and their geometry is very complex. Furthermore, the Reynolds number is very large compared with fluid flows in channels and machinery. In LES method, large-scale motion is resolved by discrete computational grid and directly computed by numerical method while motions of smaller scales are modeled. Near solid boundaries, the scale of motion is necessarily small and usually too small to be resolved by numerical grid that can be handled by easily accessible computers and need some kind of empiricism. The models to be used near solid boundaries, however, have not quite been developed to the similar level of sophistication as the subgrid scale turbulent stress models needed to model the small-scale turbulence in flows away from solid boundaries. In simulating atmospheric boundary layer, for example, Deardorff[1)] and more recently Mason[2)] assumed that the velocity profile is described by logarithmic law. While this assumption may be reasonable in flat terrain, it is by no means appropriate for winds over undulating topography such as hills and mountains, where the flow can show such variations as local acceleration, deceleration and even reverse flows. Recent advancements in simulations of flows with much smaller Reynolds numbers, suggest (eg.[3),4)]) to resolve the thin near-wall flow and apply low-Reynolds number corrections such as modification of the eddy viscosity. These near-wall corrections, however, cannot be applied in the case of very large Reynolds number flows such as wind over natural terrain.

In the present work, we try to evaluate a few different methods of representing the flow near solid wall or treatment of the boundary conditions used in LES and point out the inadequacies of such methods in high-Reynolds number applications. Then we propose new methods of representing the near ground velocity profiles and the method of imposing the appropriate ground boundary conditions. The test calculations are performed on an idealized hill for which detailed experimental data are available.

2. DESCRIPTION OF THE TEST FLOW

The flow configuration considered (Fig.1) is that past an isolated hill, a smooth two-dimensional topography, defined by an analytical expression

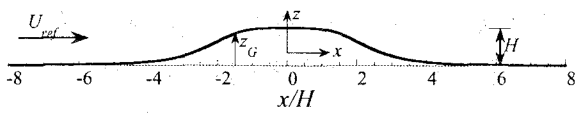


Fig. 1. Flow configuration

$\frac{z_G}{H}=\frac{1}{1+(x/2.3H)^4}$, where z_G is the elevation of the ground at horizontal position x, and H is the height of the hill. x is the horizontal distance from the center of the hill and z is the vertically upward co-ordinate.

This flow has been subjected to a detailed experiemental study as reported in Nakayama and Yokota[5]. Mean velocity and turbulent stresses have been measured for the Reynolds number based on the oncoming velocity U_{ref} and H of 13000. This is relatively a gentle topography without a flow separation. The mean velocity profile, however, deviate from the commonly assumed logarithmic law in the inner layer near the top of the hill. For more details of the experimental data, Nakayama and Yokota[5] may be referred to.

3. NUMERICAL METHODS

The basic equations used in the present LES are three-dimensional, time dependent, Navier-Stokes equations, filtered in order to separate the large scale and the small-scale motions. We consider isothermal incompressible flow and solve the filtered governing equations along with closure subgrid-stress model. As the focus of the present work is a study on the influence of boundary conditions, governing equations are not described here and they can be found in Nakayama and Noda[6] and standard text books[4),7)]. LES model chosen is the conventional Smagorinsky model in which the turbulent stress, R_{ij} is modeled as

$$R_{ij}=\frac{2}{3}k_s\delta_{ij}-2\nu_G S_{ij} \qquad (1)$$

where, k_s is the subgrid turbulent kinetic energy, δ_{ij} is the Kronecker delta, ν_G is the subgrid eddy viscosity and S_{ij} is the strain tensor. The eddy viscosity ν_G is modeled by

$$\nu_G=(C_S\Delta)^2\left[\frac{\partial u_i}{\partial x_j}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right]^{1/2} \qquad (2)$$

where, Δ is the grid size defined by the geometric average of the grid spacings in three directions, $(\Delta x_1\Delta x_2\Delta x_3)^{1/3}$, u_i is the spatially filtered velocity

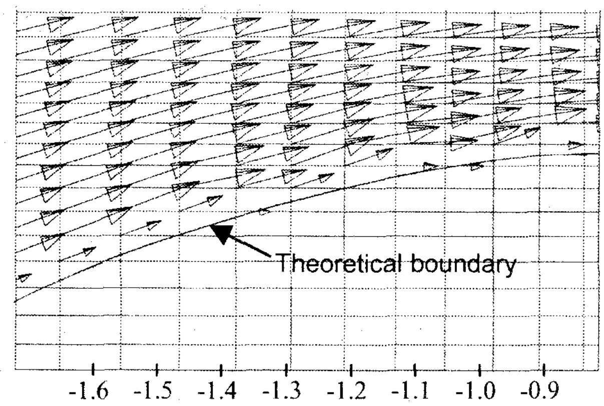


Fig. 2. Enlarged plot of velocity vector near the solid boundary

component in the x_i direction, $(x_1,x_2,x_3)=(x,y,z)$; $(u_1,u_2,u_3)=(u,v,w)$ and C_s is the model constant for which we use the value of 0.13.

(1) Calculation domain and grid

The computational region covers the test flow shown in Fig.1, from about $8.5H$ in the upstream and $14H$ in downstream in the streamwise direction, $7H$ in the cross stream-wise and $4H$ in the spanwise direction. A rectangular grid is used, which is uniformly spaced in the spanwise direction. In the streamwise direction, points are closely spaced (90 points) within $4H$ on either side from the hill summit, stretched with a factor of 1.038. In the cross stream-wise direction, the first point from the ground is placed at $0.03H$ near the bottom of the wall, stretched with a factor of 1.05 upto $0.5H$ and then compressed with a factor of 0.95 upto $1.5H$ and then placed non-uniformly with stretching factor of 1.1 until the end. This grid distribution gives $z^+=zu_\tau/\nu$ of the first node about 20 on the top of hill and about 15 at x/H=4 and thus viscous layer are not resolved. The total grid size is 128x61x21.

The curved boundary is represented by cartesian co-ordinate system with staggered mesh arrangement. The boundary conditions are applied at the mesh points closest to the real boundary, but not exactly on it. In order to find the influence of the approximate position of the boundary, example of calculated velocity vectors along with grid and test case geometry between two streamwise stations are shown in Fig.2. This figure shows that there is no such thing as step corners due to the approximation. Small deviation of the vectors from the direction tangent to the local boundary surface is seen, but this does not influence the results on the whole, with respect to this particular study is concerned.

(2) Numerical schemes

We solve the governing equations by a finite difference procedure. Non-linear convective terms in the equations are discretised by a third order upwind differencing, (UTOPIA) to avoid stability problems and viscous terms are discretised by second-order accurate central differencing scheme. Inflow conditions for the streamwise velocities are adopted from experimental data. Radiation outflow condition is applied at the downstream boundary. The periodic boundary conditions are used for the spanwise direction. In the cross flow direction, the nonslip boundary conditions are applied on the ground surface and slip conditions are applied on the top boundary. HSMAC iteration scheme is used for calculating pressure. Time advancing of the momentum equations is done by a second-order accurate explicit, Adams-Bashforth method. Performance of the code had been assessed earlier for flow past a bluff body and for the curved geometry by Nakayama and Noda[6]. All the calculations are performed with the non-dimensional time step, dtU_{ref}/H of 0.001. All calculations have been allowed to settle down until 40 non-dimensional time units, and then statistical averages over the next 40 non-dimensional time units are obtained that are presented below.

(4) Treatment of ground surface boundary conditions

In wall-bounded flows, the only correct boundary condition at the surface is the no-slip condition, but this requires solution of the flow upto the wall with sufficient grid resolution. However, as the Reynolds number increases, boundary layer thickness decreases, resulting in requirement of large number of grid points. In RANS type simulation, wall-function approach is used as one method of way out to meet this condition. But in LES the problem is severe as pointed out by Spalart et.al.[8] and no definite solution has been proposed yet. We perform calculation for the present test case, with non-slip boundary condition as a baseline solution to compare and this case is referred to as Case A.

Hino and Okumura[9] have performed flow over a wavy wall by assuming single-layer linear distribution for the velocity. This approximation is good only when viscous sublayer can be resolved. When the laminar sublayer cannot be resolved by the computational grid, artificial boundary condition may be applied at some distance from the wall. In LES, this technique is recently referred to as "Off the wall" boundary condition, (Cabot[10]). As one method, Werner-Wengle[11] proposed instantaneous two-layer linear-power law velocity distribution. This has been used quite extensively in many LES calculations, reported by Rodi et.al.[12]. However, they cannot be used in separated flows and non-equilibrium flows. This two-layer model is modified into three-layer linear-loglaw version in the format given by Von Karman[13], to specify the boundary conditions for the velocities in the tangential directions, at the first point from the wall. Nakayama et.al.[14] have tested the validity of this boundary condition, for LES of flow over an isolated hill at Re=50000. The maximum slope angle of this is 45 degrees from the horizontal direction and according to the experiment, the flow separates on the lee side and then reattaches. They found that the prediction with log-law boundary condition was closer to the experimental data compared with calculations without it. However, they indicate that the use of the log-law was too dissipative to show significant turbulent fluctuations. We test once again the three-layer linear-log law in the present model and this is referred to as Case B. In this method, an approximation to a wall law given by the following equation is used.

$$u^+ = z^+, \qquad 5 > z^+ > 0 \qquad (3)$$
$$u^+ = 5 \ln z^+ - 3.05, \qquad 30 > z^+ \geq 5 \qquad (4)$$
$$u^+ = 2.5 \ln z^+ + 5.5, \qquad z^+ \geq 30 \qquad (5)$$

where, $z^+=zu_\tau/\nu$ and $u^+=u/u_\tau$ are the non-dimensionalised vertical distance and velocity respectively. The friction velocity, u_τ is calculated from these equations with the velocities at the second point from the wall.

Natural terrain is subjected to wavy topography and the flow past it is subjected local acceleration and deceleration due to pressure gradients and resistance due to roughness of boundary. In Case C, we use the log-law modified to include local and instantaneous pressure gradient effects in the format given by Wilcox[15], to specify velocities at the first point from the wall. The wall function for the velocity modified to include pressure gradient effects is given as:

$$u^+ = z^+, \qquad 10 > z^+ > 0 \qquad (6)$$

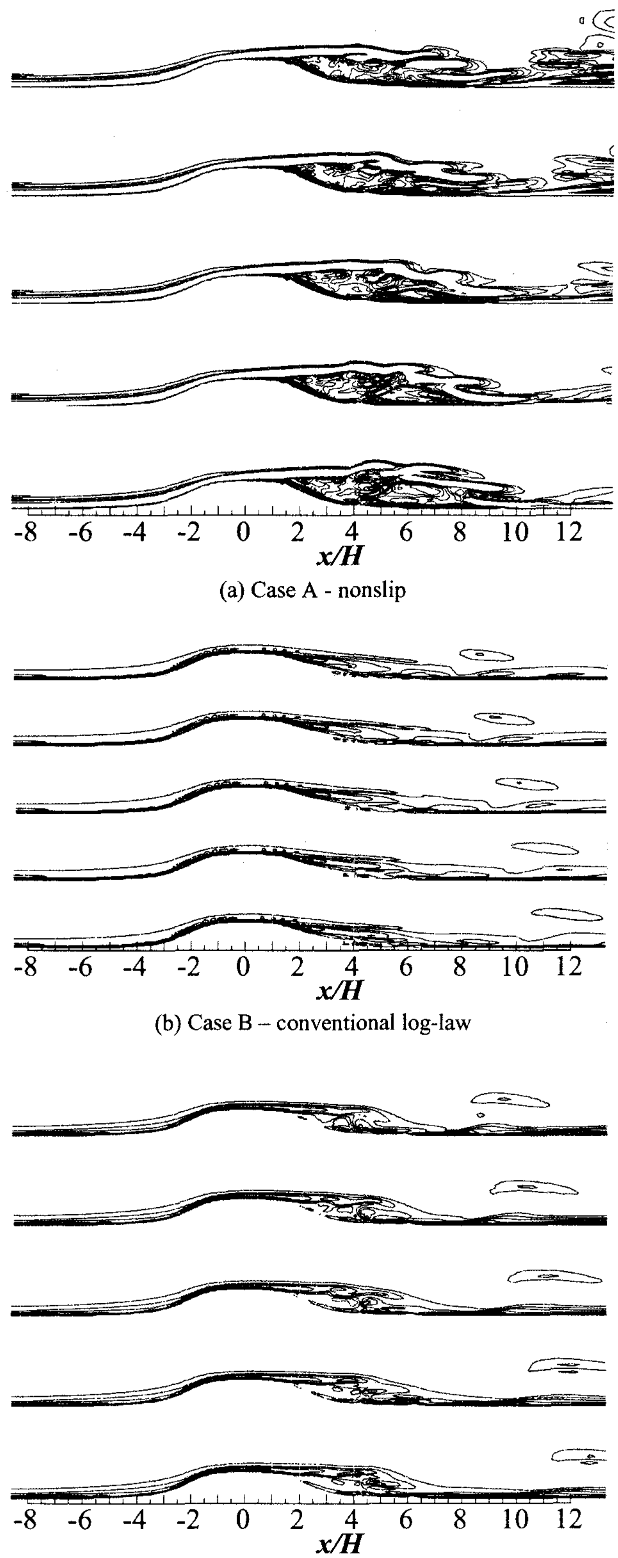
$$u^+ = 2.5 \ln z^+ + 5.5 - C_{pc} z^+ P^+, \quad z^+ \geq 10 \qquad (7)$$

where, P^+ is the dimensionless pressure-gradient parameter defined by $P^+ = \dfrac{\nu}{u_\tau^3}\dfrac{dP}{ds}$, where s is the distance along the boundary. The above equation is proposed for mean velocity profile. In order to apply it to the instantaneous velocity, the pressure correction coefficient C_{pc} is set to 0.005.

4. RESULTS AND DISCUSSION

In order to see the time evolution of the calculation, contours of the instantaneous spanwise vorticity distributions at non-dimensional time interval of 1.0 are plotted in Fig.3. For all the cases, they are shown at the same time, between tU_{ref}/H=41 to tU_{ref}/H=45 and to the same scale. In the calculation using the nonslip condition, the unsteadiness is initiated in the thin boundary layer near the top of the hill and grows into large-scale vortex structure downstream. The mean velocity plots of Fig.4 show separation around x/H=1 and a very large reverse flow region. The results of Case B with conventional log-law shows more steady behaviour and yet present separation and re-circulation region. Calculation by the proposed modification shows some kind of unsteadiness around x/H=2. Both this case and nonslip boundary condition indicate irregular fluctuations appear somewhat downstream and they are more random with small-scale fluctuations superimposed by the large scale vortex structure.

Profiles of time averaged streamwise velocity component, U_1 along specified streamwise stations computed using different boundary conditions and experimental results are plotted in Fig.4. The velocity profile by the calculations at station x/H=-4 matched to that of experiment. At x/H=0, experimental results show that the flow accelerates just near the top of the hill. At the same station, calculation with the nonslip boundary condition (Case A) shows the development of the boundary layer and the maximum velocity is drastically under-predicted. Calculations using the log-law boundary condition (Case B) show a thinner boundary layer and results by the modified log-law (Case C) are seen closer to the experiment. As observed earlier in Fig.3, Case A shows separation at x/H=2 and predicts a large re-circulation zone at further downstream. There the results using the



(a) Case A - nonslip

(b) Case B – conventional log-law

(c) Case C – log-law with pressure gradient effects

Fig.3. Time development of lateral vorticity contours

modified log-law (Case C), in which the effects of one of the locally changing parameters are included, show trends that are in better agreement with the experiment. This implies that to improve simulation results for a complex topography, one needs to include influence of locally and temporally changing parameters.

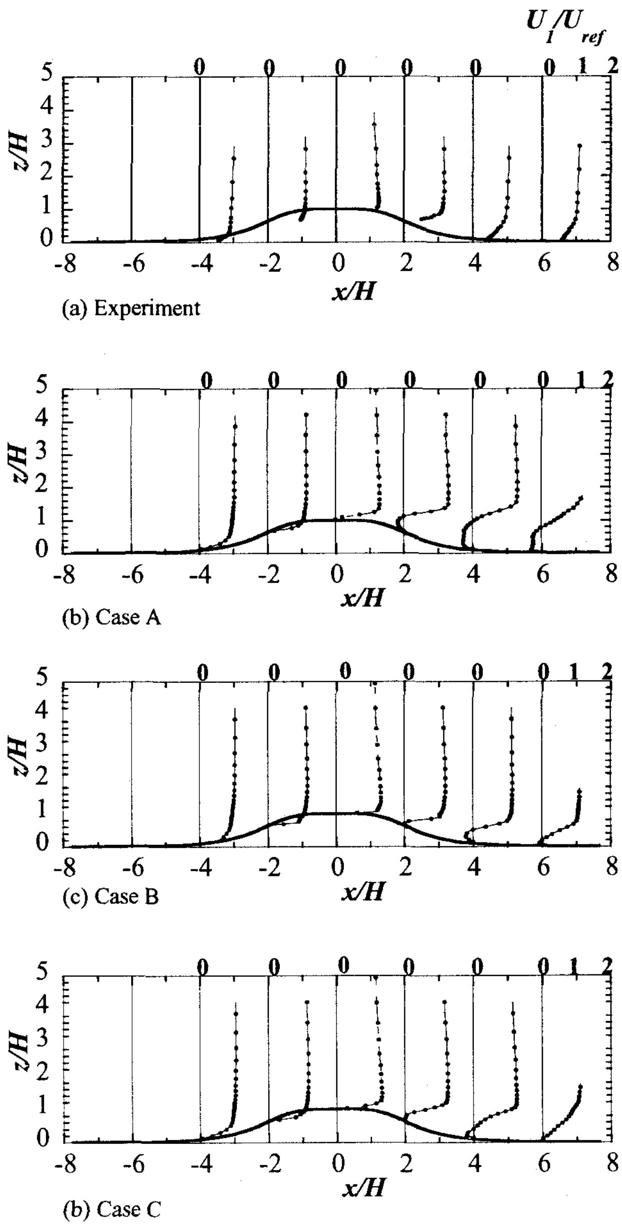


Fig.4. Velocity profiles along selected cross stream sections

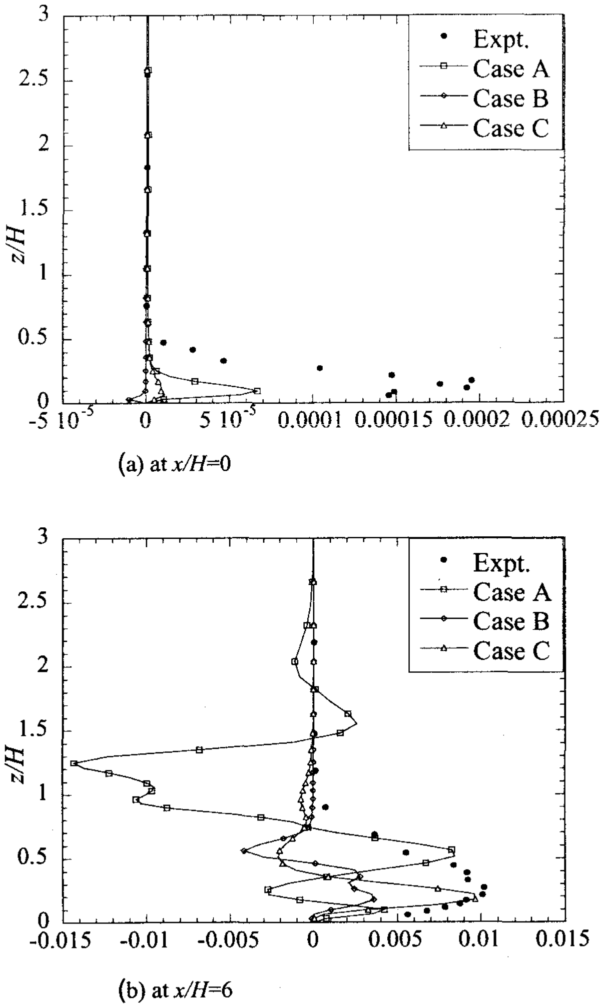


Fig.5. Shear Stress distribution

Fig.5 compares the shear stress distribution calculated by three cases along with that of experiment at x/H=0 and x/H=6 stations. Calculation results are presented after adding modeled part of the shear stress to the resolved part. On top of the hill, all three cases grossly under-predict the distribution and at farther downstream, there is a improvement in the predictions. At x/H=0, prediction with the non-slip boundary condition case is closer to the experimental values. This can be attributed to the fact that early massive separation caused is responsible for turbulent production. Prediction using the conventional log-law is the worst among and this trend substantiates the earlier observation from Fig.3 that very small turbulence is produced. At $x/H=6$, the results of Case A shows a large negative shear stress which is also due to the large separation. Here the modified boundary condition appears to give results closer to the experiment.

5. CONCLUSIONS

Turbulent flow past an idealized two-dimensional hill, with gentle slope at moderate *Re* number has been investigated, to identify the appropriate boundary condition to be used in LES of practical high Reynolds number flows and those over a complex topography. At first, two existing boundary conditions – nonslip and conventional log-law assumption are studied and their limiting behavior is elucidated. A modification in the conventional log-law to include local and instantaneous pressure gradient effect that reflects local acceleration and deceleration due to changing topography is proposed. This method improved the simulation results. It is found from the present study that for LES of the flow near ground, by considering other parameters such as the rate of temporal change and three-dimensionality, further refinements may be possible.

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