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IMPROVEMENT IN A GENETIC ALGORITHM FOR OPTIMIZATION OF RUNOFF-EROSION MODELS

Celso A. G. SANTOS[1], Koichi SUZUKI[2], Masahiro WATANABE[1]

[1]Member of JSCE, Dr. of Eng., Associate Professor, Dept. of Civil and Environmental Eng., Ehime University
(3-Bunkyo-cho, Matsuyama 790-8577, Japan)
[2]Member of JSCE, Dr. of Eng., Professor, Dept. of Civil and Environmental Engineering, Ehime University

In order to provide a robust tool to be used in runoff-erosion modeling, the present paper introduces new evolution steps in the SCE-UA genetic algorithm, which is based in the simplex theory. The new evolution steps were conceived in order to improve the efficiency of such an algorithm. Thus, they will theoretically expand the simplex in a direction of more favorable conditions, or contract it if a move is taken in a direction of less favorable conditions. Hence, these new evolution steps enable the simplex both to accelerate along a successful track of improvement and to home in on the optimum conditions. Therefore, it will usually reach the optimum region quicker than the previous version and pinpoint the optimum levels more closely. The new proposed algorithm is tested with special mathematical functions, as well as in the optimization of the erosion parameters presented in a physically-based runoff-erosion model. On the basis of these simulation results, the mean erosion parameter values are given, which agree with previous values reported to the same area. Thus, the new algorithm can be considered as a promising tool to optimize physically-based models as well as other kinds of models.

Key Words : *genetic algorithm, runoff-erosion model, physically-based model*

1. INTRODUCTION

New robust techniques for parameter calibration of physically-based erosion models have always been investigated due to the difficulties involved in such calibration. The evolutionary algorithms have proven to be robust in optimization process, because the natural evolution is a population based optimization process. Thus, simulating this process by computer results in optimization techniques that can often outperform classical methods of optimization when applied to difficult real-world problems.

Santos et al.[1)] tested a genetic algorithm named the shuffled complex evolution (SCE-UA) developed by Duan et al.[2)], which showed promising performance to optimize parameters of conceptual rainfall-runoff models. Their results showed that the SCE-UA could be used in physically-based erosion model optimization, but to assure that the method could pinpoint the optimum point faster and more closely, some improvement should be introduced; hence, making the method a more robust tool. The SCE-UA method applies a simplex downhill search scheme[3)] for the evolution of each complex; thus, in order to improve its efficiency in terms of how to reach the global minimum, new evolution steps are introduced into the search scheme.

The next sections will describe the simplex downhill search scheme present in the SCE-UA method, but including the new evolution steps, and then test if this modified method is capable of finding the global minimum of test mathematical functions. Finally, the modified method is applied to optimize the main erosion parameters of a fundamental sheet erosion model developed specially for small watershed. The selected area is a 0.48 ha experimental micro-basin located in northeastern Brazil.

2. MODIFING THE SCE-UA METHOD

The SCE-UA method can be considered a robust tool for typical optimization problems because it embodies the following desirable properties: (1) global convergence in the presence of multiple regions of attraction; (2) ability to avoid being trapped by small pits and bumps on the objective function surface; (3) robustness in the presence of differing parameter sensitivities and parameter interdependence; (4) non-reliance on the availability of an explicit expression for the objective function or the derivatives; and (5) capacity of handling high-parameter dimensionality. These properties characterize the problems encountered in model calibration, inclusive of physically-based erosion model calibration.

The SCE-UA method is based on a synthesis of four concepts: (1) combination of deterministic and probabilistic approaches; (2) systematic evolution of a "complex" of points spanning the parameter space, in the direction of global improvement; (3) competitive evolution; (4) complex shuffling. The steps of the SCE-UA method are (a) generate randomly a sample of s points $x_1, \ldots, x_s$ in the feasible space $\Omega \subset \mathbb{R}^n$, compute the function value f_i at each point x_i, rank the points according to the order of increasing criterion, and partition of the sample into p complexes $A^1, \ldots, A^p$, each containing m points where the first point in the first complex represents the point with the smallest function value, the second smallest value is in the second complex and so on; (b) evolve each complex (community) independently according to the competitive complex evolution (CCE) algorithm; (c) shuffle the complexes; (d) check if any of the pre-specified convergence criteria are satisfied, if so stop, otherwise, check the reduction in the number of complexes and continue to evolve.

The CCE algorithm, based on the Nelder and Mead[3)] simplex downhill search scheme, used by the original SCE-UA presents only three evolution steps: reflection, contraction and mutation. The simplex methods are based on an initial design of $n + 1$ trials, where n is the number of variables. Thus, the simplex is a geometric figure in an n-dimensional space; i.e., a simplex defined by three different trial conditions for two control variables has a shape of a triangle. In the same way, the shapes of the simplex in a one and three variable search space are a line and a tetrahedron, respectively. A geometric interpretation is difficult with more variables, but the basic mathematical approach can handle the search for optimum conditions. In order to improve the evolution process and to make the algorithm reach the optimum region quicker and pinpoint the optimum levels more closely, new evolution steps were introduced in this present paper. These modifications are introduced into the CCE algorithm, then the new algorithm should be called modified competitive complex evolution (MCCE) algorithm, whereas modified SCE-UA or MSCE-UA would be the best denomination for the SCE-UA that uses the MCCE. The MCCE is then presented below:

1. To initialize the process, select q, α, and β, where $2 \le q \le m$, $\alpha \ge 1$ and $\beta \ge 1$.
2. Assign weights as follows. Assign a trapezoidal probability distribution to A^k, i.e.,
$$\rho_i = \frac{2(m+1-i)}{m(m+1)}, i = 1, \cdots, m \qquad (1)$$
The point x_1^k has the highest probability $\rho_1 = 2/(m + 1)$. The point x_m^k has the lowest probability $\rho_m = 2/m(m + 1)$.
3. Select parents by randomly choosing q distinct points $u_1, \ldots, u_q$ from A^k according to the probability distribution specified above. The q points define a "subcomplex", which functions like a pair of parents, except that it may comprise more than two members. Store them in array $B = \{u_j, v_j, j = 1, \ldots, q\}$, where v_j is the function value associated with point u_j. Store in L the locations of A^k which are used to construct B.
4. Generate offspring according to the following procedure: (a) Sort B and L so that the q points are arranged in order of increasing function value and compute the centroid G using the expression:
$$G = \frac{1}{q-1}\sum_{j=1}^{q-1} u_j \qquad (2)$$
(b) Compute the new point $r = 2G - u_q$ (reflection step). (c) If r is within the feasible space Ω, compute the function value f_r and go to step d; otherwise go to step g. (d) If $f_r < f_q$, compute $e = 3G - 2u_q$ (expansion step); otherwise go to step g. (e) If e is within the feasible space Ω, compute the function value f_e and go to step f. (f) If $f_e < f_r$ replace u_q by e and go to step l; otherwise replace u_q by r and go to step l; (g) Compute $c^+ = (3G - u_q)/2$ (positive contraction step). (h) If f_{c+} is within the feasible space Ω, compute the function value f_{c+} otherwise go to step j. (i) If $f_{c+} < f_q$ replace u_q by c^+ and go to step l; otherwise go to step j. (j) Compute $c^- = (G + u_q)/2$ (negative contraction step), and compute f_{c-}. (k) If $f_{c-} < f_q$ replace u_q by c^- otherwise compute the smallest hypercube $H \subset \mathbb{R}^n$ that contains A^k, randomly generate a point z within H, compute f_z, set $r = z$ and set $f_r = f_z$ (mutation step). (l) Repeat steps a-k α times, where $\alpha \ge 1$ is the number of consecutive offspring generated by the same subcomplex.
5. Replace parents by offspring as follows: Replace B into A^k using the original locations stored in L. Sort A^k in order of increasing function value.
6. Iterate by repeating steps 2-5 β times, where $\beta \ge 1$ is the number of evolution steps taken by each complex before complexes are shuffled; i.e., how far each complex should evolve.

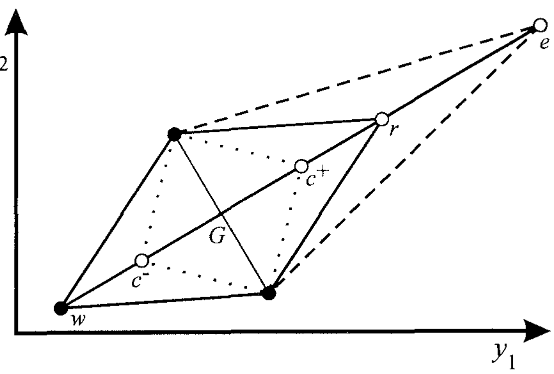

Fig. 1 Example of the evolution steps that can be taken by each complex in a two-variable control space (y_1 and y_2).

If the dimension of the subcomplex is set to $n + 1$, the subcomplex will become a simplex and the local improvement direction could be reasonably estimated by the described evolution steps. **Figure 1** summarizes the evolution steps including the new ones (expansion e and negative contraction c^-) to evolve the worst point w through the centroid G in a subcomplex defined by three different trial conditions (black dots) for two control variables (y_1 and y_2). Differently from the previous version, the mutation step takes place if all evolution steps fail to improve the criterion value, randomly selecting a point in the feasible parameter space to replace the worst point w of the subcomplex. This mutated point is selected according to a normal distribution with the best point of the subcomplex as mean value and using also the standard deviation of the population.

3. PHYSICALLY-BASED MODEL

A distributed physically-based runoff-erosion model called WESP[4)] is used to test the MSCE-UA. This model has been used for simulations in the selected area, and thus the new optimized parameters could be compared with previous studies. The model uses the Green-Ampt equation to model the infiltration:

$$f(t) = K_s\left(1 + \frac{N_s}{F(t)}\right) \tag{3}$$

where $f(t)$ is the infiltration rate (m/s), K_s is the effective soil hydraulic conductivity (m/s), N_s is the soil moisture-tension parameter (m), $F(t)$ is the cumulative depth of infiltrated water (m) and t is the time variable (s).

(1) Plane flow

The plane flow is considered one dimensional. Meaning's turbulent flow equation is given by:

$$u = \frac{1}{n_p} R_H^{2/3} S_f^{1/2} \tag{4}$$

where $R_H(x,t)$ is the hydraulic radius (m), u is the local mean flow velocity (m/s), S_f is the friction slope and n_p is the Manning friction factor of flow resistance for the planes. Thus, the local velocity equation for planes can be obtained making $R_H = h$ and using the kinematic approximation that the friction slope is equal to the plane slope ($S_0 = S_f$):

$$u = \alpha' h^{m'-1} \tag{5}$$

where h is the depth of flow (m), α' is a parameter related to surface roughness, equal to $(1/n_p)S_0^{1/2}$, and $m' = 5/3$ is a geometry parameter.

Sediment transport is considered as the erosion rate in the plane reduced by the deposition rate within the reach. The erosion occurs due to raindrop impact as well as surface shear. The sediment flux Φ (kg/m^2/s) to the flow is written as:

$$\Phi = e_I + e_R - d \tag{6}$$

where e_I is the rate of sediment by rainfall impact, e_R is the rate of sediment by shear stress, and d is the rate of sediment deposition. The rate e_I (kg/m^2/s) is obtained from the relationship:

$$e_I = K_I I r_e \tag{7}$$

in which K_I is the soil detachability parameter (kg·s/m^4), I is the rainfall intensity (m/s), and r_e is the effective rainfall (m/s), which is equal to $I - f$. The rate e_R (kg/m^2/s) is expressed by the relationship:

$$e_R = K_R \tau^{1.5} \tag{8}$$

where K_R is a soil detachability factor for shear stress (kg·m/N$^{1.5}$·s), and τ is the effective shear stress (N/m^2), which is given by:

$$\tau = \gamma R_H S_f \tag{9}$$

where γ is the specific weight of water (N/m^3), and d (kg/m^2/s) is expressed as:

$$d = \varepsilon V_s c \tag{10}$$

where ε is a coefficient that depends on the soil and fluid properties (set to 0.5 in this study), $c(x,t)$ is the sediment concentration in transport (kg/m^3) for the planes, and V_s is the particle fall velocity (m/s) given by:

$$V_s = F_o \sqrt{\frac{(\gamma_s - \gamma)}{\gamma} g d_s} \tag{11}$$

and,

$$F_o = \sqrt{\frac{2}{3} + \frac{36\nu^2}{g d_s^3\left(\frac{\gamma_s}{\gamma} - 1\right)}} - \sqrt{\frac{36\nu^2}{g d_s^3\left(\frac{\gamma_s}{\gamma} - 1\right)}} \tag{12}$$

where γ_s is the specific weight of sediment (N/m^3), v is the kinematic viscosity of water (m^2/s), d_s is the mean diameter of the sediment (m), and g is the acceleration of gravity (m/s^2).

(2) Channel flow

The concentrated flow in the channels is also described by continuity and momentum equations. The momentum equation can be reduced to the discharge equation with the kinematic approximation:

$$Q = \alpha' A R_H^{\,m'-1} \tag{13}$$

where A is the area of flow (m^2). The net sediment flux Φ_c (kg/m/s) for the channel is expressed by:

$$\Phi_c = q_s + e_r - d_c \tag{14}$$

where q_s is the lateral sediment inflow into the channel (kg/m/s), e_r is the erosion rate of the bed material (kg/m/s) obtained from the relation:

$$e_r = a(\tau - \tau_c)^{1.5} \tag{15}$$

in which a is the sediment erodibility parameter, and τ_c is the critical shear stress for sediment entrainment (N/m^2), which is given by the relationship:

$$\tau_c = \delta(\gamma_s - \gamma)d_s \tag{16}$$

where δ is a coefficient (0.047 in the present study), γ_s is the specific weight of sediment (N/m^3) and d_s is the mean diameter of sediments (m).

The deposition term d_c (kg/m/s) in equation (14) is expressed by:

$$d_c = \varepsilon_c T_W V_s C \tag{17}$$

in which ε_c is the deposition parameter for channels, considered as unity in the present case, T_W is the flow top width (m), and $C(x,t)$ is the sediment concentration in transport for the channels (kg/m^2).

4. FIELD EXPERIMENT

SUDENE (Superintendency of Northeast Development, Brazil), ORSTOM (French Office of Scientific Research and Technology for Overseas Development) and UFPB (Federal University of Paraíba, Brazil) operated an experimental basin called Sumé Experimental Watershed, which was located in northeastern Brazil in a typical semiarid area. Several micro-plots operated by simulated rainfall, four micro-basins, nine experimental plots, one sub-basin, and several micro-plots subjected to natural rainfall composed the facilities of such experimental basin. In order to evaluate the runoff and sediment yield, the surface conditions, as well as the slope, for each either micro-basin or experimental plot were maintained differently. Four standard rain gauges and two recording rain gauges were installed close to the micro-basins and plots so that rainfall data could be provided. At the outlet of the basins, a rectangular collector for the measurement of sediment discharge was settled, terminating with a 90° triangular weir for the measurement of flow discharges. The collector could hold all the surface runoff and sediment discharges from most of the low to medium rainfall events, thereby providing a means for accurate runoff and sediment measurement[5)].

One of the four micro-basins of this experimental basin was selected to be used in this work because it was maintained always bare and thus the influence of human intervention, as well as the desertification process, could be also examined. Its mean slope, area and perimeter are 7.1%, 0.48 ha, and 302 m, respectively.

Based on the work of Santos et al.[1)], 45 events were selected between 1987 and 1991, because it was the period in which the surface of the micro-basin was actually maintained bare.

5. TESTING THE MODIFIED SCE-UA WITH MATHEMATICAL FUNCTIONS

This section describes a number of test functions used in assessing the performance of the Modified SCE-UA Algorithm. These functions are drawn from the literature on genetic algorithms, evolutionary strategies and global optimization.

(1) Setting of the genetic parameters

The genetic algorithm contains many probabilistic and deterministic components that are controlled by some algorithmic parameters. For the method to perform optimally, these parameters must be chosen carefully. The first one is m, the number of points in a complex ($m \geq 2$), which should be neither too small, to avoid the search to proceed as an ordinary simplex procedure, nor too large, to avoid an excessive use of computer processing time while no certainty in effectiveness is taken. Then the default value, $m = 2n + 1$, was selected. For the number of points in a subcomplex q ($2 \leq q \leq m$), the value of $n + 1$ was selected because it would make the subcomplex a simplex; this defines a first-order approximation (hyperplane) to the objective function surface and will give a reasonable estimate of the local improvement direction. The number of consecutive offspring generated by each subcomplex α ($\alpha \geq 1$), was set to one to avoid the search becoming more strongly biased in favor of the local search of the parameter space. The number of evolution steps taken by each complex β ($\beta > 0$) was set to $2n + 1$ to avoid a situation in which complexes would be shuffled frequently if set to a small value or to avoid it shrinking into a small cluster if a large value is used. The number of complexes p was set to 2 based on the nature of the problem, and the minimum number of complexes required in the population $p_{\min}$ ($1 \leq p_{\min} \leq p$) was set to p because it gave the best overall performance in terms of effectiveness and efficiency.

Since there are two control variables, n is equal to 2 and the number of points in a complex m is equal to 5 because $m = 2n + 1$. The number of points in a subcomplex q is equal to $n + 1$, thus $q = 3$. The number of consecutive offspring generated by each subcomplex α is set to 1. The number of evolution steps taken by each complex β is equal to 5 because $\beta = 2n + 1$. The number of complexes p is set to 2 thus the population becomes equal to 10, and finally the minimum number of complexes required in the population $p_{\min}$ is set to p.

(2) Mathematical functions

Three test functions were selected to perform the tests: The Rosenbrock, Goldstein-Price[6] and Six-Hump Camel-Back functions[7].

Rosenbrock's valley is a classic optimization problem, also known as the Banana function. The global optimum is inside a long, narrow, parabolic shaped flat valley. To find the valley is trivial, however convergence to the global optimum is difficult and hence this problem has been repeatedly used in assessing the performance of optimization algorithms.

$$f_{\text{Rosen}} = 100\left(y_2 - y_1^2\right) + \left(1 - y_1\right)^2 \qquad (18)$$

in which the control variables are as $-2.048 \leq y_1 \leq 2.048$ and $-2.048 \leq y_2 \leq 2.048$. The global minimum is located at $(y_1, y_2) = (1, 1)$ where the function value is $f_{\text{Rosen}}(y_1, y_2) = 0$.

The Goldstein-Price function f_{Gold} is also a global optimization test function used to test global optimization techniques, which is defined as:

$$f_{\text{Gold}}(y_1, y_2) = \text{Term}_1 \times \text{Term}_2 \qquad (19)$$

where:

$$\text{Term}_1 = 1 + \left(y_1 + y_2 + 1\right)^2 \times \text{Term}_3$$

$$\text{Term}_3 = \left(19\text{-}14y_1 + 3y_1^2\text{-}14y_2 + 6y_1y_2 + 3y_2^2\right)$$

$$\text{Term}_2 = 30 + \left(2y_1\text{-}3y_2\right)^2 \times \text{Term}_4$$

$$\text{Term}_4 = \left(18\text{-}32y_1 + 12y_1^2 + 48y_2\text{-}36y_1y_2 + 27y_2^2\right)$$

in which the control variables are as $-2 \leq y_1 \leq 2$ and $-2 \leq y_2 \leq 2$. The global minimum is located at $(y_1, y_2) = (0,-1)$ where the function value is $f_{\text{Gold}}(y_1, y_2) = 3$.

The 2-Dimensional Six-hump camel back function was another global optimization test function. Within the bounded region are six local minima; two of them are global minima.

$$f_{\text{Sixh}}\left(y_1, y_2\right) = \left(4 - 2.1y_1^2 + y_1^{4/3}\right)y_1^2 + \text{Term}_5 \qquad (20)$$

where:

$$\text{Term}_5 = y_1y_2 + \left(-4 + 4y_2^2\right)y_2^2$$

in which the control variables are as $-3 \leq y_1 \leq 3$ and $-2 \leq y_2 \leq 2$. The global minimum is located at $(y_1, y_2) = (-0.0898, 0.7126)$ or $(y_1, y_2) = (0.0898, -0.7126)$ where the function value is $f_{\text{Sixh}}(y_1, y_2) = -1.0316$.

In spite of the difficulty involved in finding these function global minima, the Modified SCE-UA showed a promising performance in terms of efficiency and effectiveness, because it located the global optimum for each function or pinpoint it more closely within few evolutions; e.g., the final criterion value for the f_{Rosen} was equal to 0.213×10^{-6} after 24 evolutions whereas with the previous version, the correspondent value was 0.131×10^{-5} with 23 evolutions.

Table 1 Recommended values for the genetic parameters.

Genetic Parameters	m	q	α	β	p	$p_{\min}$
General	$2n$+1	n+1	1	$2n$+1	2	p
$n = 2$	5	3	1	5	2	2
$n = 3$	7	4	1	7	2	2

6. APPLICATION WITH FIELD DATA

(1) Selection of the genetic parameters

The genetic algorithm parameters used for this application were set with the same values as used in the application of the method with the mathematical functions as described in section (5.1). That is, n was equal to 3 because there were three control variables. The number of points in a complex m became 7 because $m = 2n + 1$. The value of q, which is the number of points in a subcomplex, was set to $n + 1$, then $q = 4$. The number of consecutive offspring generated by each subcomplex α was set to 1. The number of evolution steps taken by each complex β was equal to 7 because $\beta = 2n + 1$, and the number of complexes p was set to 2 thus the population became equal to 14. Finally the minimum number of complexes required in the population $p_{\min}$ was set to 2, which is equal to the number of complexes p.

(2) Optimization of the physically-based model

Firstly, a scheme of planes and channels was selected to represent the studied area. The schematization of the micro-basin in 10 elements has been reported[8] to be the best scheme to represent the

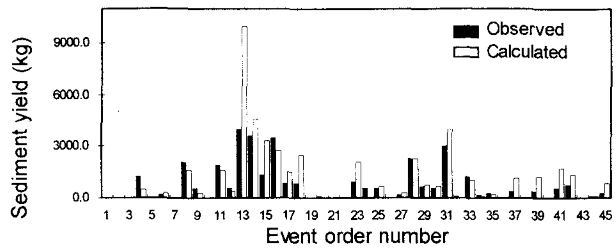

Fig. 2 Observed and simulated sediment yield.

area, thus this schematization was selected in this study. There were four parameters in the WESP model to be determined by optimization. The first was the soil moisture-tension parameter N_s in equation (3), which could be calibrated simply by adjusting the computed runoff depth with the observed value. The remaining three parameters were related to the erosion process, so the optimization had to be done according to the adjustment of computed and observed sediment yield data. Since there were three erosion parameters (a, K_R and K_I) to be calibrated, the Modified SCE-UA method was then used.

The initial values of these parameters were set as $a = 0.0144$ kg·m^2, $K_R = 2.174$ kg·m/N$^{1.5}$·s and $K_I = 5.0 \times 10^8$ kg·s/m^4, and the objective function J to be minimized was:

$$J = \left| \frac{E_o - E_c}{E_o} \right| \tag{21}$$

where E_o is the observed sediment yield (kg) and E_c is the calculated one (kg). The optimization for the 45 events agreed 100% with each event, in which some cases the efficiency improved more than two times. The mean values of the erosion parameters are computed as $a = 0.008$ kg·m^2, $K_R = 2.524$ kg·m/N$^{1.5}$·s, and $K_I = 5.632 \times 10^8$ kg·s/m^4 and they were used then to run new simulations. **Figure 2** shows the simulation results for the sediment yield with some acceptable degree of agreement, except for a few events, which can be attributed to some errors in the observed data as well as to the fact that mean parameter values were used for such runs.

7. CONCLUSION

In order to develop a robust tool to be used in optimization of physically-based erosion models, new evolution steps were introduced to the SCE-UA genetic algorithm, which evolves the point communities according to the simplex search scheme. These new evolution steps were intended to expand the simplex, theoretically, in a direction of more favorable conditions, or contract it if a move is taken in a direction of less favorable conditions. Hence, these steps enable the simplex both to accelerate along a successful track of improvement and to home in on the optimum conditions. Therefore, it will usually reach the optimum region quicker and pinpoint the optimum levels more closely. The tests using special mathematical functions showed that the new algorithm could find their global optimum point, thus proving that it could be used in the optimization of physically-based models. Hence, final tests were performed in the optimization of the main erosion parameters of the WESP model, which is a distributed physically-based runoff-erosion model, and the results showed that the Modified SCE-UA can be considered as a promising tool for further optimizations.

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REFERENCES

1) Santos, C.A.G., Suzuki, K., Watanabe, M. and Srinivasan, V. S.: Optimization of a runoff-erosion model through a genetic algorithm, *Annual J. of Hydraulic Engineering, JSCE*, Vol. 43, p. 557-561, 1999.

2) Duan, Q., Sorooshian, S. and Gupta, V.: Effective and efficient global optimization for conceptual rainfall-runoff models. *Water Resour. Res.*, Vol. 28, No. 4, 1015-1031, 1992.

3) Nelder, J.A., and Mead, R.: A simplex method for function minimization, *Comput. J.*, 7(4), 308-313, 1965.

4) Lopes, V.L.: *A numerical model of watershed erosion and sediment yield*, Ph.D. Thesis, University of Arizona, Tucson, Arizona, 1987.

5) Cardier, E., and Freitas, B.J.: Bacia Representativa de Sumé, primeira estimativa dos recursos de água (Representative Basin of Sumé, first estimation of the water resources), Recife-PE, Brazil, SUDENE, 1982. (in Portuguese)

6) Goldstein, A.A., Price, I.F.: On descent from local minima. *Math. Comput.*, Vol. 25 (115), 1971.

7) Dixon, L.C.W., Szego, G.P.: The optimization problem: An introduction. In: Dixon, L.C.W., Szego, G.P. (Eds.), Towards Global Optimization II, New York, North Holland, 1978.

8) Santos, C.A.G., Suzuki, K., Watanabe, M. and Srinivasan, V. S.: Scale effects of basin elements on coefficients in runoff-erosion modeling, *Annual J. of Hydraulic Engineering, JSCE*, Vol. 38, p. 83-88, 1994.

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