FREE-SURFACE-PRESSURIZED FLOW SIMULATIONS BY F.D.S. SCHEME

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Flux-difference splitting scheme of Roe for free-surface flow simulations is combined with the Preissmann slot to simulate flows in a closed conduit wherein the flow may change from free-surface to pressurized flow and vice-versa. The model can simulate conduits with uniform cross-sections of arbitrary shape, with bed slope and bed friction. The model is verified against available experimental data on free-surface-pressurized flow. Thereafter, the model is tested against some exacting sample problems. It is demonstrated that the model yields very reasonable results in all the cases considered. A sensitivity analysis is performed for the size of the slot and useful conclusions are drawn from the study for the simulation of free-surface-pressurized flows.

Key Words : *free-surface-pressurized flow, Priessmann slot, flux-difference splitting*

1. INTRODUCTION

Flow in a closed conduit can be free-surface or pressurized or free-surface in some reaches while pressurized in others. Flows in a conduit with transitions from free-surface to pressurized flow, and vice-versa are called free-surface-pressurized flows. Such flows may occur in sewers, tailrace tunnel of a hydropower plant, tunnels of morning glory spillway, diversion tunnels etc. Since free-surface and pressurized flows are governed by different equations, the simulation of free-surface-pressurized flows become problematic. However, a comparison of the governing equations for free-surface and pressurized flows reveals that the equations are identical if the depth of flow in the equation for free-surface flows is assumed equal to piezometric head in the case of pressurized flow[1)].

Following this similarity, Priessmann developed a technique wherein a very narrow slot is assumed at the top of the conduit in such a way that it does not add to the wetted perimeter and its contribution to the flow area is negligible. This interesting concept facilitates computation of free-surface-pressurized flow by the shallow water equation alone.

Wiggert[2)] computed free-surface-pressurized flow by a shock-fitting model and verified numerical results with his experimental data. Baines et al.[3)] did some preliminary works on application of Roe's upwind TVD scheme to flows with steep waves in plant channels. They computed only one case of free-surface-pressurized flow and termed their outcome as inconclusive. Capart et al.[4)] used Pavia Flux Predictor scheme to compute flow in sewer pipes and verified their model with experimental and field data. The model was found to accurately compute the considered cases. Garcia-Navarro et al.[5)] presented an implicit method for computing flow in channels and pipes. The model was reported to yield reasonable results for transient flows, particularly flows with continuous or discontinuous steady states.

In this paper, flux-difference splitting scheme of Roe[6)], well studied and found accurate in case of free-surface flow simulations[7,8)], is combined with the Preissmann slot to simulate free-surface-pressurized flows. The model includes bed slope and bed friction. The model is successfully verified against experimental data of Capart et al.[4)] and its applicability is tested with problems of surge propagation in rectangular conduits. Finally, a sensitivity analysis is performed for the width of the slot with respect to the width of the conduit.

2. GOVERNING EQUATIONS

The governing equations for one-dimensional free-surface flows can be written as

$$\frac{\partial \mathbf{U}}{\partial t}+\frac{\partial \mathbf{E}}{\partial x}+\mathbf{S}=0 \tag{1}$$

Where **U** is the vector of unknowns, **E** is the flux vectors and **S** is the vector containing source and sink terms. The vectors are given by

$$\mathbf{U}=\begin{pmatrix}A & uA\end{pmatrix}^T \tag{2a}$$

$$\mathbf{E}=\begin{pmatrix}uA & u^2A+gF_h\end{pmatrix}^T \tag{2b}$$

$$\mathbf{S}=\begin{pmatrix}0 & -gA(S_o-S_f)\end{pmatrix}^T \tag{2c}$$

where A = flow area, u = flow velocity, g = acceleration due to gravity, S_o and S_f are bed and friction slopes, respectively, and F_h is hydrostatic pressure term defined as the first moment of the flow area about the free surface. The flux vector **E** is related to **U** through it's Jacobian **J** as

$$\mathbf{J}=\frac{\partial \mathbf{E}}{\partial \mathbf{U}}=\begin{bmatrix}0 & 1\\ gA/W(\eta)-u^2 & 2u\end{bmatrix} \tag{3}$$

where W(η) is the conduit width at distance η from conduit bottom. The governing equations are known to be hyperbolic, which means that **J** has a complete set of independent and real eigenvectors expressed as

$$\mathbf{e}^{1,2}=\begin{pmatrix}1\\ u\pm c\end{pmatrix} \tag{4}$$

$$c=\sqrt{\frac{gA}{W(h)}} \tag{5}$$

where c = celerity. The eigenvalues of **J** are given by

$$\lambda^{1,2}=u\pm c \tag{6}$$

Roe[6] constructed an approximate Jacobian in place of **J**, which makes the resulting scheme conservative. The approximate Jacobian uses following average values of velocity

$$u_{i\pm1/2}=\frac{A_{i+1}^{1/2}u_{i+1}+A_i^{1/2}u_i}{A_{i\pm1}^{1/2}+A_i^{1/2}} \tag{7}$$

and with the following definition of operators

$$\Delta(\bullet)_{i+1/2}=(\bullet)_{i+1}-(\bullet)_i \tag{8}$$

average celerity is given as

$$c_{i\pm1/2}^2=g\frac{\Delta(F_h)_{i\pm1/2}}{\Delta A_{i\pm1/2}} \tag{9}$$

3. NUMERICAL SCHEME

Roe's first-order accurate flux difference splitting scheme for one-dimensional transient free surface flows can be written as

$$\mathbf{U}_i^{t+1}=\mathbf{U}_i^t-\gamma\left[\mathbf{F}_{i+1/2}^t-\mathbf{F}_{i-1/2}^t\right] \tag{10}$$

where i and t = space and time indices, respectively; γ = Δt/Δx; Δt = time increment and Δx = finite difference grid size in space. The treatment for the source term will be discussed later in the paper. All variables are computed at known time level t, if not indicated otherwise. $\mathbf{F}_{i+1/2}$ and $\mathbf{F}_{i-1/2}$ are called numerical fluxes and are expressed as

$$\mathbf{F}_{i\pm1/2}=0.5\left(\mathbf{E}_i+\mathbf{E}_{i\pm1}\right)-.5\sum_{k=1}^{2}\left|\lambda_{i\pm1/2}^k\right|\alpha_{i\pm1/2}^k\mathbf{e}_{i\pm1/2}^k \tag{11}$$

where α = wave strength, defined as

$$\alpha_{i\pm1/2}^{1,2}=\mathbf{e}_{i\pm1/2}^{-1}\Delta\mathbf{U}_{i\pm1/2} \tag{12}$$

$$\Delta\mathbf{U}_{i+1/2}=\mathbf{U}_{i+1}-\mathbf{U}_i \tag{13}$$

Roe's scheme is conservative and consistent with the governing equations. However, it violates energy inequality condition in case of a rarefaction wave. The most common remedy for this problem is to replace the modulus of λ in Eq.11 by a small positive quantity δ whenever the modulus of λ is less than δ. Value of δ can be set by trial but this paper uses the formula suggested by Harten and Hyman[9]

4. SOURCE TERM

This term in the present study includes bed friction as well as bed slopes. The bed friction is computed by the Manning's formula as

$$S_f=\frac{Q|Q|n^2}{A^2R^{4/3}} \tag{14}$$

where n = Manning's coefficient, Q= discharge and R = hydraulic mean radius defined as R=A/P, P being the wetted perimeter. In case of pressure flow, the following formulas can be used to compute flow depth/piezometric level, hydrostatic pressure term and wetted perimeter. Let A_f = full cross-section area of the conduit, P_f= wetted perimeter at full flow, h_f = maximum height (diameter, d in case of circular pipe) of the conduit, and b_s = width of the slot.

$$h = h_f + \frac{A - A_f}{b_s} \tag{15}$$

$$F_h = A_f\left(0.5h_f + \frac{A - A_f}{b_s}\right) + \frac{(A - A_f)^2}{2b_s} \tag{16}$$

$$P = P_f \tag{17}$$

The bed slope term contains derivative of bed level with respect to independent variable x. Following Roe[10], the bed slope term should be upwinded in the same way as the flux term **E**, the details of which are referred to Jha et al.[11].

5. NUMERICAL STABILITY

The scheme presented herein must satisfy the well known CFL condition for stability. Therefore, the new time step is computed at the end of each step by the following formula

$$\Delta t \le C_n \frac{\min(\Delta x)}{\max(u + c)} \tag{18}$$

where C_n=the Courant number

6. NUMERICAL RESULTS

The model is first verified against available experimental data taken from Capart et al.[4] The experiments were conducted in a 12.74m long closed conduit of circular cross-sections connecting two tanks. The pipe diameter was 0.145m, constant along its length, but had three different longitudinal slopes in three sections as given below;

$0 \le x \le 3.48$	0.01954 m/m
$3.48 \le x \le 9.23$	0.01704 m/m
$9.23 \le x \le 12.74$	0.01225 m/m

A constant discharge of 0.0042 m^3/s was provided from the upstream end throughout the experiment. This results in supercritical flow throughout the pipe. The water level in the downstream tank was then raised by means of an outflow control weir, a jump was eventually formed which traveled upstream in the pipe leaving pressurized flow behind. Just before the jump could reach the upstream end of the pipe, the water level in the downstream tank was drastically reduced which allowed the flow to return to its initial free-surface flow. The measured water level downstream is used as the downstream boundary condition in the

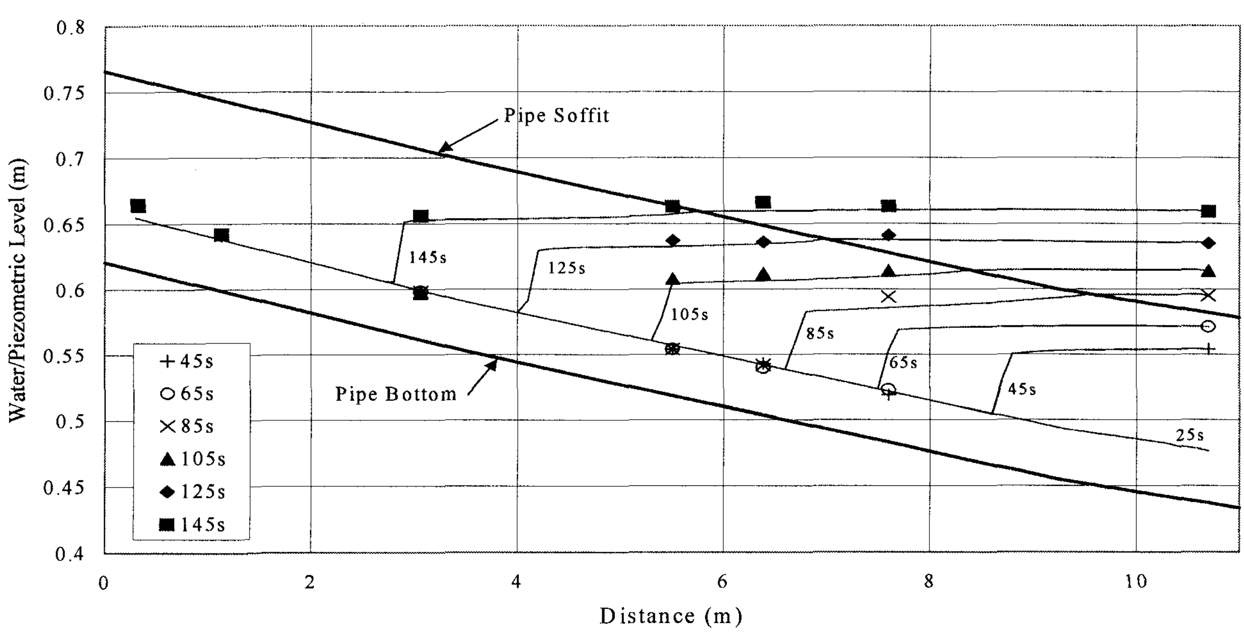

Fig.1 Surge moving upstream in a closed pipe. Line – Computed, Symbol – Observed.

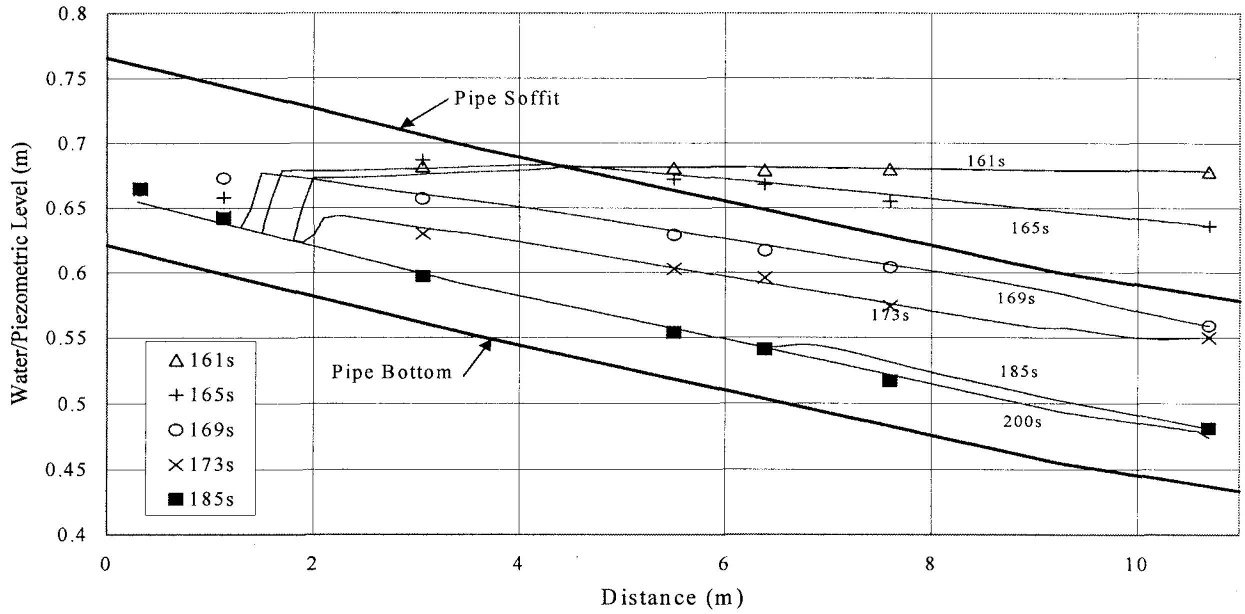

Fig.2 Surge receding downstream following lowering of water level in the downstream tank.
Line – Computed, Symbol – Observed.

numerical computations. The water/piezometric levels were recorded at seven points along the pipe at x-0.325m, 1.135m, 3.06m, 5.505m, 6.835m and 7.6m. Other details of the experiment may be referred to Capart et al.[4)]

The computations are carried out with $\Delta x = 0.1$m and the Courant number equal to 0.6. The slot width is specified as 10% of the pipe diameter. Fig.1 shows the computed and recorded water surface profiles at 25s, 45s, 65s, 85s, 105s, 125s and 145s. During these times, the jump forced by the raising of water level in the downstream tank advances upstream. Soon after 65s, flow in the lower parts of the pipe becomes pressurized while in the rest of the pipe free-surface flow prevails. It can be seen from the figure that the model correctly computes the surge height and the celerity, both in the free-surface and in the pressurized zone.

The water level in the downstream tank rises till 162s, the time when the surge is close to the inlet but yet to cause drowning of the inlet. Then the downstream water level is lowered suddenly. The surge begins to recede towards the downstream end and eventually the initial flow is restored. The water surface profiles during this depressurization phase are shown in Fig.2. The profile at 169s returns to fully free-surface flow and at 200s the initial flow profile has been fully restored. The computed profile again compares very well with the observed data. It may be noted that the experimental data used in this figure are different from Capart et al.[4)]. It has been confirmed through personal communications that the data given in Capart et al.[4)] is partly in error. We have obtained the correct data from the first author for use in this paper.

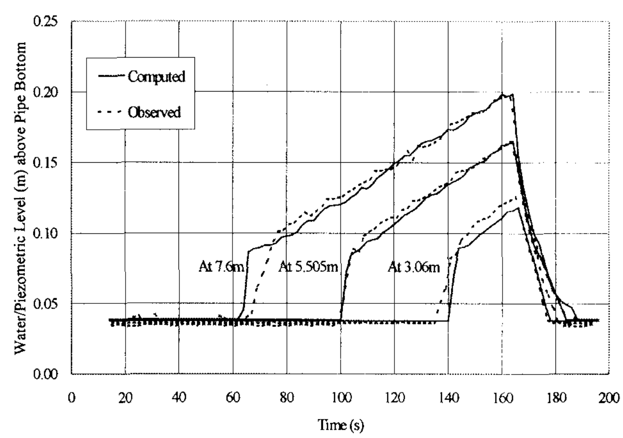

Fig.3 Plot of Piezometric levels at locations along the pipe.

Fig.3 compares computed and observed depth hydrographs at three locations along the pipe. The observed depth hydrographs were obtained by piezometers. The model results compare reasonably well with the recorded data in this figure as well.

The model is now applied to conduits of rectangular cross-sections. The computations are carried out for conduit of length 100m and the base width is assumed to be 1m. The Δx is 1m. The conduit lies horizontal and is frictionless. The computations for these cases are also carried out with a slot width equal to 10% of the conduit width.

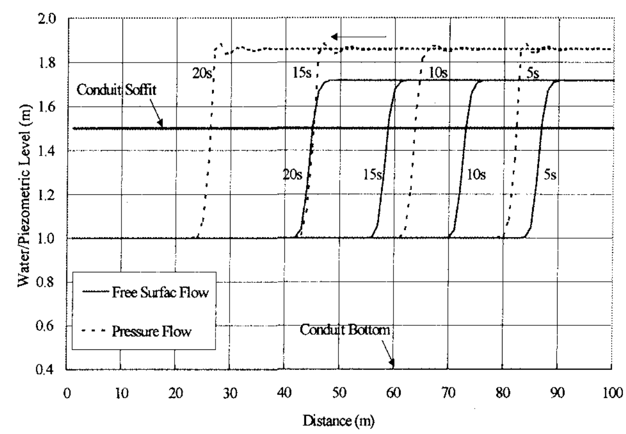

Fig.4 Reflected surge in a rectangular conduit.

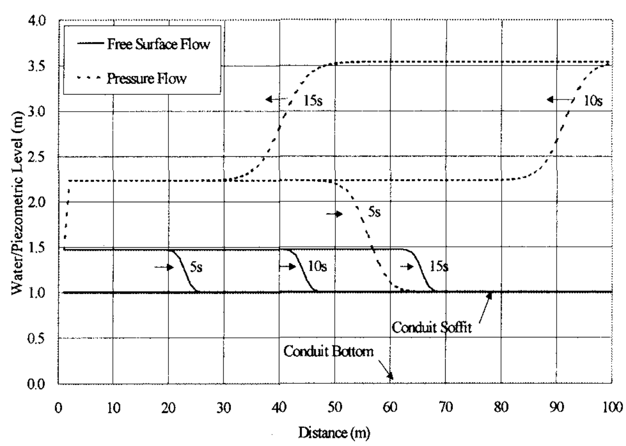

Fig.5 Surge entry and pressurization from upstream.

In the first case, a uniform flow of 1m depth and velocity 2 m/s flows through the conduit. At the start of the computation, a zero outflow condition is imposed at the downstream end that simulates sudden closure of the conduit. A surge is formed which travels upstream leaving still water behind. For the first run, it is assumed that the conduit is open at the top, so that the flow is never pressurized. Thereafter, the conduit is closed at 1.5m height. This causes the conduit to pressurize from the downstream end as the surge is formed. The water surface profiles, for open as well as closed conduit, at 5s, 10s, 15s and 20s are shown in Fig.4. It may be noted that for an open conduit, the analytical solution is also available. The numerical results perfectly agree with analytical solution but the analytical solutions are not shown in the figure for the sake of clarity. As can be seen from Fig.4, the piezometric level rises higher than the fully open conduit case, which is expected. It is also noticed that at the interface between free-surface and pressurized zone, there are some oscillations.

In the second case, the conduit has 1m deep still water in the beginning and is closed at the downstream end. A constant discharge of 2.0 m^3/s is imposed at the upstream end, which creates a surge that travels downstream. As in the previous case, the result for open conduit is obtained first. Thereafter, the conduit is closed at 1m height. This generates pressurized flow with larger piezometric heads and faster celerity. The results are shown in Fig.5. It can be seen that the pressurized flow reaches the downstream end much faster than the case of fully free-surface flow and at 10s and 15s, the pressurized flow is travelling upstream after being reflected from the downstream end. The analytical solution for fully free-surface flow case is again not shown in the figure for the sake of clarity but it is noted that the computed results very well agree with analytical solution for this case. The model reasonably computes pressurized flow and its reflection.

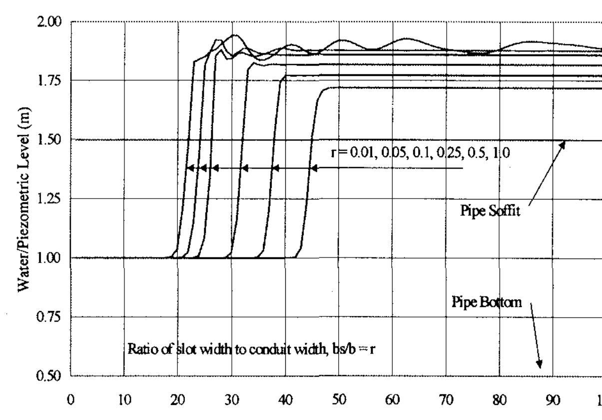

Fig.6 Sensitivity analysis for slot width

Finally, a sensitivity analysis is carried out to examine effect of the size of the slot on the solution. The conduit is 1m wide and 1.5m high with 1m deep water flowing at 2 m/s as initial condition. The propagation of reflected surge following sudden closure of the downstream end is simulated with different ratio r of slot width, b_s to conduit width, b. Five values of r, 0.5, 0.25, 0.1, 0.05, and 0.01, are used. The computed results at 20s are shown in Fig.6. It is seen from the figure that the piezometric level does not change much when r is reduced further from 0.1 and the celerity also becomes only slightly faster. On the other hand, it is seen that oscillations originating at the interface between free-surface and pressurized flow propagate throughout the surge and their amplitude increases considerably. Based on the results, it is felt that 0.1 is the most reasonable value for r. However, this needs to be verified more extensively before a final conclusion can be drawn.

7. CONCLUSIONS

Roe's flux-difference splitting scheme has been applied to free-surface-pressurized flows utilizing the concept of the Preissmann slot. The resulting model has been verified with experimental data for flow in pipes. The model's applicability has been tested with applications to propagation and reflection of surge in rectangular conduits. The slot size used in the model is 10% of the width of the conduit. This value, besides giving accurate results, has been found to be most reasonable through sensitivity analysis. This work shows that the Roe's scheme, with all its good features ascertained in case of free-surface flows, can also be a very good tool for simulating free-surface-pressurized flows. The work is continuing to enhance the formal accuracy of the present model and include internal boundary conditions for simulating branching of channel as well as free-surface-pressurized flow in pipe networks.

ACKNOWLEDGEMENTS

We sincerely thank professor H. Capart for clarifying some experimental conditions in Capart et al.[4] and for providing revised experimental data for the same.

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(Received September 30, 1999)