ANALYSIS ON SPATIAL DISTRIBUTION OF ANNUAL WATER BUDGET AND DAILY RUNOFF ALONG RIVER NETWORK IN A BASIN

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Annual and daily runoffs at most locations of a basin are unknown, which are estimated by the proposed methodology. First a squared region covering the Fuji River basin is divided into 9,450 grids of nearly 1 km^2. Annual or daily precipitation data of 27 gauges in the region are used to derive the precipitation at each grid, by applying a spatial interpolation algorithm called spline function. Monthly or daily mean temperatures at grids are estimated by a step-wise regression method. Second annual water yield at any grid is obtained with Hamon and Pike formulas and the water budget equation. Runoff distribution in the river network drawn out using digital elevation data is estimated. Finally, based on daily rainfall and temperature, the daily runoffs along the rivers are simulated by a distributed hydrologic model. All runoff estimations are checked by the observations in several sites.

Key Words: *Spatial distribution, Rainfall, Temperature, Spline function, Regression, River network, Runoff*

1. INTRODUCTION

Information of water resources and its spatial distribution in a region is of vital importance to flood-drought control, efficient utilization and management of water resources. Flood prediction, dam planning, water conservation and other practices need the knowledge of runoff characteristics (annual, daily, flood-drought runoffs) not only at one or two gauges in a basin but also at many locations without gauges. Some regional-scale studies have been implemented. River flows were derived for southern Africa using grids of 0.5^0 longitude by 0.5^0 latitude (Reynard et al., 1997). Renewable water resources in the European Union were estimated using 10 km grids (Rees et al., 1997). However, high resolution (1 km or smaller grid-cell) distribution of water resources has not been well evaluated, and this resolution of 1 km is more effective to basin-scale problems. Therefore a new and easily-applied method is needed to derive the distribution from the meteorological-hydrological observatories sparsely allocated in a basin. These observed data should be utilized as frequently as possible, although they are sparse in most areas of the world.

The remote-sensing technology can provide spatial information on land cover and soil moisture, but can not provide high resolution distribution of water resources at present stage. On the other hand, GIS (geographical information system) is becoming more and more applicable in environmental planning, resources development and various researches, by providing standard databases of topography, vegetation, soil, geology and landuse. These databases should be also utilized in estimating the distribution of water resources. In fact Estrela et al. (1997) utilized runoff maps and digital terrain models (1 km cell) in a derivation of flow discharges in Spain.

The upper and middle basin (2,179 km^2) of the Fuji River (**Fig. 1**), located at central-southern Japan, is selected as the testing basin. Topography of the basin is low in central and southern parts and high in surrounding mountains. In order to estimate rainfall in space, weather gauges both in and around the basin are taken into consideration, totally 27 gauges are included (18 out of them have temperature data). The region represented by these gauges is a rectangular square with its latitude range being 35.175^0-36.05^0 and longitude range being 137.875^0-139.0^0, and is covering the river basin. A small grid size of latitude 30" by longitude 45" (about 1 by 1 km, the accurate grid area is 1.01659 km^2) is selected, and the whole region is divided into 9,450 grids. Grids system and gauge locations are illustrated in **Fig. 1**.

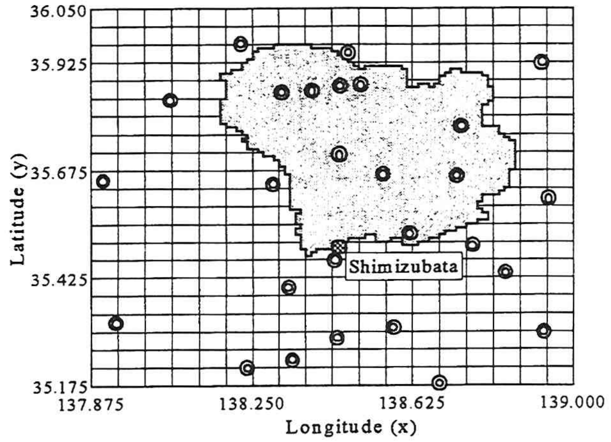

Fig.1 Fuji Basin and gauges

The purpose of this study is to estimate water budget items (rainfall, evapotranspiration, runoff) at each grid, to obtain those budget items within the basin, and then to estimate discharge volume in the river which connects the basin grids. The most important variables are rainfall and temperature, because the evapotranspiration of grids can be estimated from temperature and rainfall, and annual runoff can be estimated from evapotranspiration and rainfall. Therefore a spline-function interpolation method is developed for spatial rainfall estimation, and a step-wise regression is applied for temperature allocation. Derived runoff at grids are used to derive discharge distribution along the river network. Observed discharge data at three sites are used to check the estimated results.

2. CREATION OF GRID RAINFALL

Spatial variation of rainfall in a basin or region is random and irregular, and has not been well understood or quantified. At present a physically-based meteorological model can not provide satisfied rainfall at 1 km scale, although it is applied in meso- or global scales. Radar technique is being used to monitor cloud and rainfall, however a high accuracy in radar rainfall data is not assured. The Thiessen method is simple, but is too rough to give smoothly continuous distribution.

Some efforts were made to analyze the variation by using an interpolation way. Creutin and Obled (1982) made an earlier review of six methods to rainfall fields, namely the nearest neighbor method, the arithmetic mean, spline-surface fitting, the so-called optimal interpolation, the kriging method and interpolation based on empirical orthogonal functions. They pointed out the applicability of each method to independent total event rainfall and the limitation of these methods. In a research by Estrela et al. (1995), rainfall values in cells are obtained with a method called the squared inverse distance. Rubel (1996) interpolated the 12-hour precipitation fields of 55 km resolution for the Baltic Sea Experiment area by using the method of optimal averaging with normalized weights. Rees et al. (1997) used a simple linear interpolation method to obtain annual rainfall values on the 10 km grids.

As for applications of spline functions, Watazu et al. (1980) developed a cubic spline function under tension to estimation of air pollution. Hashino and Kitazawa (1984) used a B-spline function to interpolate the central pressure, moving direction and moving velocity of typhoons. Obled et al. (1994) computed hourly rainfall surfaces of 500m mesh size with the spline surface fitting.

Spline function can be an useful and practical interpolation way, as it can produce a less oscillatory approximate function than a polynomial, and it needs usually less data series than the statistical methods. Therefore the spline method is selected for rainfall distribution. It is simply explained below.

There is a region R in the x-y plain, and some points (x_i, y_i) on the plain correspond to the rainfall data z_i. Then a spline function $Z(x, y)$ which passes through these data points (x_i, y_i, z_i) is to be searched for. An objective index is defined as

$$J(Z) = \int_R [(\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 Z}{\partial y^2})^2 + \sigma(\frac{\partial Z}{\partial x} + \frac{\partial Z}{\partial y})^2]dxdy \tag{1}$$

where σ is the tension parameter. And the cubic function $Z(x, y)$ that minimizes $J(Z)$ in equation (1) is just what to be searched for.

For the region of study, x coordinate is the longitude and y coordinate is the latitude, and rainfall observations of 27 gauges are the data points (Fig. 1). By searching and using the spline function, rainfall values at all grids other than the gauges can be calculated or interpolated. As an initial step, the average annual rainfall for 16 years (1979-1994) is considered. Estimated rainfall field is plotted in **Fig. 2**. Direct test of the estimated results could not be

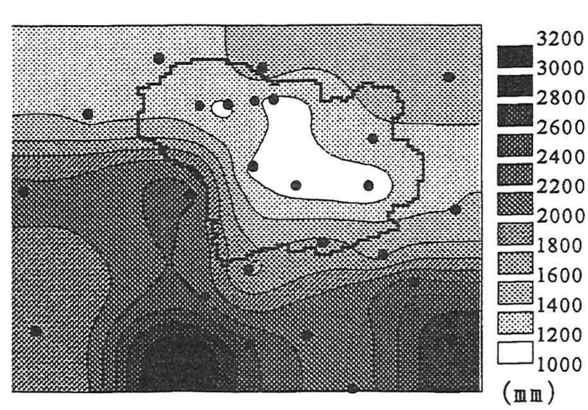

Fig.2 Spatial annual rainfall

done since all rain gauges in the region have been included in the spline calculation.

3. CREATION OF GRID TEMPERATURE

Apart from rainfall, temperature is another important variable (or forcing factor) controlling the water cycle and water budget in a region. It is relatively easier to know the spatial distribution of temperature than rainfall, as the temperature at a site is strongly related to the elevation of site. Statistical relationship of temperature to geographical factors was found out and spatial temperature distribution could be well implemented (Yao et al., 1998; Yao and Terakawa, 1999). In this study a similar and improved method is to be used.

The spatial variability of temperature in a region or basin is mainly controlled by the difference of geographical features such as latitude, longitude and altitude. Other factors like hill slope also have minor influence but were not included, because there is not the corresponding data. The temperature gets gradually lower when latitude and/or altitude get larger. This tendency proves clear especially in case of monthly mean or annual mean values. Ten gauges named Ooizumi, Nirasaki, Katsunuma, Ootsuki, Kamikuishiki, Nakatomi, Kawaguchiko, Yamanaka, Nanbu and Kofu, having temperature data out of the 21 rain gauges, are taken to validate the tendency between temperature and geography. Sixteen years (1979-1994) of monthly-mean temperature are available, and they are further treated into 16-year averages for each of twelve months and for each observatory gauge. These averages are basic data sets for regression analysis.

The Kofu gauge is located at center of these gauges and is operated as a basic weather station, therefore it is taken as a reference site. For each month the regression relationship of temperature to geography is expressed as following multivariate regression equation:

$$T(i,j) - T_0(j) = b_0(j) + b_1(j)[X_1(i) - X_{10}] + b_2(j)[X_2(i) - X_{20}] + b_3(j)[X_3(i) - X_{30}] \quad (2)$$

where i is the number of nine gauges besides Kofu (i=1, 2, ……, 9), j is the month number (j=1, 2, ……, 12), $T(i, j)$ is the monthly temperature at any gauge, $X_1(i)$ is the latitude of any gauge, $X_2(i)$ is the longitude, $X_3(i)$ is the altitude or elevation, $T_0(j)$, X_{10}, X_{20} and X_{30} are the correspondent values at reference Kofu, b_0, b_1, b_2 and b_3 are coefficients. This regression implies that meteorological difference between any site and Kofu can be explained by the difference in geographical features between the two sites. The coefficients in formula (2) are determined with a step-wise regression algorithm (Enslein et al., 1977; Yao et al., 1997).

Obtained coefficients for each month are listed in **Table 1**, giving high correlation ratios (r). The longitude itself should have no effects on temperature, however the regression algorithm found a weak correlation between them, and therefore it was used in equation (2).

Table 1 Coefficients results

Month	b_0	b_1	b_2	b_3	r
1	1.437	0.308	-0.124	-0.643	0.961
2	1.320	0.165	-0.088	-0.473	0.981
3	1.144	0.092	-0.051	-0.244	0.986
4	1.035	0.087	-0.016	-0.137	0.987
5	1.006	0.079	-0.010	-0.101	0.987
6	1.009	0.065	-0.018	-0.080	0.989
7	1.013	0.043	-0.014	-0.065	0.987
8	1.011	0.047	-0.014	-0.067	0.990
9	1.041	0.026	-0.020	-0.076	0.987
10	1.077	0.024	-0.030	-0.103	0.982
11	1.136	0.021	-0.048	-0.138	0.966
12	1.174	0.167	-0.046	-0.301	0.942

Eight gauges out of 18 with temperature data, which were not used to calculate regression coefficients, and a very high observatory (3,775m high) on the top of Fuji Mount in the south-east corner of the region, are used to test the accuracy of regression formula. Estimations of 16-year averages of temperature (in Celsius degree) in nine sites are compared with the observed ones in **Fig. 3**. They have good agreement, with small errors of less than 5%, showing the applicability of regression method to spatial distribution of temperature in the region.

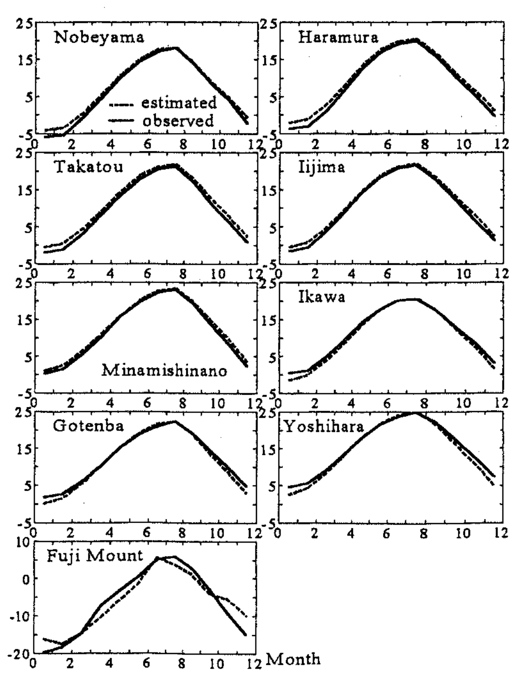

Fig.3 Tests of temperature at nine sites

As a result, the regression (2) and coefficient values in **Table 1** can be applied to give temperatures at all grids. Required latitude, longitude and altitude values of grids are picked out from a national GIS database of Japan. 16-year mean temperatures of 12 months are derived respectively. As an example, the distribution of annual-mean temperature (the mean of 12 months' temperatures) is plotted on **Fig. 4**. The temperature is higher in central and southern plain parts than in surrounding mountains, corresponding well to the elevation variation.

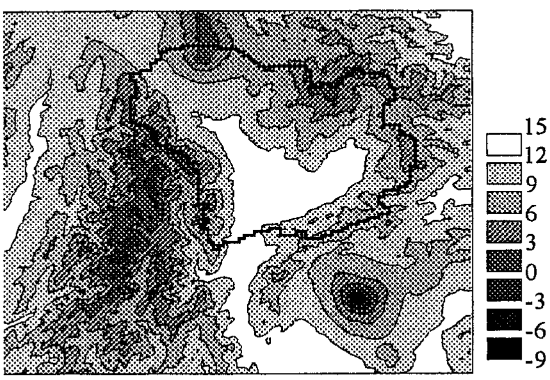

Fig.4 Spatial mean temperature

After obtaining rainfall and temperature values, annual water budgets of grids can be evaluated as follows.

4. WATER BUDGTS OF GRIDS

Annual water budget for a grid or a watershed is typically expressed as

$$R = P - E \qquad (3)$$

in which P is annual rainfall, E is evapotranspiration, and R is runoff generated in the grid. P has been already estimated above.

A simple experiential formula for annual evapotranspiration was first proposed by Turc and modified by Pike (1964), and applied by some researchers (Rees et al., 1997; Estrela et al., 1995). It is written below.

$$E = P / (\sqrt{1 + (P / E_P)^2}) \qquad (4)$$

where E_P is the annual potential evapotrans This formula was proved applicable in Europe and should be applicable to Japan where rainfall is more than the Europe, although many affecting factors such as land use, climate trip are not explicitly included in the formula. This formula is selected for use as formulas for annual evapotranspiration are rarely available.

E_P is the sum of 12 month evapotranspirations that can be obtained with the Hamon formula.

$$E_P = \sum_{m=1}^{12} E_{Pm} \qquad E_{Pm} = 0.14 D^2 \cdot P_t \cdot M$$

$$P_t = 4.5 \times 10^{\frac{7.5 T_m}{T_m + 273}} \qquad (5)$$

where E_{Pm} is monthly potential evapotranspiration, D is maximum sunshine duration dependent on the month and latitude, M is day number in a month, and P_t is absolute saturation air humidity determined by month-mean temperature T_m which was estimated before.

Therefore, based on grid-oriented temperature in **Fig. 4**, potential evapotranspiration is estimated with formula (5), actual evapotranspiration is estimated with formula (4) using grid rainfall, and finally runoff of each grid is obtained with formula (3).

Furthermore, those 2,160 grids belonging to the Fuji River basin are identified from 9,450 grids of the whole region, and the corresponding rainfall, evaporation and runoff values are also picked out. Then the 16-year averaged annual budget items are derived spatially in the basin and illustrated in **Fig. 5**. Speaking roughly, grid rainfall gets smaller from south to north and is most less in the central Kofu plain of the basin; evapotranspiration is larger in southern and central parts than in the surrounding mountains; and distribution of grid runoff is similar to that of rainfall, less in the center. Annual discharges (in unit of million tons) along the river network is also displayed in the figure, which will be explained in the following.

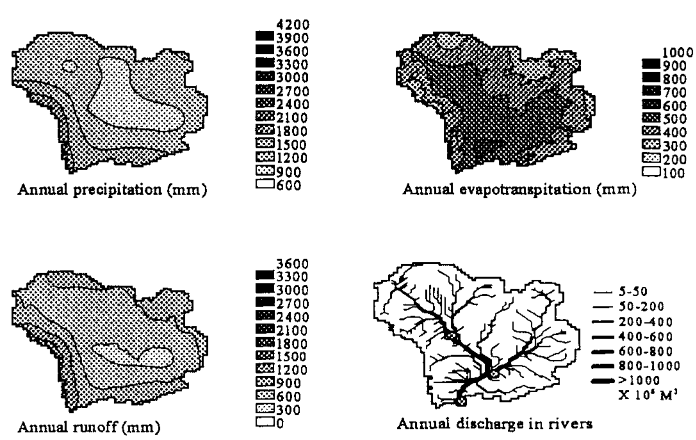

Fig.5 Distribution of water budget

5. CREATION OF RIVER DISCHARGES

A simplified and approximate river network for the real river system is produced by means of digital elevation method, and is shown in **Fig. 6**. At first it is supposed that there must be a stream link within any grid of 1 km size, and the link collects the runoff generated on this grid and drains the runoff out off grid. Then the drainage direction, e.g. where should the link be connected, is determined from elevations of eight grids neighboring this center grid. In general, the largest gradient direction between this grid and a neighboring one is the drainage direction (water flow direction). Finally all links of all grids formulate the river network.

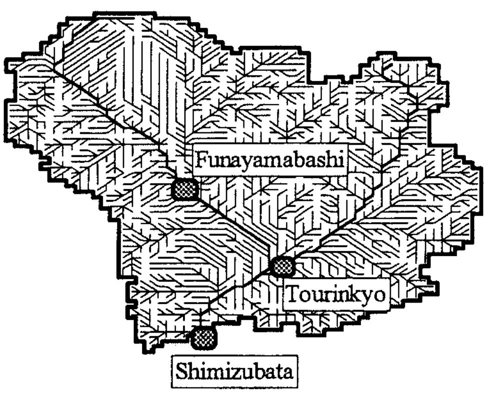

Fig.6 River network

There are three discharge gauges in the basin (also **Fig. 6**). The Shimizubata gauge is located at the outlet of the basin, having a controlling area of 2,179 km^2, the Funayamabashi gauge is at the left upper reach with a drainage area of 495 km^2, and the Tourinkyo gauge is at the right upper reach with an area of 909 km^2. Their discharge data are used to test the estimations of river discharge.

For situation of annual discharge, the routing process in river links is done as follows. In principle, the discharge at any river link is simply the integration or summary of all runoffs from those grids that drain into this link. All the 2,160 links are separated into ordered groups, the first-order links have only one link or itself, the second-order links collect streamflows of first-order links and transport them to the third-order links, and so on. Finally all discharge integrate together at the outlet point. In this way spatial distribution of river discharges are obtained and shown in **Fig. 5**. The thickness of link lines in the figure represent the discharge volume.

Estimated discharge volumes at the Shimizubata, Tourinkyo and Funayamabashi are 1,929, 692 and 475 million cubic meters respectively, having small errors of 3.2 %, 13.0% and 2.5% against the observed 1,994, 796 and 464 million cubic meters.

Similarly, rainfall and water budgets for each of the 16 years are also calculated. They give good river discharge results. The percent errors of estimated discharges against the observed at Tourinkyo are 0.5% to 21.4%, errors at Shimizubata are 0.2% to 23.4%.

6. DAILY DISCHARGE BY MODEL

Daily discharge is needed both to short time analysis (flood, drought) and long time analysis (resource management). The distribution along rivers is implemented in two steps.

Daily rainfall at a grid is obtained by the spline method, using daily data of the 27 gauges. The year 1990 is selected for analysis, and distributions in each of two days (Aug. 9 and Aug. 10) are estimated and illustrated in **Fig. 7**. Daily temperature is simply obtained by using the regression formula (2) to those days within a month.

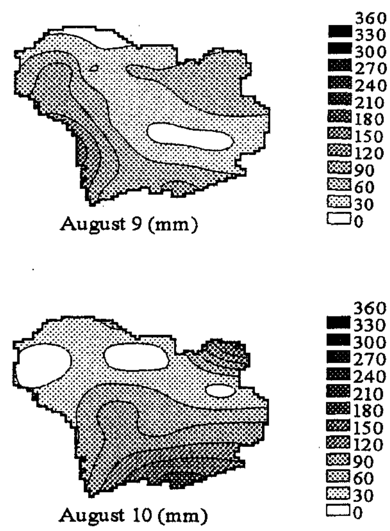

Fig.7 Daily rainfall distribution

A distributed hydrological model is applied. It is a grid-orientated and GIS-based model (Yao and Terakawa, 1999; Yao et al., 1998), and improved with the flow routing algorithm in river network. The model accepts distributed daily rainfall, temperature and other inputs, estimates evapotranspiration, soil moisture and runoff generation in grids, and routes these runoffs in the digitized river network. Estimated daily discharges in 1990 are compared with the observed, having good agreements as shown in **Fig 8**. Distributions of discharge volumes for the two days are plotted in **Fig. 9**.

By comparing Fig. 7 and Fig. 9, it is seen that the rain during Aug. 9 is larger at the left part of basin, and is larger at the central and right part for Aug. 10. Correspondingly, the discharge on Aug. 9 concentrates relatively on the left-lower part, while the discharge on Aug. 10 concentrates on the right-lower part.

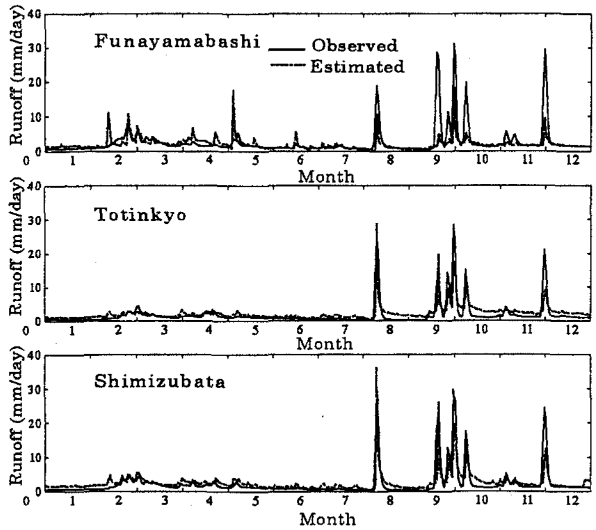

Fig.8 Comparison of daily runoffs

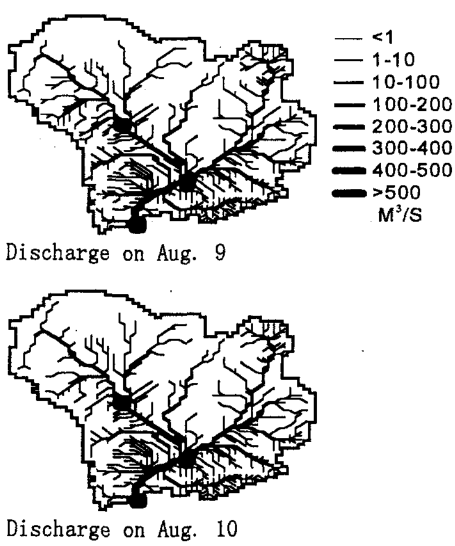

Fig.9 Daily discharges along river network

7. CONCLUSION

The methodology of spatial water budget analyses is briefly summarized. Annual rainfall (or daily) and its distribution are derived by a cubic spline method using 27 gauges data; monthly or daily temperatures are spread onto grids by a step-wise regression method using field data and GIS data; evapotranspirations of grids are then gained from the distributed rainfall and temperature; and finally runoffs of grids and discharges in rivers are estimated using water balance method or model tool. The applications to Fuji River basin show a good reproduction of annual and daily discharges.

As for further research, daily and hourly rainfall are to be distributed by the spline method, and hourly hydrologic processes will be simulated. These kinds of information would be very useful to flood-drought prediction, water management and assessment of environmental impacts.

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(Received September 30, 1999)