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OPTIMIZATION OF A RUNOFF-EROSION MODEL THROUGH A GENETIC ALGORITHM

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The difficulties involved in calibration of physically-based erosion models have been partly attributable to the lack of robust optimization tools. Recently, a global optimization method known as the SCE-UA has shown promise as an effective and efficient optimization technique for calibrating watershed models. This paper presents the essential concepts of the SCE-UA method and a physically-based erosion model, and then presents the optimization results in which the WESP model was calibrated. On the basis of these results, the recommended erosion parameter values are given, which should also help to provide guidelines to estimate them in other similar areas.

Key Words : *genetic algorithm, runoff-erosion model, semiarid*

1. INTRODUCTION

Several optimization methods have been tested in the past years in the calibration of physically-based erosion models but it is difficult to assure that they are not trapped in a local minimum. Thus, robust algorithms are being applied to such models. The most robust algorithms used nowadays are the evolutionary algorithms, which are an umbrella term used to describe computer-based problem solving systems which use computational models of evolutionary processes as key elements in their design and implementation. A variety of evolutionary algorithms have been proposed, e.g., genetic algorithm, evolutionary programming, evolution strategies, classifier systems, and genetic programming. They all share a common conceptual base of simulating the evolution of individual structures via processes of selection, mutation, and reproduction. In the parameter calibration process, the most used is the genetic algorithm, which is basically a model of machine learning that derives its behavior from a metaphor of the process of evolution in nature. The genetic algorithm selected here is known as the shuffled complex evolution (SCE-UA), which was developed by Duan et al. [1)] in order to solve problems in the application of conceptual rainfall-runoff models. The same kind of problems in the optimization process with physically-based erosion models have been reported and the authors have been testing some optimization techniques[2)]. Thus, this powerful new global optimization procedure was chosen to be applied to the watershed erosion simulation program (WESP) developed by Lopes[3)]. Descriptions of the method and model are briefly presented in the following sections as well as the calibration results.

2. THE SCE-UA METHOD

The typical optimization problems that characterize the problems encountered in physically-based erosion model calibration are (1) global convergence in the presence of multiple regions of attraction; (2) ability to avoid being trapped by small pits and bumps on the objective function surface; (3) robustness in the presence of differing parameter

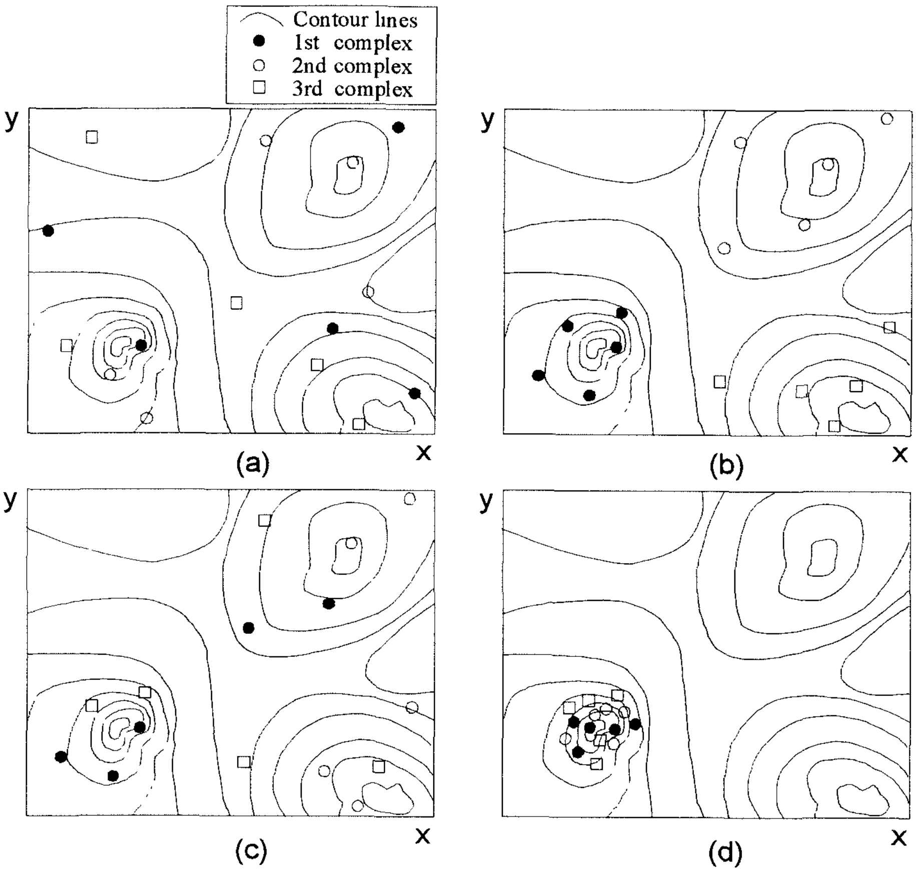

Fig. 1 Illustration of the shuffled complex evolution (SCE-UA) method.

sensitivities and parameter interdependence; (4) non-reliance on the availability of an explicit expression for the objective function or the derivatives; and (5) capacity of handling high-parameter dimensionality.

The SCE-UA method embodies the desirable properties described above and is based on a synthesis of four concepts: (1) combination of deterministic and probabilistic approaches; (2) systematic evolution of a 'complex' of points spanning the parameter space, in the direction of global improvement; (3) competitive evolution; (4) complex shuffling. The synthesis of these elements makes the SCE-UA method effective and robust, and also flexible and efficient. The SCE-UA method is explained in **Fig. 1**, by use of a two-dimensional example, where the contour lines represent a function surface with a global optimum and two local optima. The steps of the SCE-UA method are (a) randomly generate a sample of s points, rank the points according to the order of increasing criterion, and partition the sample into p complexes (communities) with the first point in the first complex, the second point in the second complex and so on (**Fig. 1-a**); (b) evolve each complex independently according to the competitive complex evolution (CCE) algorithm (**Fig. 1-b**); (c) shuffle the complexes (**Fig. 1-c**); and (d) check if any of the pre-specified convergence criteria are satisfied, if so stop (**Fig. 1-d**), otherwise, check the reduction in the number of complexes and continue to evolve.

One key component of the SCE-UA method is the CCE algorithm. The algorithm, based on the Nelder and Mead[4] Simplex downhill search scheme, is presented briefly as follows: (i) construct a subcomplex by randomly selecting q points from the complex according to a trapezoidal probability distribution. A subcomplex functions like a pair of parents, except that it may comprise more than two members; (ii) identify the worst point of the subcomplex and compute the centroid of the subcomplex without including the worst point; (iii) attempt a reflection step by reflecting the worst point through the centroid. If the newly generated point is within the feasible space, go to Step iv; otherwise, randomly generate a point within the feasible space and go to Step vii; (iv) if the newly generated point is better than the worst point, replace the worst point by the new point. Go to Step vii. Otherwise, go to Step v; (v) attempt a contraction step by computing a point halfway between the centroid and the worst point. If the contraction point is better than the worst point, replace the worst point by the contraction point and go to Step vii. Otherwise, go to Step vi; (vi) randomly generate a point within the feasible space. Replace the worst point by the randomly generated point; (vii) repeat Steps ii-vi α times, where $\alpha \geq 1$ is the number of consecutive offspring generated by the same subcomplex; and (viii) repeat Steps i-vii β times, where $\beta \geq 1$ is the number of evolution steps taken by each complex before complexes are shuffled.

3. WESP MODEL

The selected runoff-erosion model to be applied in this work is the WESP model, developed by Lopes[3)], because it was developed for small basins. The model uses the Green-Ampt equation to model the infiltration:

$$f(t) = K_s\left(1 + \frac{N_s}{F(t)}\right) \tag{1}$$

where $f\ (t)$ is the infiltration rate (m/s), K_s is the effective soil hydraulic conductivity (m/s), N_s is the soil moisture-tension parameter (m), $F(t)$ is the cumulative depth of infiltrated water (m) and t is the time variable (s).

(1) Overland flow

The overland flow is considered one-dimensional. Manning's turbulent flow equation is given by:

$$u = \frac{1}{n_p} R_H^{2/3} S_f^{1/2} \tag{2}$$

where $R_H(x,t)$ is the hydraulic radius (m), u is the local mean flow velocity (m/s), S_f is the friction slope and n_p is the Manning friction factor of flow resistance for the planes. Thus, the local velocity equation for planes can be obtained making $R_H = h$ and using the kinematic approximation that the friction slope is equal to the plane slope ($S_0 = S_f$):

$$u = \alpha' h^{m'-1} \tag{3}$$

where h is the depth of flow (m), α' is a parameter related to surface roughness, equal to $(1/n_p)S_0^{1/2}$, and $m' = 5/3$ is a geometry parameter.

Sediment transport is considered as the erosion rate in the plane reduced by the deposition rate within the reach. The erosion occurs due to raindrop impact as well as surface shear. The sediment flux Φ (kg/m^2/s) to the flow is written as:

$$\Phi = e_I + e_R - d \tag{4}$$

where e_I is the rate of sediment by rainfall impact, e_R is the rate of sediment by shear stress, and d is the rate of sediment deposition. The rate e_I (kg/m^2/s) is obtained from the relationship:

$$e_I = K_I I r_e \tag{5}$$

in which K_I is the soil detachability parameter (kg·s/m^4), I is the rainfall intensity (m/s), and r_e is the effective rainfall (m/s), which is equal to $I - f$. The rate e_R (kg/m^2/s) is expressed by the relationship:

$$e_R = K_R \tau^{1.5} \tag{6}$$

where K_R is a soil detachability factor for shear stress (kg·m/N$^{1.5}$·s), and τ is the effective shear stress (N/m^2), which is given by:

$$\tau = \gamma R_H S_f \tag{7}$$

where γ is the specific weight of water (N/m^3), and d (kg/m^2/s) is expressed as:

$$d = \varepsilon V_s c \tag{8}$$

where ε is a coefficient that depends on the soil and fluid properties (set to 0.5 in this study), V_s is the particle fall velocity (m/s), and $c(x,t)$ is the sediment concentration in transport (kg/m^3).

(2) Channel flow

The concentrated flow in the channels is also described by continuity and momentum equations. The momentum equation can be reduced to the discharge equation with the kinematic approximation:

$$Q = \alpha' A R_H^{m'-1} \tag{9}$$

where A is the area of flow (m^2). The net sediment flux Φ_c (kg/m/s) for the channel is expressed by:

$$\Phi_c = q_s + e_r - d_c \tag{10}$$

where q_s is the lateral sediment inflow into the channel (kg/m/s), e_r is the erosion rate of the bed material (kg/m/s) obtained from the relation:

$$e_r = a(\tau - \tau_c)^{1.5} \tag{11}$$

in which a is the sediment erodibility parameter, and τ_c is the critical shear stress for sediment entrainment (N/m^2), which is given by the relationship:

$$\tau_c = \delta(\gamma_s - \gamma)d_s \tag{12}$$

where δ is a coefficient (0.047 in the present study), γ_s is the specific weight of sediment (N/m^3) and d_s is the mean diameter of sediments (m).

The deposition term d_c (kg/m/s) in equation (10) is expressed by:

$$d_c = \varepsilon_c T_W V_s C \tag{13}$$

in which ε_c is the deposition parameter for channels, considered as unity in the present case, T_W is the flow top width (m), $C(x,t)$ is the sediment concentration in transport (kg/m^2) and V_s is as defined in equation (8).

4. THE STUDIED AREA

A bare micro-basin, which is one of the four micro-basins of the Sumé Experimental Watershed, in Brazil at Paraíba State, was selected to be applied in this work. Its mean slope, area and perimeter are

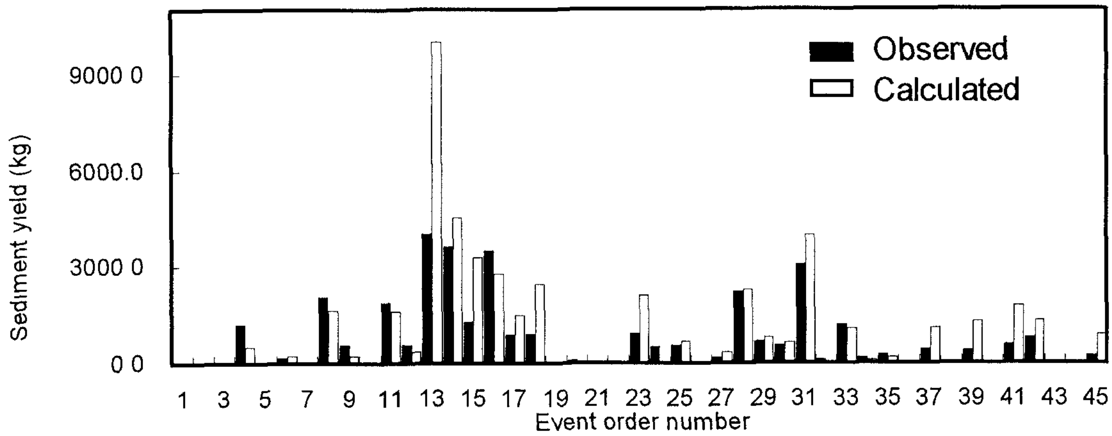

Fig.2 Observed and calculated sediment yield.

7.1%, 0.48 ha, and 302 m, respectively. This experimental watershed has been operated since 1972 by SUDENE (Superintendency of Northeast Development, Brazil), ORSTOM (French Office of Scientific Research and Technology for Overseas Development) and UFPB (Federal University of Paraíba, Brazil)[5].

Based on the work of Santos et al.[2], 45 events were selected between 1987 and 1991, because during this period there was no vegetation cover. The runoff and erosion data were measured after each rainfall event and the rainfall data were obtained from a recording rain gauge installed close to the selected micro-basin.

5. APPLICATION OF SCE-UA METHOD

(1) Selection of SCE-UA algorithm parameters

The SCE-UA method contains many probabilistic and deterministic components that are controlled by some algorithmic parameters. For the method to perform optimally, these parameters must be chosen carefully. The first one is m, the number of points in a complex ($m \geq 2$), which should not be neither too small to avoid the search to proceed as an ordinary simplex procedure nor too large to avoid an excessive use of computer processing time while no certainty in effectiveness is taken. Then the default value, $m = 2n + 1$, was selected, where n is the number of parameters to be optimized on. For the number of points in a subcomplex q ($2 \leq q \leq m$), the value of $n + 1$ was selected because it would make the subcomplex a Simplex; this defines a first-order approximation (hyperplane) to the objective function surface and will give a reasonable estimate of the local improvement direction. The number of consecutive offspring generated by each subcomplex α ($\alpha \geq 1$), was set to one to avoid the search becoming more strongly biased in favor of the local search of the parameter space. The number of evolution steps taken by each complex β ($\beta > 0$) was set to $2n + 1$ to avoid a situation in which complexes would be shuffled frequently if set to a small value or to avoid it shrinking into a small cluster if a great value is used. The number of complexes p was set to 2 based on the nature of the problem, and the minimum number of complexes required in the population p_{min} ($1 \leq p_{min} \leq p$) was set to p because it gave the best overall performance in terms of effectiveness (the ability to locate global optimum) and efficiency (the speed to locate global optimum).

(2) Optimization of the erosion parameters

In order to start the calibration process, the micro-basin had to be represented as a scheme of planes and channels. The authors have discussed which schematization would be the best to represent the area[6], and the schematization in 10 elements was chosen here. The first parameter to be calibrated in the WESP model was the soil moisture-tension parameter N_s in equation (1) and it could be calibrated by a simple optimization method because it was necessary just to fit the computed runoff depth with the observed value. However, after this step the WESP model contains more three erosion parameters (a, K_R and K_I) which should be calibrated; thus, the SCE-UA method was used for such a task.

The initial values of the erosion parameters were $a_0 = 0.0144$ kg·m^2, $K_{R0} = 2.174$ kg·m/N$^{1.5}$·s and $K_{I0} = 5.0 \times 10^8$ kg·s/m^4, and the following objective function J was used:

$$J = \left| \frac{E_o - E_c}{E_o} \right| \tag{8}$$

where E_o is the observed sediment yield (kg) and E_c is the calculated one (kg). The optimization for the 45 events gave the mean values of the erosion parameters as a = 0.008 kg·m^2, K_R = 2.585 kg·m/N$^{1.5}$·s, and $K_I = 6.222 \times 10^8$ kg·s/m^4. The values

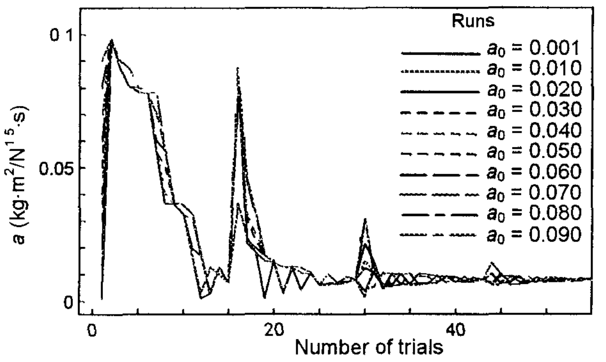

Fig.3 Ten independent runs with different initial values of a_0

were used then to run new simulations, and **Fig. 2** shows the simulation results for the sediment yield with some acceptable degree of agreement, except for a few events.

In several optimization methods, the initial parameter values have a strong influence on the final optimized values; thus, 10 independent runs were carried out with several initial values for the parameter a. However, as shown in **Fig. 3**, all of them converged to the same optimized value, showing then that this genetic algorithm has no sensitivity to these initial values.

6. CONCLUSION

In order to calibrate the parameters in the distributed physically-based erosion WESP model, the genetic algorithm SCE-UA was used. The results showed that this genetic algorithm could be used as a powerful method for such calibration and the values of the calibrated parameters could be also used as a guideline for further calibration of the WESP model to the studied area as well as to other similar areas in northeastern Brazil. The SCE-UA proved to be insensitive to the initial value of the sediment erodibility parameter a; i.e., independent of the initial value of each erosion parameter, the final optimized values were the same for each run.

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