SPECTRAL ANALYSIS OF TURBULENT BOTTOM SHEAR STRESS UNDER IRREGULAR WAVES

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A method to compute turbulent bottom shear stress from free stream velocity variation under irregular waves has been proposed for plane bed condition. The correlation has been assessed through spectral analysis of free stream velocity and bottom shear stress obtained from generated irregular waves and from k-ε model result respectively. A significant improvement in the prediction has been achieved compared to that proposed by the authors earlier. Turbulent friction factor and phase difference obtained through spectral analysis corresponds quite well to those from experimental data for sinusoidal waves and with corresponding available smooth turbulent friction factor descriptions. Proposed method presents a high degree of accuracy and is very convenient to use from the viewpoint of practical application.

Key words: *turbulent oscillatory flow, k-ε model, bottom shear stress, estimation.*

1. INTORDUCTION

Waves propagating into water of finite depth manifest the influence of sea bottom through the establishment of a boundary layer in the immediate vicinity of the bottom. Flow in this layer is strongly shared through turbulent dissipation and governs the flow dynamics. Although many studies have been reported on regular sinusoidal and short crested wave boundary layers, consideration of irregular wave to investigate bottom boundary layer properties is relatively recent.

Several researchers have studied irregular wave bottom friction considering spectral wave dissipation and from statistical consideration[1]. Samad and Tanaka[2),3)] had studied the time varying character of bottom boundary layer for both laminar and turbulent cases. It has been reported that for laminar flow, high flow inertia causes a double peaked shape in bottom shear stress, while for turbulent motion, turbulent mixing reduces flow inertial effects, therefore, bottom shear stress becomes almost instantaneous with free stream velocity variation.

To estimate time variation of bottom shear stress Hasselmann and Collins[4], and Kabling and Sato[5] have suggested a relationship based on the definition of wave friction factor. Applicability of such a relationship to irregular waves was studied by Samad et al.[6] along with the comparison of k-ε model results. Though their result proved encouraging, several modifications seemed essential; specially concerning the assessment of friction factor and the inclusion of phase difference between free stream velocity and bottom shear stress.

In the present paper a detailed spectral analysis has been made to evaluate correlation characters between free stream velocity and turbulent plane bed bottom shear stress computed through k-ε model. The analysis has resulted in a systematic improvement in the estimation and accordingly the earlier proposed method has been modified. When applied along with average turbulent phase difference, the modified method could predict turbulent bottom shear stress from free stream velocity with significant accuracy. It has also been observed that the turbulent irregular wave friction factor corresponds quite well to those from sinusoidal waves.

2. GOVERNING EQUATIONS AND METHODOLOGY

(1) Irregular Wave

The spectral density for irregular water surface elevation has been computed using Bretschneider-Mitsuyasu spectral density formulation. Applying small amplitude wave theory, free stream velocity transfer function has been computed which is later applied to obtain free stream velocity spectrum. To generate free stream velocity time variation, consideration has been made that irregular waves can be resolved as a sum of infinite number of wavelets with small amplitudes and random phases. Details of irregular wave generation can be found in Ref.3).

(2) k-ε Model

Generated irregular wave free stream velocity time variation has been used for turbulence computation in the boundary layer applying Jones and Launder[7] original low Reynolds number k-ε model. A full description of the modeling system can be found in Sana and Tanaka[8].

(3) Non-Dimensional Parameters

Reynolds Number : The wave Reynolds number for sinusoidal waves (*Re*) as given by Jonsson[9] is:

$$\mathrm{Re} = \frac{U_0 a_m}{\nu} = \frac{U_0^2}{\nu\omega} \qquad (1)$$

where U_0 is free stream velocity amplitude, a_m maximum bottom orbital displacement, ω wave frequency and ν is kinematic viscosity.

Irregular wave Reynolds number has been defined in terms of significant wave properties and has been made analogous to *Re*, given by:

$$\mathrm{Re}_{1/3} = \frac{{U_{1/3}}^2}{\nu\omega_{1/3}} \qquad (2)$$

with

$$U_{1/3} = \frac{\pi H_{1/3}}{T_{1/3}} \frac{1}{\sinh 2\pi h / L} \quad \text{and} \quad \omega_{1/3} = \frac{2\pi}{T_{1/3}} \qquad (3)$$

where $U_{1/3}$, $H_{1/3}$, $T_{1/3}$ and $\omega_{1/3}$ are significant quantities of free stream velocity, wave height, wave period and wave frequency respectively, h water depth and L is wave length corresponding to $\omega_{1/3}$.

3. COMPUTATION RESULT AND SPECTRUM ANALYSIS

(1) Input Parameters

The input wave parameters specified for computation have been the significant wave height and period, water depth, and a normalizing depth, z_h, where the flow is same with the free stream velocity. Four turbulent cases have been considered here and the parameters are presented in Table 1.

(2) Spectral properties of Turbulent Bottom Shear Stress and Free Stream Velocity

Considering the definition of bottom shear stress

Table 1: Parameters for turbulent irregular wave computation.

Run	h cm	z_h cm	$T_{1/3}$ sec	$H_{1/3}$ cm	$U_{1/3}$ cm/s	$Re_{1/3}$
Case1	1000	100	10	430	184.0	5.39x10^6
Case2				480	205.3	6.71x10^6
Case3				505	216.1	7.43x10^6
Case4				530	226.7	8.18x10^6

several researchers[4),5)] have suggested a generalized relation to compute the instantaneous bottom shear stress in the from:

$$\frac{\tau_0(t)}{\rho} \propto U(t)\left|U(t)\right|^{n-1} \qquad (4)$$

where, $\tau_0(t)$ and $U(t)$ are instantaneous bottom shear stress and free stream velocity respectively, ρ water density and n is a correlating exponent. In Eq.(4) generally the constant of proportionality and the exponent can be replaced introducing the wave friction factor.

In this study, in order to achieve maximum correlation, cross-spectrum and coherence have been studied from k-ε model computed bottom shear stress and free stream velocity for different values of 'n'. Fig.1 shows the spectral density for n=1.68 (or $U|U|^{0.68}$) for Case1. Corresponding coherence and phase difference between bottom shear stress and free stream velocity is presented in Fig.2. Average coherence and phase difference then have been computed between frequency range corresponding to 99% of velocity spectrum (as indicated in Fig.1) and plotted against the exponent values as shown in Figs.3 and 4 respectively. Fig.3 shows that the average coherence achieved a maximum value when n=1.68, which means that maximum correlation between τ_0 and U can be achieved using 1.68 as the exponent. The average phase difference shows very small variation except for Case1 (Fig.4) and at n=1.68 it has been found to be 0.16 rad (≈9.2 deg.).

4. PREDICTION METHODS

Following Hasselmann and Collins[4)] the instantaneous bottom shear stress can be evaluated from Eq.(4) along with wave friction factor as:

$$\tau_0(t) = \frac{\rho}{2} f_w U(t)\left|U(t)\right| . \qquad (5)$$

The turbulent friction factor, f_w, can be computed following Fredsøe and Deigaard[10)] expression for sinusoidal wave which is given by:

$$f_w = 0.035\,\mathrm{Re}^{-0.16} \qquad (6)$$

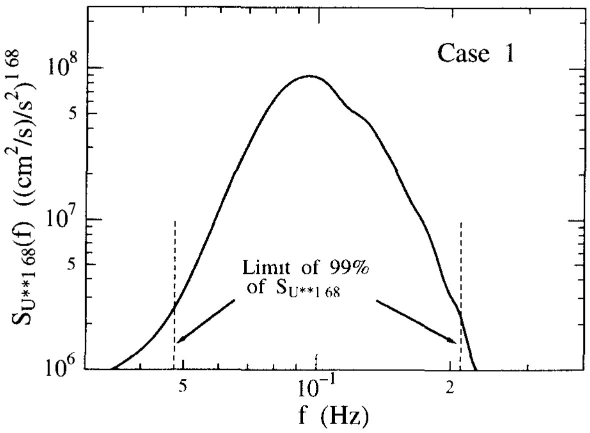

Fig.1: Spectral density of $U|U|^{0.68}$, Case 1.

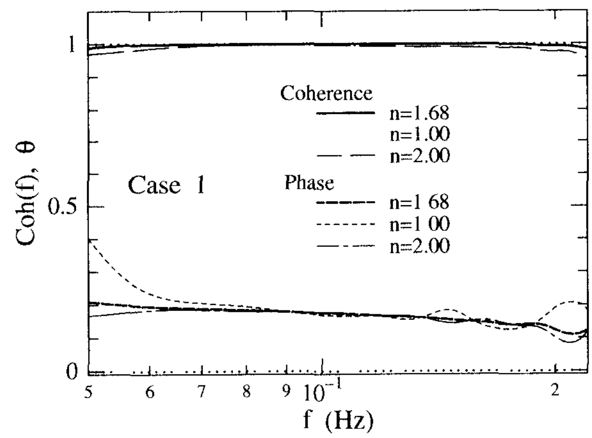

Fig.2: Coherence and Phase for Case 1.

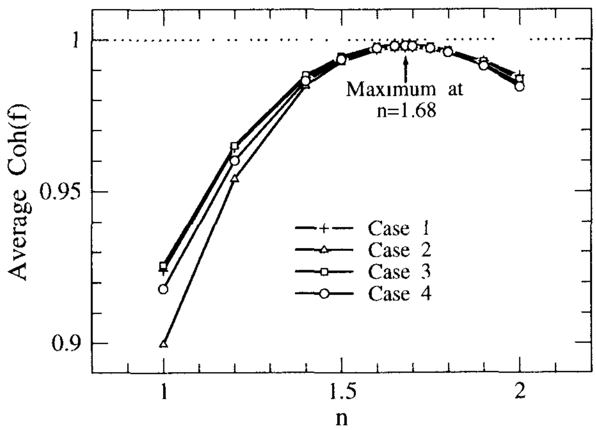

Fig.3: Average coherence as a function of *n*.

where *Re* can be considered as the Reynolds number (*RE*) corresponding to instantaneous free stream velocity such that:

$$RE = \frac{U(t)^2}{\omega \nu}. \tag{7}$$

Substituting Eq.(7) in Eq.(6) and replacing f_w in Eq.(5) one can get:

$$\tau_0(t) = \frac{\rho}{2} 0.035(\omega \nu)^{0.16} U(t)|U(t)|^{0.68}. \tag{8}$$

Eq.(8) has already been presented by Samad et al.[6]. It is interesting to note that the exponent obtained through spectral analysis matches exactly with that from the equation proposed by Hasslemann and Collins if the turbulent friction factor is assessed through Fredsøe and Deigaard formulation.

A further assessment of the coefficient in Eq.(8) can be made by defining a transfer function between bottom shear stress and free stream velocity such as:

$$H_{\pi}(f) = \frac{\rho}{2} A\,(\omega \nu)^{0.16} \tag{9}$$

where *A* is a coefficient that can be obtained from the comparison of corresponding transfer function from *k-ε* model result. The later is defined as:

$$H_{\pi}(f) = \frac{S_{\tau}(f)}{S_{U^{1.68}}(f)} \tag{10}$$

with $S_{\tau}(f)$ and $S_U^{1.68}(f)$ are being shear stress and velocity spectral densities respectively. From Eq.(9) and Eq.(10) the coefficient has been evaluated as A=0.041. The comparison is presented in Fig.5 along with different methods to evaluate ω which are elaborated later. The transfer function so defined resulted in a friction factor relation of:

$$f_w = 0.041\,\mathrm{Re}^{-0.16}. \tag{11}$$

Obtained friction factor formulation (Eq.11) and phase difference has been compared with those from sinusoidal waves in Figs.6 and 7 along with several experimental data and smooth turbulent friction factor expressions by Jonsson[9], Kajiura[11], Tanaka and Thu[12], and Fredsøe and Deigaard. Considering the exponent and coefficient of *Re* from spectral analysis, Fig.6 suggests that irregular wave friction factors and phases are in very good agreement with those from sinusoidal waves. Although lack of experimental data in the range of high Reynolds numbers restricts any detailed comparison, Eq.(11) shows a very good agreement with computed results. Similarly the average phase difference (ψ) obtained through spectral analysis also corresponds well with that from sinusoidal wave (Fig.7) for the range of Reynolds number considered.

Therefore, an alternative to Eq.(8) to compute irregular wave instantaneous bottom shear stress could be proposed as:

$$\tau_0\left(t - \frac{\psi}{\omega}\right) = \frac{\rho}{2} 0.041(\omega \nu)^{0.16} U(t)|U(t)|^{0.68}. \tag{12}$$

The use of Eq.(12) poses difficulty in selecting a representative wave period. In this respect two options could be of interest, firstly to consider period for individual waves and secondly to apply

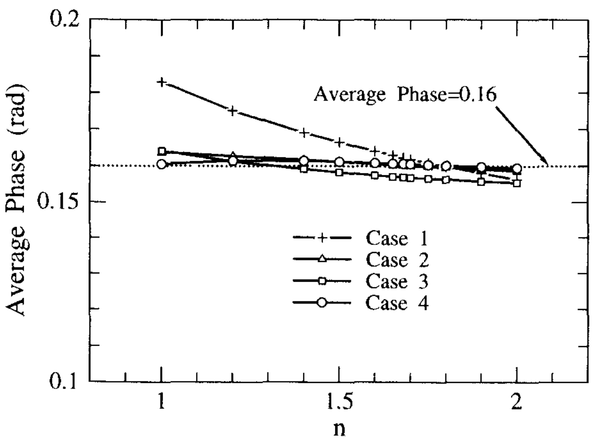

Fig.4: Average phase for turbulent cases.

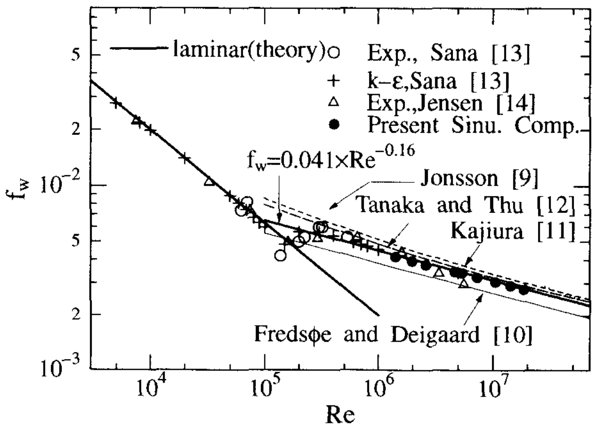

Fig.6: Proposed and sinusoidal wave friction factor.

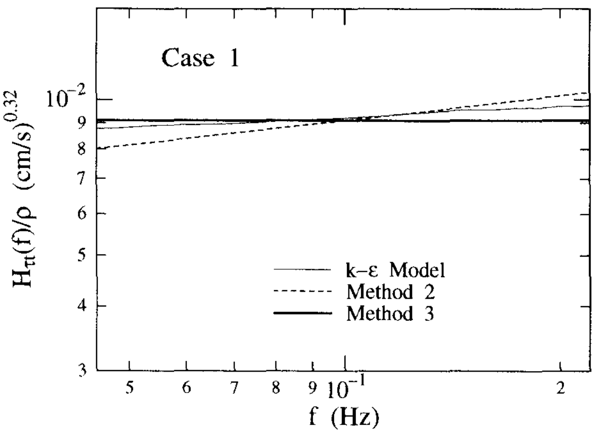

Fig.5: Comparison of bottom shear stress transfer function, Method 2 and 3, Case 1.

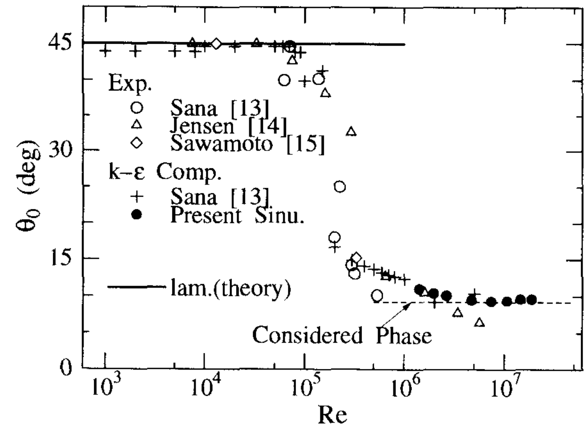

Fig.7: Phase difference for sinusoidal wave.

the significant wave period.

As such, three possible methods to compute instantaneous bottom shear stress could be identified. These are:

- Eq.(5) with f_w computed from Eq.(11) for individual waves (Method1),
- Eq.(12) considering individual wave periods (Method2) and
- Eq.(12) with significant wave period (Method3).

Method3 presents the most easy to use formula from practical application viewpoint.

(1) Accuracy of Prediction

Bottom shear stress obtained through proposed methods are compared with *k*-ε model results. Although *k*-ε models have mainly been proposed for unidirectional flows and its use in oscillatory flows is relatively recent, the results obtained by different researchers[16),8),13)] suggest its suitability.

Figure 5 shows comparison of proposed transfer functions for Case1. The agreement between proposed methods and that from *k*-ε model are generally good and shows a better agreement with Method3.

Comparison of bottom shear stress time series for all the methods are shown in Figs. 8 and 9. Accuracy comparisons for Method2 and Method3 are also presented in Figs. 10 and 11.

The quantitative accuracy of proposed methods has also been assessed through the following accuracy factor.

$$F_a = \frac{\tau_{0comp}}{\tau_{0k-\varepsilon}}. \tag{13}$$

The standard deviation has been defined as:

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(F_{ai} - F_m\right)^2} \tag{14}$$

with

$$F_m = \frac{1}{N}\sum_{i=1}^{N} F_{ai} \tag{15}$$

where τ_{0comp} and $\tau_{0k-\varepsilon}$ are proposed and *k*-ε model computed bottom shear stresses respectively, F_m mean accuracy factor, N total number of data and i is index subscript. During flow reversal bottom shear stress become very small and, as such, use of Eq.(13) could result in unrealistically small or large values for F_a. Therefore, bottom shear stresses with values less than 10% of the significant bottom shear stress has not been considered for accuracy factor computation. Table 2 shows a summary of accuracy analysis for both instantaneous ($\tau_0(t)$) as well as maximum (τ_c) bottom shear stresses in a wave cycle. For all the cases, on average, a total of about 9000 data pairs have been considered with a time step of 0.1sec. covering about 140 wave cycles. It shows that both Method2 and Method3 have high predictive ability with about 65% $\tau_0(t)$ lying in ±10% of the accuracy range. For τ_c it is over 80%. It can be seen that Method3 is not only easy to use for practical purposes but also the most accurate.

5 CONCLUSIONS

A detailed spectral analysis has been made on irregular wave k-ε model results for turbulent bottom shear stress and free stream velocity for plane bed condition. Following the definition of wave friction factor three methods for the determination of instantaneous bottom shear stress could be identified based on the spectral analysis.

Obtained exponent and coefficient values in Eq.(4) for turbulent cases provide description for irregular wave friction factor which corresponds very well with those obtained from sinusoidal waves. The average phase difference also falls in the same range of corresponding sinusoidal wave.

Based on spectrum analysis modifications has been made to the prediction method for turbulent instantaneous bottom shear stress that was proposed earlier by the authors. Generally much improved prediction has been achieved. Among the three methods proposed Method3 shows the best agreement when compared with k-ε model result.

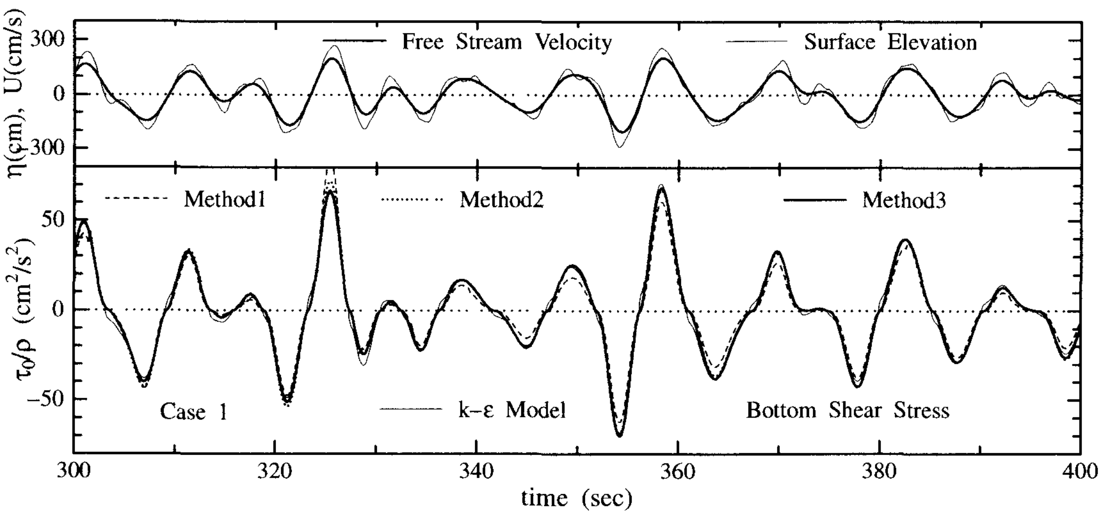

Fig.8: Comparison of predicted and k-ε model bottom shear stress time series, Case 1.

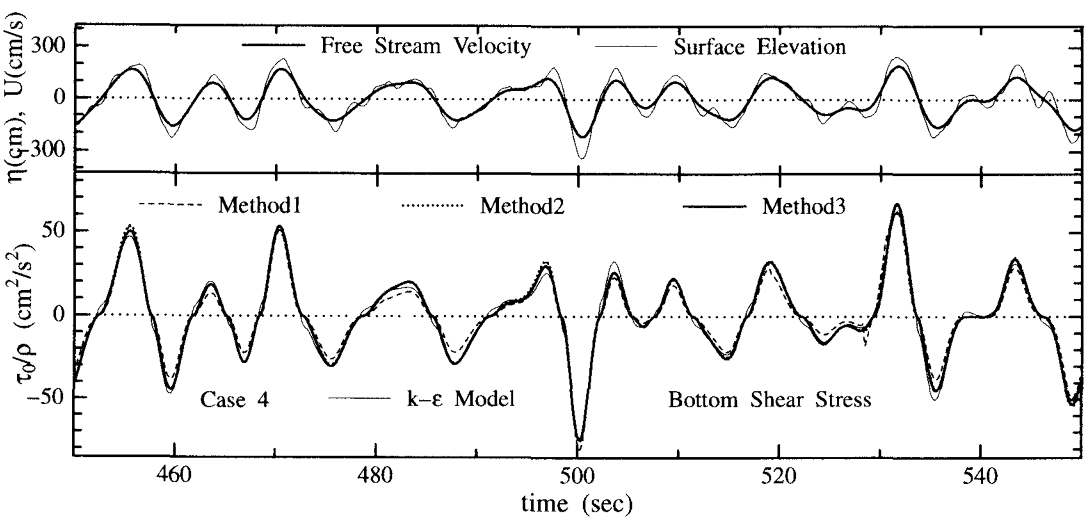

Fig.9: Comparison of predicted and k-ε model bottom shear stress time series, Case 4.

Table 2 : Accuracy of predicted bottom shear stress : % of data in the accuracy range of ±10% and ±20% and Standard Deviation.

Model Runs	Method 1						Method 2						Method 3					
	Instantaneous			Maximum			Instantaneous			Maximum			Instantaneous			Maximum		
	Accuracy Range			Accuracy Range			Accuracy Range			Accuracy Range			Accuracy Range			Accuracy Range		
	0.9-1.1	0.8-1.2	S.D.	0.9-1.1	0.8-1.2	S.D.	0.9-1.1	0.8-1.2	S.D.	0.9-1.1	0.8-1.2	S.D.	0.9-1.1	0.8-1.2	S.D.	0.9-1.1	0.8-1.2	S.D.
Case1	23.0	51.5	0.19	40.1	71.2	0.23	64.3	88.2	0.14	76.5	95.5	0.10	66.1	89.2	0.14	82.6	95.5	0.10
Case2	25.1	52.7	0.22	40.1	66.9	.022	64.9	88.2	0.14	82.2	96.8	0.09	66.8	89.8	0.13	86.7	98.1	0.09
Case3	21.2	51.5	0.22	29.9	69.5	0.26	62.7	89.1	0.15	76.8	97.4	0.08	67.2	89.9	0.14	86.1	98.0	0.07
Case4	20.8	50.5	0.24	32.3	67.7	0.24	63.3	88.4	0.14	81.1	96.8	0.09	66.4	89.6	0.14	86.2	96.9	0.09

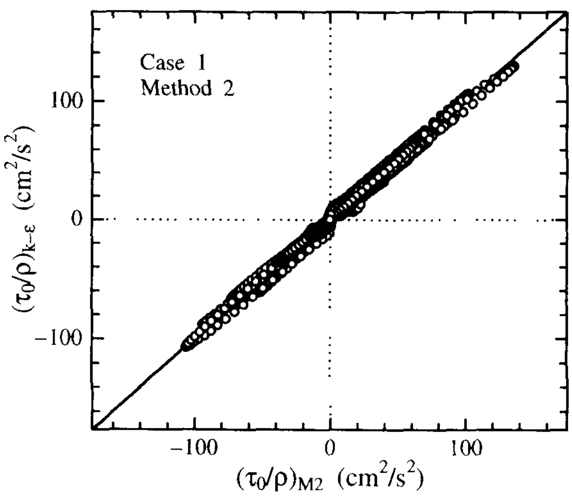

Fig.10: Accuracy check for predicted bottom shear stress, Method 2, Case 1.

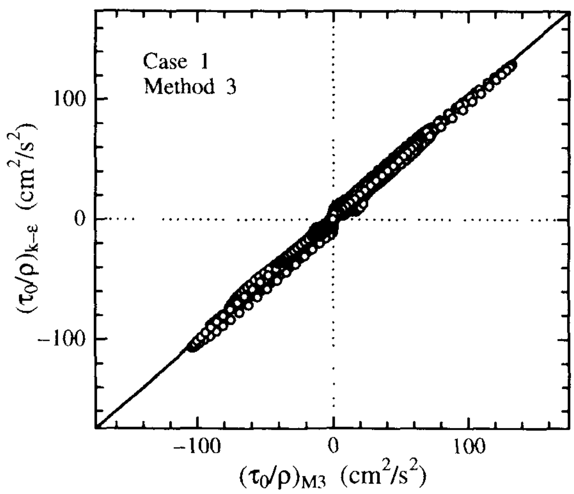

Fig.11: Accuracy check for predicted bottom shear stress, Method 3, Case 1.

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