A CONCEPTUAL SOIL EROSION MODEL

Celso A. G. SANTOS[1], Masahiro WATANABE[2] and Koichi SUZUKI[3]

[1]Member of JSCE, Dr. of Eng., JSPS Postdoctoral Fellow, Dept. of Civil and Environmental Engineering, Ehime University (3-Bunkyo-cho, Matsuyama 790-77, Japan)
[2]Member of JSCE, Dr. of Eng., Associate Professor, Dept. of Civil and Environmental Eng., Ehime University
[3]Member of JSCE, Dr. of Eng., Professor, Dept. of Civil and Environmental Engineering, Ehime University

Soil erosion models have been moved from empirically-based models towards physically-based mathematically complicated models. The empirical models seem to be the more practical since they are mathematically simple, but they are limited to the area where they have been developed, and they are based on standard runoff plots on uniform slopes. Thus, hereby a conceptual model, which uses an empirical erosion equation, is presented and tested with erosion data from a bare micro-basin located in the Brazilian semiarid area. The results showed that the model could be used as a practical and simple tool for prediction of erosion in semiarid areas.

Key Words : *erosion modeling, conceptual model, infiltration*

1. INTRODUCTION

Bare land without vegetation is widely distributed throughout semiarid areas in northeastern Brazil because the amount of rainfall impedes the establishment of good surface cover. Reduction of the agricultural productivity of the land in this ecosystem due to the fertile topsoil loss by water erosion has become one of the severest problems to be solved in the near future, because the erratic rainfalls result this region prone to erosion.

Many analytical models, of various complexity, that evaluate sediment yield from the bare soil slopes due to erosion have been put into development over the last years to obtain a good tool for evaluation of soil erosion. Soil erosion modeling has moved from empirically-based and simple mathematical models towards physically-based and mathematically much more complicated models[1)]. However, no useful model to predict the sediment yield precisely and practically has been developed, because the erosion process inherently contains many uncertainties that are difficult to solve analytically.

In order to develop a useful and practical soil erosion model, the authors have studied the mechanism of the erosion process on bare slope surfaces. They have calibrated a physically-based, distributed model and used it to produce synthetic data in order to establish a sheet erosion empirical equation for the Brazilian semiarid. The sheet erosion equation was then tested with data from nine different plots within the region[2)].

Basically, there are three categories of soil erosion models, which are empirical, conceptual, and physically-based model. The empirical models are based on data from field observations, and mostly standard runoff plots on uniform slopes. The main limitation of this type of models is their limited applicability outside the range of conditions for which they have been developed. Their adaptation to a new environment requires a major investment of resources and time to develop the database required to drive the model. Thus, in order to collect a set of data large enough to calibrated such equation, the authors used a physically-based model to produce the synthetic data, then they could reduce the investment of resources and time needed for such kind of calibration. The empirical equation proved to be a practical tool to evaluate the soil erosion on plots with different kinds of vegetation covers as well as degree of slope. In this paper, this proposed empirical equation is used in the development of a conceptual model, which lies somewhere between the empirically and physically-based models, and it is applied to a bare micro-basin in northeastern Brazil on nonuniform slopes using several events selected between 1987 and 1991. The micro-basin is represented as a cascade of planes, thus the main step forward by the development of this model was the introduction of the grouping of the studied area into a number of elements in order to describe the spatial variations in erosion and deposition.

2. TEST FIELD

The test field is located in northeastern Brazil in a typical semiarid area. It is an experimental watershed named Sumé Experimental Watershed, which has been in operation since 1972 by SUDENE (Superintendency of Northeast Development), ORSTOM (French Office of Scientific Research and Technology for Overseas Development), and UFPB (Federal University of Paraíba)[3]. The Sumé Experimental Watershed has four micro-basins with areas between 0.5 to 1 ha, nine experimental plots, one sub-basin with 10 km^2 operated by natural rainfall and several micro-plots with 1 m^2 operated by simulated rainfall. The equipment are conceived to measure the runoff and sediment yield from most of the low to heavy rainfall events. Brown non calcic "vertic" soil covers more than 85% of the watershed and this soil is typical of most of the Brazilian semiarid regions. The surface conditions as well as the slopes for each of either micro-basin or plot are different.

Fig. 1 shows the micro-basin number 3 which was selected to be studied here because it had no vegetation. This micro-basin has a mean slope of 7.1%, area of 5200 m^2 and a perimeter of 302 m. At the outlet of the micro-basin, a rectangular collector for the measurement of sediment discharge is settled, terminating with a 90° triangular weir for the measurement of flow discharges. The collector holds all the surface runoff and sediment discharges from most of the low to medium rainfall events, thereby providing a means for accurate runoff and sediment measurement, and a recording rain gauge is installed close to the micro-basin to provide the necessary precipitation data.

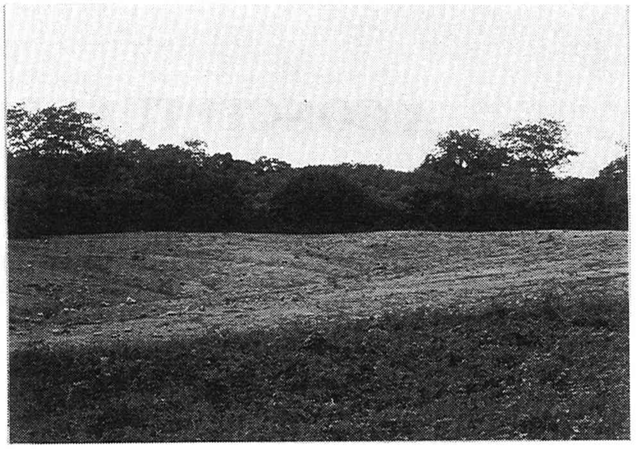

Fig.1 Micro-basin 3 with bare surface.

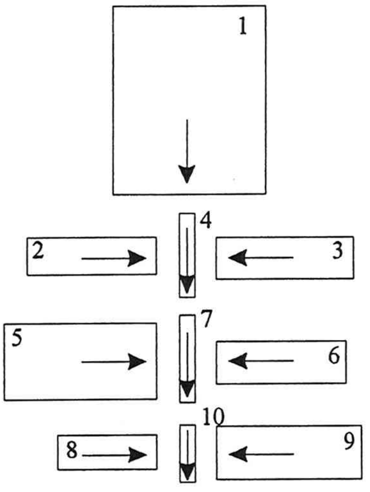

Fig.2 Schematic representation of micro-basin 3 in 10 elements.

3. CONFIGURATION OF THE BASIN

In order to start the modeling it was necessary to represent the micro-basin as a cascade of planes. As distortion errors may occur when representing a basin in plane and channel cascade, these distortions must be reduced in order to achieve accurate results. Segmentation of the micro-basin was made from its topographic mapping according to the delineation of the overland flow planes. The plane boundaries are either streamlines or contour lines. During the discretization process an attempt was made to minimize geometric distortion by preserving the areas and length of flow paths for each plane element. The micro-basin was represented in three different configurations, which were 4 elements, 10 elements and 23 elements. Since Santos et al.[4] proved that the 10 elements configuration **(Fig. 2)** was the best configuration to be used, only the results for this configuration will be presented here.

4. THE CONCEPTUAL EROSION MODEL

(1) Equations

It was assumed that all abstractions arise from infiltration and a method for determining the ponding time and infiltration under a variable intensity rainfall was developed based on Green & Ampt infiltration equation as presented by Chow et al.[5]

The basic principles used for determining ponding time is that in the absence of ponding, cumulative infiltration is calculated from cumulative rainfall; the potential infiltration rate at a given time is calculated from the cumulative infiltration at that time; and ponding has occurred when the potential infiltration rate is less than or equal to the rainfall intensity.

Then, the first step is to calculate the current potential infiltration rate f_t from the known value of cumulative infiltration F_t:

$$f_t = K_s\left(1+\frac{N_s}{F_t}\right) \quad (1)$$

where K_s is the effective soil hydraulic conductivity (m/s) and N_s is the soil moisture-tension parameter (m).

The result f_t is compared to the rainfall intensity i_t. If f_t is less than or equal to i_t, case (1) arises and there is ponding throughout the interval. In this case, the cumulative infiltration at the end of the interval, $F_{t+\Delta t}$, is calculated from

$$F_{t+\Delta t} - F_t - N_s \ln\left(\frac{F_{t+\Delta t}+N_s}{F_t+N_s}\right) = K_s \Delta t \quad (2)$$

Both cases (2) and (3) have $f_t > i_t$ and no ponding at the beginning of the interval. Assume that this remains so throughout the interval; then, the infiltration rate is i_t and a tentative value for cumulative infiltration at the end of the time interval is

$$F'_{t+\Delta t} = F_t + i_t \Delta t \quad (3)$$

Next, a corresponding infiltration rate $f'_{t+\Delta t}$ is calculated from $F'_{t+\Delta t}$. If $f'_{t+\Delta t}$ is greater than i_t, case (2) occurs and there is no ponding throughout the interval. Thus $F_{t+\Delta t} = F'_{t+\Delta t}$ and the problem is solved for this interval.

If $f'_{t+\Delta t}$ is less than or equal to i_t, ponding occurs during the interval (case(3)). The cumulative infiltration F_p at ponding time is found by setting $f_t = i_t$ and $F_t = F_p$ in Eq. (1) and solving for F_p to give

$$F_p = \frac{K_s N_s}{i_t - K_s} \quad (4)$$

The ponding time is then $t + \Delta t'$, where

$$\Delta t' = \frac{F_p - F_t}{i_t} \quad (5)$$

and the cumulative infiltration $F_{t+\Delta t}$ is found by substituting $F_t = F_p$ and $\Delta t = \Delta t - \Delta t'$ in Eq.(2). The excess rainfall values are calculated by subtracting cumulative infiltration from cumulative rainfall, then taking successive differences of the resulting values. The excess rainfall rate is also calculated by subtracting infiltration rate from rainfall intensity.

If the excess occurs, then the sediment yield for each plane element for that interval can be computed by an empirical sheet erosion equation.

$$E = \alpha L^{\beta_1} S^{\beta_2} r_e^{\beta_3} \quad (6)$$

where E is the sediment yield (kg/m^2/hr), L is the element length (m), S is the element slope (%), r_e is the rainfall excess rate (mm/hr), α is the coefficient depending mainly on the vegetation condition of soil surface and on soil characteristics, and β_1, β_2 and β_3 are the constant exponents. Finally, the sediment yield in kg can be computed by multiplying the sediment yield in kg/m^2/hr by the element area and time interval Δt.

(2) Parameters

In order to calibrate Eq.(6), it is necessary to make a large database, but to minimize the investments to develop such database, synthetic data produced by a physically-based model could be used, as it was proposed by Santos et al.[2] They have calibrated this equation for this specific micro-basin; thus, the values of these parameters α, β_1, β_2 and β_3 were obtained to be 1.91 x 10^{-5}, 0.90, 1.04 and 1.91, respectively. Then, this equation can be written as:

$$E = 1.91\text{x}10^{-5} L^{0.90} S^{1.04} r_e^{1.91} \quad (7)$$

The parameters for the infiltration are K_s equal to 5 mm/hr based on field tests, and the N_s must be different for each rainfall event because the moisture condition changes; however, if the correct value for this parameter is unknown, it can be determined by adjusting the observed and calculated sediment yield values.

Any time interval Δt (s) can be used, but this interval can be computed based on the maximum rainfall intensity:

$$\Delta t < \frac{\Delta x}{\alpha' m\left[\frac{(i_{\max} - K_s)L}{\alpha'}\right]^{\frac{m-1}{m}}} \quad (8)$$

where L is the length of element at the lower end of the characteristic length (m), m is a parameter of geometry equal to 5/3, i_{max} is the maximum rainfall intensity for the correspondent event (m/s), α' is a parameter related to surface roughness set as $(1/n)S^{1/2}$ where n is Manning friction factor of flow resistance of element at the lower end of the characteristic length set to 0.02 for the planes and 0.03 for the channels, and Δx (m) is given by:

$$\Delta x = \frac{L}{N-1} \quad (9)$$

where N is the maximum value between 3 and $15L/L_{max}$, in which L_{max} is the characteristic length (m). The characteristic length is defined as either the sum of the lengths of the longest cascade of planes in the system or the longest single channel whichever is greater.

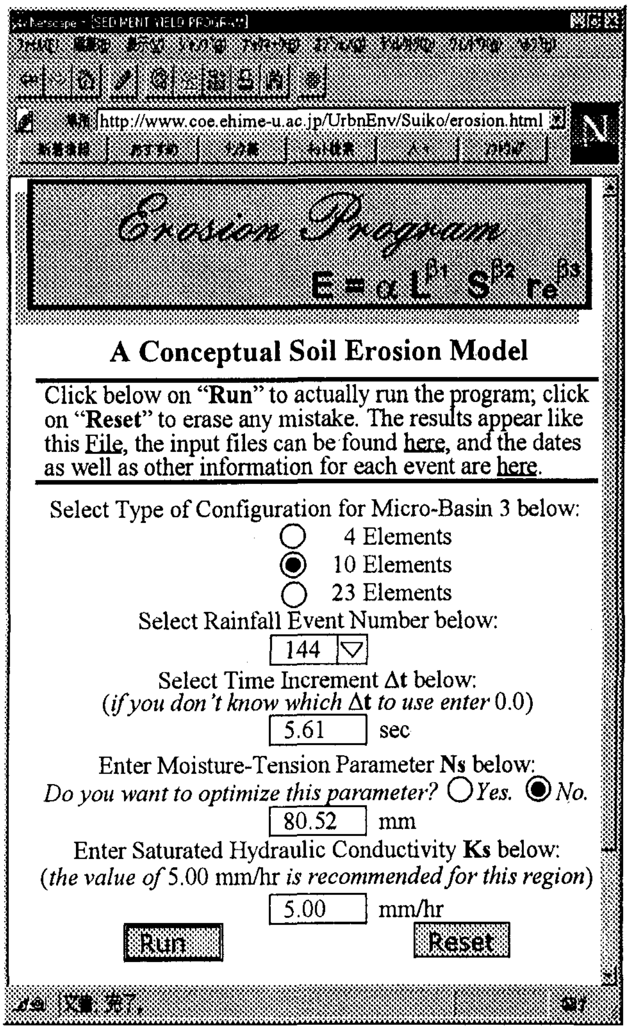

Fig.3 The layout of the program in the WWW.

Table 1 Excess rainfall hyetograph and sediment yield.

Time (min)	Rain (mm/hr)	f (mm/hr)	r_e (mm/hr)	Erosion (kg)
1.00	13.50		0.00	0.00
1.10	13.50	12925.14	0.00	0.00
1.20	13.50	6465.07	0.00	0.00
1.30	13.50	4311.71	0.00	0.00
1.40	13.50	3235.04	0.00	0.00
1.49	13.50	2589.03	0.00	0.00
1.59	13.50	2158.36	0.00	0.00
1.69	13.50	1850.73	0.00	0.00
1.79	13.50	1620.02	0.00	0.00
1.89	13.50	1440.57	0.00	0.00
1.99	13.50	1297.01	0.00	0.00
(...)				
10.31	84.00	96.99	0.00	0.00
10.40	84.00	93.09	0.00	0.00
10.50	84.00	89.51	0.00	0.00
10.60	84.00	86.20	0.00	0.00
10.70	84.00	83.15	0.00	0.00
10.80	**84.00**	**81.70**	**0.85**	**0.03**
10.90	**84.00**	**79.64**	**2.30**	**0.21**
11.00	**60.24**	**77.36**	**4.36**	**0.71**
(...)				
19.02	0.00	58.54	0.00	0.00
19.12	0.00	58.54	0.00	0.00
19.22	0.00	58.54	0.00	0.00
19.32	0.00	58.54	0.00	0.00
19.41	0.00	58.54	0.00	0.00
19.51	0.00	58.54	0.00	0.00
19.61	0.00	58.54	0.00	0.00
19.71	0.00	58.54	0.00	0.00
19.81	0.00	58.54	0.00	0.00
19.91	0.00	58.54	0.00	0.00
20.01	0.00	58.54	0.00	0.00

5. RESULTS

The program is today available in the World Wide Web (WWW), as shown in **Fig. 3**, at http://www.coe.ehime-u.ac.jp/UrbnEnv/Suiko/erosion.html, which makes it available for the researchers in UFPB and Ehime University as well as in other universities and centers. In order to run it, the user must choose the micro-basin configuration, the event number, and then enter the Δt in seconds, N_s in mm and K_s in mm/hr. However, if the user does not know which Δt to use in the computation, the value zero can be entered, and then the program will compute a new value based on Eq.(8). The value for N_s can be set freely, but there is an option for this parameter to be optimized, then the optimum value, which gives the closest value of sediment yield to the observed one, is found. The value of 5 mm/hr is recommended to K_s for this area based on field tests, but the user can use any value as well. As result, the program gives the infiltration rate, rainfall excess rate, and sediment yield according to the time. Sediment yield for each element is also given, then the user is able to compare the production of sediment for each element. **Table 1** shows the principal outputs for the simulation of event number 156 for the first, tenth (with rainfall excess) and last minute. The total computed sediment yield is 0.94 kg with 0.42 kg in element 1, 0.06 in element 2, 0.08 in element 3, 0.16 kg in element 5, 0.08 kg in element 6, 0.03 in element 8, and 0.11 in element 9.

The N_s parameter controls the infiltration rate, which controls the rainfall excess amount. **Fig. 4** shows the hyetograph of event number 156 and the infiltration rate when N_s was set to 57.56 mm, which is the optimized value. If a smaller value is used instead, e.g., N_s = 40.00 mm, the infiltration rate curve will drop and the rainfall excess will become greater as shown in the figure.

Table 2 shows the final results for the events between 1987 and 1991. It can be observed that the calculated sediment yield and observed ones agree very well because the model optimization was made in order to fit these values. Since the N_s parameter

Table 2 Final results for the computed sediment yield.

Event	Date (m/d/y)	Δt (s)	N_s (mm)	E_o (kg)	E_c (kg)
144	02/07/87	5.61	80.52	4.09	4.09
146	02/12/87	10.25	13.78	19.19	19.19
148	03/02/87	8.99	27.32	0.97	0.98
150	03/10/87	5.94	36.37	21.54	21.54
151	03/12/87	9.19	6.43	29.73	29.73
155	05/02/87	9.17	21.75	1214.61	1214.60
156	06/01/87	5.94	57.56	0.94	0.95
159	06/29/87	9.91	3.14	168.18	168.18
160	07/04/87	11.42	3.35	5.27	5.27
161	07/09/87	7.57	9.04	2823.75	2823.74
162	01/20/88	6.13	120.73	2061.86	2061.86
165	02/23/88	10.98	3.32	568.49	568.48
166	02/24/88	5.61	74.84	3334.78	3334.78
167	02/25/88	11.25	1.20	1000.78	1000.79
168	03/02/88	9.96	19.66	571.51	571.51
169	03/09/88	5.77	86.05	448.01	448.01
171	03/12/88	12.23	5.32	4.38	4.38
172	03/14/88	8.04	10.77	1875.53	1875.52
173	03/15/88	5.20	187.60	3499.10	3499.09
174	03/19/88	8.40	8.17	580.07	580.07
175	03/21/88	5.22	68.66	3514.92	3514.92
177	03/24/88	5.11	98.47	4019.04	4019.03
178	04/05/88	5.34	83.28	3615.40	3615.40
179	04/08/88	5.34	49.30	1286.65	1286.65
183	04/19/88	5.77	27.19	3504.55	3504.56
184	04/20/88	9.73	2.65	441.75	441.74
185	04/22/88	8.04	7.82	2694.00	2694.02
186	04/30/88	6.06	53.81	887.47	887.47
187	05/06/88	5.77	46.08	898.47	898.47
190	07/13/88	14.92	3.91	40.73	40.72
191	07/16/88	15.66	0.06	83.56	83.56
192	07/25/88	10.50	6.33	0.49	0.50
194	01/14/89	9.91	12.11	0.04	0.05
197	03/01/89	7.57	45.04	918.50	918.50
200	03/27/89	9.13	5.29	527.50	527.50
204	04/02/89	11.84	1.05	538.84	538.85
205	04/05/89	19.59	2.55	2.46	2.45
206	04/06/89	14.16	0.67	190.68	190.69
208	04/08/89	8.63	8.69	2229.68	2229.68
210	04/21/89	6.59	15.20	674.21	674.21
211	04/23/89	10.82	1.39	551.97	551.97
212	04/27/89	4.92	11.77	3074.86	3074.86
214	05/04/89	12.94	2.56	128.99	129.00
215	05/09/89	7.57	2.93	1186.17	1186.17
216	05/11/89	14.49	0.93	167.67	167.66
217	05/12/89	8.63	0.09	286.72	286.73
223	07/03/89	12.71	4.52	0.40	0.40
228	07/08/89	9.62	8.01	415.01	415.00
237	02/08/90	6.86	30.36	1.77	1.77
239	02/10/90	8.23	2.23	404.36	404.37
244	04/28/90	12.71	2.52	0.57	0.56
245	04/30/90	10.19	11.89	567.56	567.56
247	05/28/90	5.61	36.48	793.95	793.95
250	07/06/90	14.85	3.08	0.06	0.07
253	10/19/90	5.77	49.92	1443.65	1443.65
263	05/18/91	9.41	12.44	54.40	54.40
264	05/19/91	11.42	3.07	245.54	245.53

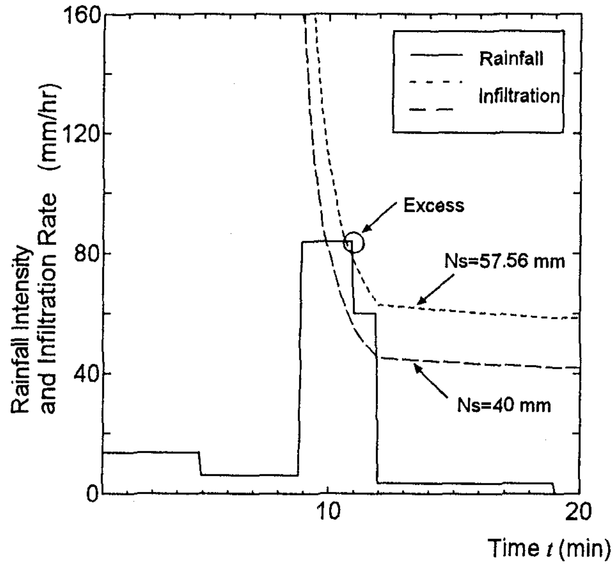

Fig.4 Rainfall hyetograph and infiltration rate for event No. 156.

depends on the moisture soil conditions in the beginning of the rainfall, it is difficult to estimate such parameter without a permanent measurement system on site. It is important to observe that each event has a particular value for N_s, which is rather related to the previous weather conditions than to the present rainfall characteristics.

6. CONCLUSION

A conceptual soil erosion model was developed based on the data from an experimental watershed located in northeastern Brazil in a semiarid region, and the following conclusions are presented:

(a) Empirical models are practical tools to predict erosion but due to their limitation to be transferred outside the range of conditions for which they have been developed they become impractical.

(b) In order to reduce the investment of resources and time to develop the database required to calibrate such an equation, physically-based models can be used to produce the synthetic data.

(c) The empirical sheet erosion equation, which Santos et al.[2] proposed, proved to be practical to be used in the development of this conceptual model.

(d) Since the conceptual soil erosion model considers nonuniform slopes and complex areas, it was proved that it could be used to predict erosion in watershed using the minimum of investment of resources and time needed.

(e) A practical tool to predict erosion should be simple as an empirical equation and based physically as fundamental models. Thus, it was proposed a conceptual model which lies between the empirically and physically-based models. The proposed model is limited to the applied area (as an empirical model) but it considers complex areas with different slopes and treats the infiltration process separately (as a fundamental model). The results showed then the practicality of the presented model.

(f) Finally, since the model is in the WWW, it is possible to the researcher groups of UFPB and Ehime University use it in their joint research projects keeping new model versions as well as maintaining a large database.

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REFERENCES

1) Lørup. J. K. and Styczen, M.: Soil erosion modelling. In: M.B. Abbott and Refsgaard, J. C. (eds.), Distributed Hydrological Modelling, Kluwer Academic Publishers, pp.93-120, 1996.

2) Santos, C. A. G., Suzuki, K., Watanabe, M. and Srinivasan, V. S.: Developing a sheet erosion equation for a semiarid region, Human Impact on Erosion and Sedimentation, *Proceedings of Rabat Symposium*, IAHS Publication no. 245, pp.31-38, 1997.

3) Cardier, E. and Freitas, B. J.: Bacia Representativa de Sumé, primeira estimativa dos recursos de água (Representative Basin of Sumé, first estimation of the water resources), Recife, Brazil, SUDENE, 1982. (in Portuguese)

4) Santos, C.A.G., Suzuki, K. Watanabe, M. and Srinivasan, V. S.: Scale effects of basin elements on coefficients in runoff-erosion modeling, *Annual Journal of Hydraulic Engineering*, JSCE Vol. 38, pp. 83-88, 1994.

5) Chow, V. T., Maidment, D. R. and Mays, L. W.: Applied Hydrology, McGraw-Hill International Editions, Civil Engineering Series, Chapter 5, pp.127-174, 1988.

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