A NEW FORM OF DISPERSION COEFFICIENT MODEL FOR THE POROUS MEDIA

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Mixing length theory concept can successfully comprehend the dispersion phenomena inside the porous medium. Based on this theory, dispersion coefficient is expressed as a function of average pore water velocity; with mixing length of the water paths and coefficient of variation of pore water velocity as model parameters[10),18),19)]. In this study, characteristics of coefficient of variation of the pore water velocity, a model parameter is discussed based on Van Genuchten's suction- moisture content and hydraulic conductivity-moisture content relationships. Numerical analysis shows that the coefficient of variation of pore water velocity can be expressed as a function of only parameter ***m*** of Van Genuchten's suction-moisture content relation. Based on this result, and the characteristics of mixing lengths, a new model for the dispersion coefficient is derived. The applicability of the proposed model is checked using the published data of previous researches.

Key Words : *porous media, dispersion coefficient, mixing length, pore water velocity, degree of saturation, suction, hydraulic conductivity*

1. INTRODUCTION

Planning and management of groundwater resources includes pollution control and abatement. It requires a tool for predicting the response of the aquifer system to the planned activities. A model is a simplified version of reality that is useful as a tool. A successful model strikes a balance between ideal and practicality. Finally the model should be verified by experimental data[1)].

Advection dispersion model is usually used to describe the flow of a solute through the porous medium because of its simpler form and few parameters to be evaluated experimentally. Many attempts have been made to relate the dispersion coefficient incorporated in that model to measurable flow variables empirically. Dispersion coefficient models proposed by Rumer[2)], Fried and Combarnous[3)], Passioura[4)], and Bear and Verruijt[5)] for saturated porous media and models used by Kirda et al.[6)], Yule and Gardner[7)] and De Smedt et al.[8)] for unsaturated porous media can be taken as some of the examples. However, existing empirical dispersion coefficient models for unsaturated condition are unable to describe the decreasing phenomena of dispersion coefficient, in the higher moisture content range of the porous media, for examples that observed by Gupta et al.[9)] and Matsubayashi et al.[10)].

Matsubayashi et al.[10)] and Devkota[11)] applied the mixing length theory to comprehend the observed dispersion processes inside the porous media. This is based on the analogy between the mixing phenomena of the solute inside the porous medium and the mixing of particles in turbulent flow in surface water regime. They concluded that mixing length theory is capable to comprehend the observed dispersion phenomena successfully both for the saturated and unsaturated conditions. Based on mixing length theory, dispersion coefficient, ***D***, can be expressed as:

$$D = \sigma l = \lambda l \bar{v} \qquad (1)$$

where, l is the mixing length of water paths, σ and $\bar{v}$ are respectively the standard deviation and the average of the pore water velocity inside the porous medium at a particular saturation. $\lambda = \sigma/\bar{v}$, is the coefficient of variation of the pore water velocity, which can be estimated from suction-degrees of saturation *(ψ-S)* and hydraulic conductivity-degrees of saturation ***(K-S)*** relationships of the porous media[10),11)]. Several relations can be found in the literature relating matric suction to the degrees of saturation. Some examples among them are: Brooks and Corey[12)], Campbell[13)], Van Genuchten[14)], and Fayer and Simmons[15)]. Since Van Genuchten's[14)] *ψ-S* model is popularly used to describe suction-moisture content relation, and also it has corresponding ***K-S*** model, characteristics of the pore water velocity is studied, in this research, based on Van Genuchten's models. A new dispersion coefficient model is then derived as a function of parameter "***m***" of Van Genuchten's *ψ-S* model, degrees of saturation and the grain size of the medium. The applicability of the proposed model is checked using the published data[9),10),16),17),18)]. Comparison of the proposed model with the empirical model, used by previous researchers, is also made.

2. THEORETICAL CONSIDERATION

(1) Modeling of coefficient of variation

Van Genuchten[14)] proposed a relation for suction head, ψ, and the degree of saturation, S, as:

$$S = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \left[1 + (a\psi)^n\right]^{-m} \tag{2}$$

where θ, θ_S and θ_r are the water content, saturated water content and residual water content respectively. ***a, m*** and ***n*** are the parameters. Here ***a*** is somewhat related to inverse of the bubbling pressure and ***n*** to the pore size distribution Kosugi[20)]. Further, ***m*** $(0<m<1)$ is related to ***n*** as

$$m = 1 - 1/n$$

The relationship for hydraulic conductivity and degree of saturation is

$$K = K_s S^{1/2}\left[1 - \left(1 - S^{1/m}\right)^m\right]^2 \tag{3}$$

where K_S is the saturated hydraulic conductivity of the medium.

Figures 1(a) and **1(b)** show the *ψ-S* and ***K-S*** relations given by Eqs. (2) and (3) respectively with different parameters. It shows that the effect of parameter ***m*** or ***n*** is on the shape of the curves while the effect of parameters ***a*** and K_S is on their positions.

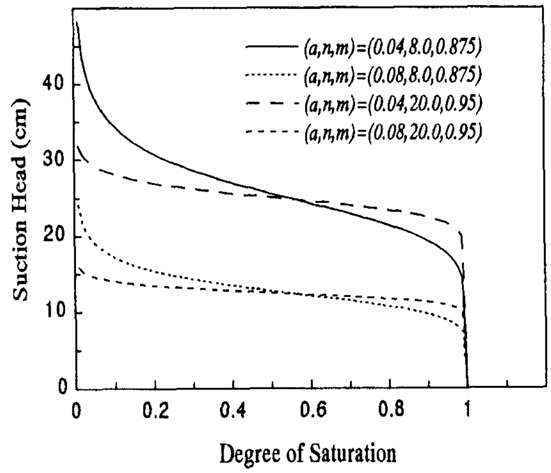

Fig. 1(a) Van Genuchten's suction head and degree ofsaturation relationships.

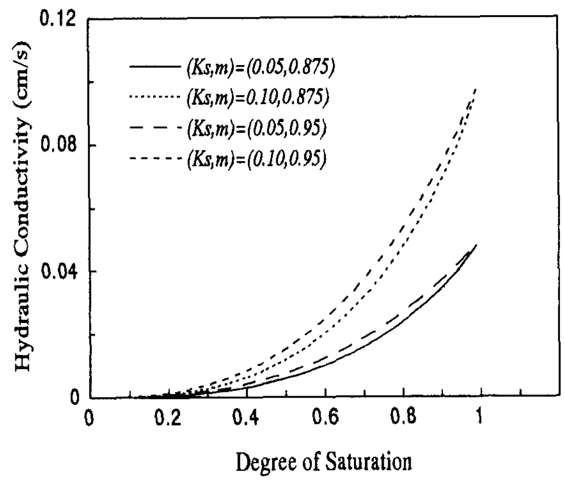

Fig. 1(b) Van Genuchten's hydraulic conductivity and degree of saturation relationships.

The coefficient of variation of the pore water velocity was determined numerically based on Eqs. (2) and (3). The procedure for determining λ can be found elsewhere[10),11),17),18)]. However, it is worthwhile to mention here that when ***S=1***, ψ was taken as ψa (air entry value) to avoid the singular point problem encountered in the analysis. This is reasonable since the change in the moisture content when suction changes from ψa to $\psi = 0$ is zero (see Devkota et al.[18)] for detail discussion on this point).

Numerical analysis found λ to be dependent on m or n but not on a or K_S. In **Figs. 2(a)** and **2(b)**, λ is plotted against m and n respectively. Relationship between λ and m is more simple than the relation between λ and n. λ-m relation shown in **Fig. 2(a)** can be expressed as

$$\lambda = 2.082 - 1.340m \tag{4}$$

Since parameter m in ψ-S relation is constant for a particular porous medium, λ is constant irrespective of moisture content.

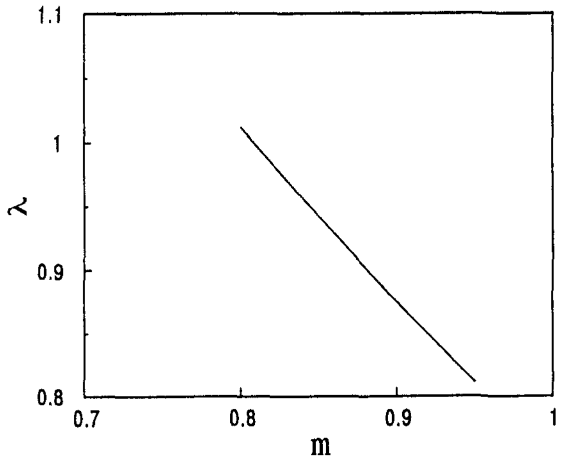

Fig. 2(a) Relationship between coefficient of variation of the pore water velocity and coefficient m.

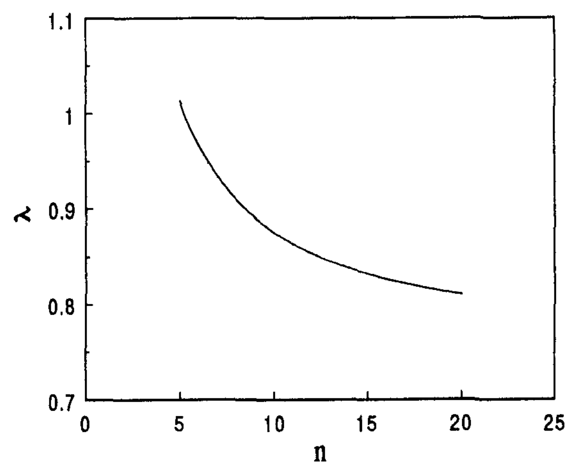

Fig. 2(b) Relationship between coefficient of variation of the pore water velocity and the coefficient n.

(2) Characteristics of mixing lengths

Mixing length reported in Devkota et al.[19] is reproduced in **Fig. 3**. **Figure 3** shows that mixing length is almost constant for a particular medium and about 8 times the grain diameter, d, of the porous media, until the degree of saturation of the media reaches a certain value. After this degree of saturation, mixing length starts to decrease, approaching to that of the saturated one. It can also be seen that saturated mixing length is constant for a given medium, about 1.8 times the grain diameter. It is assumed that after a critical value of saturation, S_C, the mixing length starts to decrease linearly until the porous media attains full saturation. Based on this assumption, l can be expressed by Eq. (5).

$$l = 8.0\text{d}, \qquad S < S_c \tag{5a}$$

$$l = 1.8d + \frac{6.2d}{(1-S_c)}(1-S), \quad S_c < S \le 1 \tag{5b}$$

Devkota et al.[19] and Devkota[11] had disscussed the observed characteristics of mixing lengths shown in **Fig. 3** under different degrees of saturation and for different grain sizes of the media. They argued that with increasing moisture content larger pores get filled with water resulting higher velocities in them, consequently mixing distances of adjacent water paths may not alter significantly. However, this behavior can be expected up to a certain level of closeness of the water paths (particular value of the saturation) inside the porous media. Beyond this particular value of the saturation mixing length starts to decrease, approaching that of the saturated one. It is also argued that larger mixing length in larger grained medium may be attributed to the larger distance the water paths have to travel to meet another one.

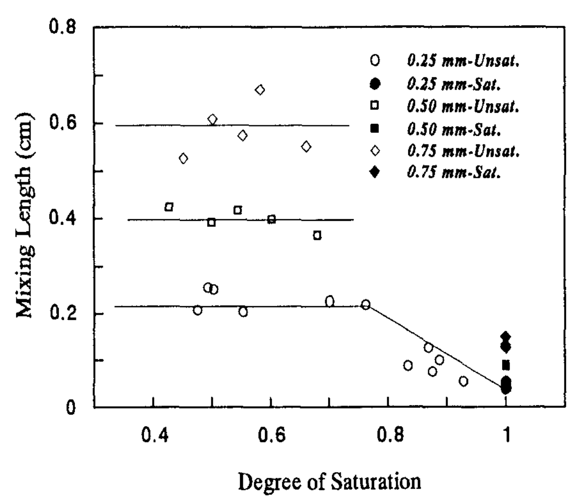

Fig. 3 Mixing length at different degrees of saturation.

(3) Proposed dispersion coefficient model

Inserting Eqs. (4) and (5) into Eq. (1), we will have

$$D = 8d\{2.082 - 1.340m\}\bar{v} \quad S < S_c \qquad (6a)$$

and

$$D = \left\{1.8d + \frac{6.2d}{(1-S_c)}(1-S)\right\}. \{2.082 - 1.340m\}\,\bar{v} \quad S_c < S \le 1 \qquad (6b)$$

Above relations are obtained without considering diffusion effect. If the pore water velocity is small, contribution of the diffusion coefficient should be taken in to account. Therefore, general form of the dispersion model becomes

$$D = D_o + 8d\{2.082 - 1.340m\}\bar{v} \quad S < S_c \qquad (7a)$$

$$D = D_o + \left\{1.8d + \frac{6.2d}{(1-S_c)}(1-S)\right\} \{2.082 - 1.340m\}\,\bar{v} \quad S_c < S \le 1 \qquad (7b)$$

(4) Estimation of parameter *m*

To apply the above model, ***m*** value should be estimated from ***ψ-S*** relationships. Van Genuchten[14] proposed the following empirical relation for the estimation of parameter ***m***.

$$m = 1 - \exp(-0.8S_p) \qquad 0 < S_p \le 1$$

$$m = 1 - \frac{0.5755}{S_p} + \frac{0.1}{S_p^2} + \frac{0.025}{S_p^3} \qquad S_p > 1$$

where S_p is the absolute value of the slope of *S* with respect to *logψ*, evaluated at point *P* of the soil-water retention curve, i.e.,

$$S_p = \left|\frac{dS}{d\log\psi}\right|$$

The slope S_p can be determined graphically from the experimental soil-water retention curve. According to Van Genuchten[14] the best location on the ***ψ-S*** curve for the evaluation of the slope is at *S*=50%.

The ***ψ-S*** relations of the glass beads of three different sizes were determined by Soil Column Method (*DOUCHYUHOU, in Japanese*, Nakano et al.[21], pp. 77-79). Fittings of the ***ψ-S*** relationship by the observed data are shown in **Fig. 4** while fitted parameters are given in **Table 1**. Parameters ***a*** and ***n*** are determined as given in Van Genuchten[14].

Table 1 Parameters of Van Genuchten's fittings.

Size of Glass Beads	*a*	*m*	*n*
0.25 mm	0.041	0.939	16.4
0.50 mm	0.066	0.899	9.90
0.75 mm	0.091	0.890	9.09

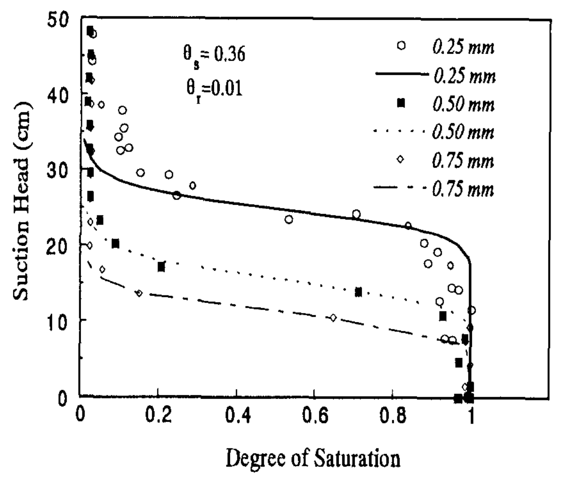

Fig. 4 Van Genuchten's suction head and degree of saturation relation fitted by observed data.

(5) Verification of the model

Observed dispersion coefficient in Devkota [18] and Matsubayashi et al.[10),17] are plotted against estimated ones by the proposed model in **Fig. 5**. This figure shows that proposed model is quite capable to describe the dispersion phenomena inside the porous medium.

To compare the proposed model with the empirical model (***D=α*** $\bar{\mathbf{v}}$, where ***α*** is the dispersivity of the media) used by previous researchers, dispersion coefficients obtained from the experiments of 0.25 mm glass beads[10], carried up to the higher moisture content conditions, are plotted against average pore water velocities in **Fig. 6**. In this figure open circles are the observed data while the dotted lines represent the estimated values based on the proposed model and solid line represents the result obtained from empirical model. From this figure it is clear that proposed model estimates are far better than the usual curve fitting technique, especially for observations in the higher moisture content range, which corresponds to higher pore water velocity range.

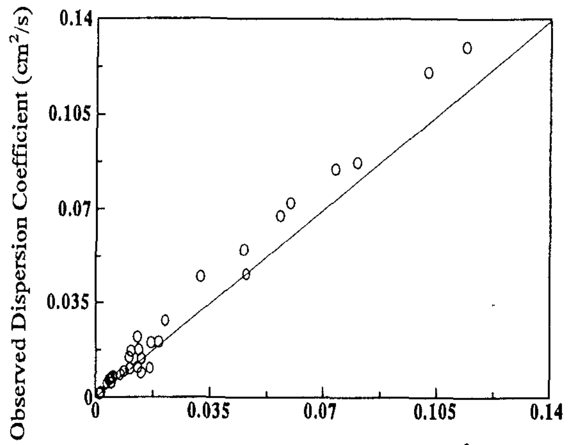

Fig. 5 Comparision between observed and estimated dispersion coefficients.

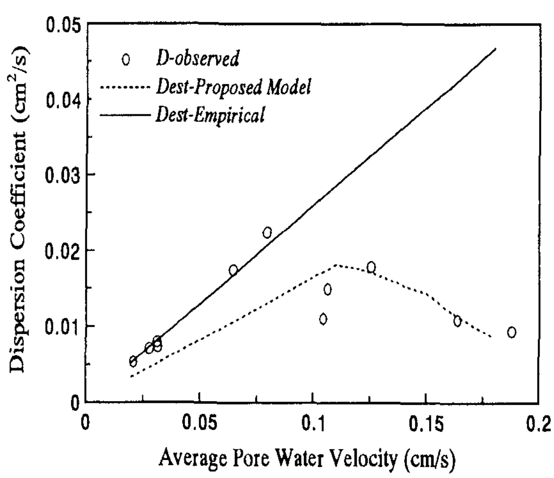

Fig. 6 Comparisons of different models for dispersion coefficient.

Gupta et al.[22] has used glass beads of size less than 0.125 mm. De Smedt and Wierenga[16] used *50%-50%* mixture, by volume, of two types of glass beads, diameter ranging from 0.088 to 0.125 mm and from 0.074 to 0.105 mm respectively. Average diameter was about 0.1 mm. Because there is no ψ-S relationship available for their data, ψ-S relationship of size 0.1 mm glass beads as obtained in this experiment is assumed valid for their glass beads (**Fig.** 7). Unsaturated dispersion coefficients observed by Gupta et al.[9] are plotted against dispersion coefficients estimated by proposed model in **Fig. 8**. Dispersion coefficients observed by De Smedt and Wierenga[16] are plotted against dispersion coefficients estimated by proposed models in **Fig.** 9. Their data are, however, of saturated experiments.

Figures 8 and **9** show that the proposed models are quite adequate to describe dispersion inside their porous media. Observed values of dispersion coefficients are little bit higher than the estimated ones. This may be attributed to the presence of immobile water in Gupta et al.[9] and due to the presence of trapped air which results in larger mixing length than the saturated one in De Smedt and Wierenga[16].

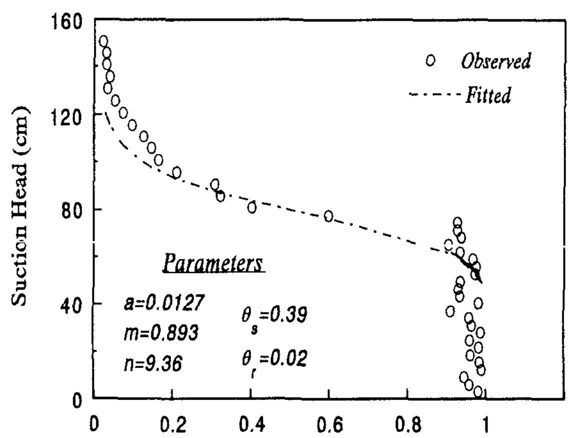

Fig. 7 Suction head and degree of saturation relationship of 0.1 mm glass beads.

3. CONCLUSIONS

Based on the preceding sections, the following conclusions can be made.

1. Coefficient of variation, a parameter of dispersion coefficient model, can be expressed as a function of only parameter ***m*** of Van Genuchten's ψ-S relation.

2 A new model for dispersion coefficient is presented. Verification of the model is made from the published results of Devkota et al.[18], Matsubayashi et al. [10),17)], De Smedt and Wierenga[16], and Gupta et al.[9]. It is found that model proposed in this study for dispersion coefficient is far better to estimate dispersion coefficient, especially in the

higher moisture content condition of the porous media.

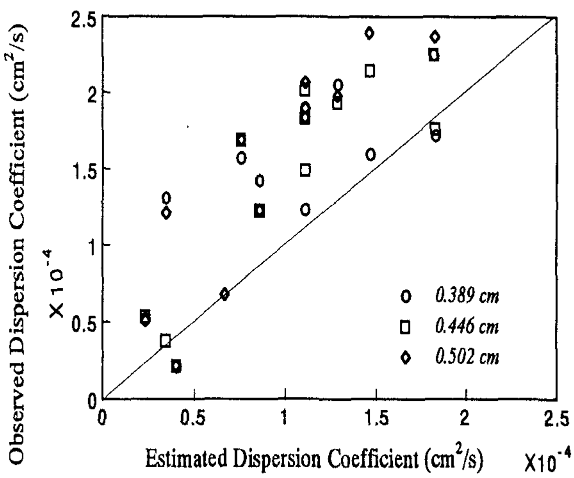

Fig. 8 Observed dispersion coefficients (Gupta et al., 1973b) vs. dispersion coefficients estimated by proposed model.

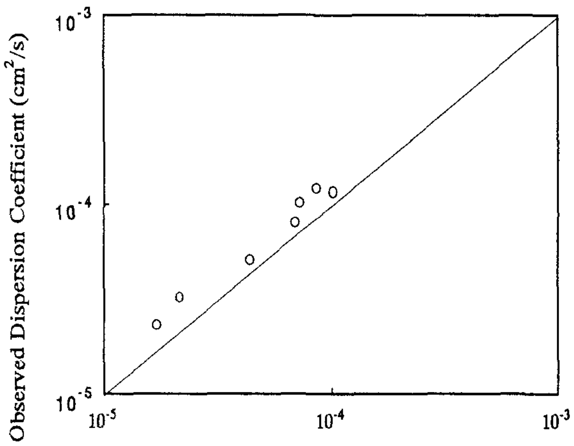

Fig. 9 Dispersion coefficients observed by De Smedt and Wierenga (1984) vs. estimated ones by proposed model.

REFERENCES

1) Bear, J., 1972. *Dynamics of fluids in porous media*, American Elsevier, New York.

2) Rumer, R. R., 1962. Longitudinal dispersion in steady and unsteady flow. *J. of the Hydraulics Division , Proc. of the ASCE, HY4*: 147-172.

3) Fried, J. J. and Combarnous, M. A., 1971. Dispersion in porous media. *Advances in Hydroscience*, 7:169-282.

4) Passioura, J. B., 1971. Hydrodynamic dispersion in aggregated media. 1. Theory. *Soil Sci.*, 111: 339-344.

5) Bear J. and Verruijt A., 1987. *Modeling groundwater flow and pollution*. D.Reidel Publishing Company, Tokyo.

6) Kirda, C., Nielsen, D. R. and Biggar, J. W., 1973. Simultaneous transport of chloride and water during infiltration. *Soil Sci. Soc. Am. Proc.*, 37: 339-345.

7) Yule, D. F. and Gardner, W. R., 1978. Longitudinal and transverse dispersion coefficient in unsaturated Plainfield sand. *Water Resou. Res.*, 14: 582-588.

8) De Smedt, F., Wauters, F. and Sevilla, J., 1986. Study of tracer movement through unsaturated sand. *J. Hydrol.*, 85: 169-181.

9) Gupta, R. K., Millington, R. J. and Klute, A., 1973b. Hydrodynamic dispersion in unsaturated porous media II. The stagnant zone concept and the dispersion coefficient. *J. Indian Soc. Soil Sci.* 21: 121-128.

10) Matsubayashi, U., L. P. Devkota and Takagi, F., 1997. Characteristics of the dispersion coefficient in miscible displacement through glass beads media. *J. Hydrol.*, 192: 51-64.

11) Devkota, L.P., 1996. Mixing length theory approach to dispersion phenomena in miscible displacement through glass beads media, Doctoral Thesis, Department of Civil Eng., Nagoya University, Japan.

12) Brooks, R. H. and Corey, A. T., 1964. Hydraulic properties of porous medium. *Hydrol papers*, Colorado State Univ., 3: 1-27.

13) Campbell, G. S., 1974. A simple method for determining unsaturated conductivity from moisture retention data. *Soil Sci.*, 117: 311-314.

14) Van Genuchten, M. Th., 1980. A closed-form equation for predicting the hydraulic conductivity of unsaturated soils. *Soil Sci. Soc. Am. J.*, 44: 892-898.

15) Fayer, M. J. and Simmons, C. S., 1995. Modified soil water retention functions for all matric suctions. *Water Resou. Res.*, 31: 1233-1238.

16) De Smedt, F. and Wierenga, P. J., 1984. Solute transfer through columns of glass beads. *Water Resou. Res.*, 20: 225-232.

17) Matsubayashi, U., L. P. Devkota and Takagi, F., 1996. Characteristics of mixing length in dispersion through unsaturated glass beads media. *Annual J. Hydraulic Engineering, JSCE*, 40:425-430.

18) Devkota, L. P., Matsubayashi, U. and Takagi, F., 1996. Mixing length approach to dispersion through saturated porous media and its relation to grain sizes, *J. Japan Society of Hydrol. and Water Res.*, 9: 551-558.

19) Devkota, L. P., Matsubayashi, U. and Takagi, F., 1997. Modeling of dispersion coefficient for the porous medium using mixing length theory concept, submitted to J. Hydrologic Engineering, ASCE.

20) Kosugi, K., 1994. Three-parameter lognormal distribution model for soil water retention. *Water Resou. Res.*, 30: 891-901.

21) Nakano, M., Miyazaki, T., Shiozawa, S. and Nishimura, T., 1995. Physical and environmental analysis of soil, University of Tokyo Press, Japan.

22) Gupta, R. K., Millington, R. J. and Klute, A., 1973a. Hydrodynamic dispersion in unsaturated porous media I. Concentration distribution during dispersion. *J. Indian Soc. Soil Sci.*, 21: 1-7.

(Received September 30, 1997)