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Analysis of Geomorphologic Properties Extracted from DEMs for Hydrologic Modeling

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Abstract

Flow accumulation method is the most common technique used in extracting river networks from digital elevation model (DEM) data. This method allows the extraction of networks with arbitrary density, that is, different threshold areas can be used to generate channel network with different density. Here, the effect of threshold area on width function and area function is examined. The scaling structures of width function and area function are studied using their singularity spectra. With the aid of singularity spectrum, it's easy to identify the differences in river networks resulting from different threshold areas. This paper discusses the stability of width function and area function and the limiting threshold areas for deriving catchment area function.

Keywords: river network, width function, area function, singularity spectrum, hydrologic model

1. Introduction

1.1 Background

It has long been recognized that catchment geomorphologic properties can be used as predictors of catchment flood characteristics. For example, mean channel length is commonly used in empirical formulae predicting the time of concentration of a catchment, and the mean annual flood is often related to the drainage area of the catchment or to the drainage density of the river network. *Rodriguez-Iturbe* and *Valdes* [1979][1] introduced the geomorphologic instantaneous unit hydrograph (GIUH) concept to address the effect of complex stream network geometry on catchment response.

The river network used in distributed hydrologic modeling is usually generated from digital elevation models (DEMs). Flow accumulation method, which is based on the steepest descent flow routing, is the most common technique to extract river networks from DEMs. This method allows the extraction of networks with arbitrary density, that is, resolution of networks to an arbitrary scale. An important aspect of flow accumulation extracting procedure is to decide where to begin the river. Traditional method is to specify a critical support area (a threshold area) that define the minimum drainage area required to initiate a river. Recently, many researchers examined the effects of threshold area selection on various stream ratios and fractal dimension, *Montgomery* and *Foufoula-Georgiou* [1993][2] *Ichoku* and *Karnieli*, et al. [1996][3].

An alternative form of representing the geomorphology, firstly by *Mesa* and *Mifflin* [1986][4], is by means of the width function. The width function W(x) is defined as the frequency distribution of streams with respect to flow distance from the outlet. It is an approximate representation of the area function A(x) under the assumption of a uniform constant of river maintenance throughout the drainage basin. The area function is defined as the frequency distribution of accumulative area with respect to flow distance from the outlet. Considering the total cumulative areas $A_C(x)$ which drains as overland flow into any point at distance x, then the area function is precisely defined as: $A(x) = dA_C(x) / dx$.

1.2 Use of Width Function and Area Function in Hydrologic Modeling

Mathematically describing the catchment spatial heterogeneities and solving the river network flood routing are important problems in hydrologic modeling. In the general case of surface hydrology, extensive computational effort is required to solve equations describing processes in two dimensions. This complexity can be greatly simplified by reducing the lateral dimension from two to one.

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Area function gives us a mathematical description of accumulative catchment area distribution. With the aid of GIS, area function can be modified to include other spatial variations, such as rainfall, soil type, and land use. These descriptions (functions) are useful for simplifying surface flow modeling. Supposing $f_h(t)$ is the slope element response, the network lateral inflow can be lumped as

$$q(x,t) = f_h(t)A(x) \tag{1}$$

Then, modeling catchment hydrologic response is reduced to model the response of a equivalent single river with the lateral inflow q(x, t).

Width function gives the stream distribution of river network. Not only the number of streams, but the order, slope, and width of each stream also can be estimated using GIS, and a single "equivalent" river can be developed. Then network flood routing can be solved using any river routing model, such as kinematic wave and dynamic wave models.

This paper examines the stability of width function and area function under varying threshold areas and the relation between width function and area function. In general, derivation of width function is easier than that of area function. Therefore limiting threshold areas which permit the use of width function instead of area function is investigated.

2. Width Function and Area Function for Different Catchment

2.1 Study area

Fourteen catchments located in Hokuriku, Kanto, Shikoku, and Kyushu region of Japan, are selected for this study. Catchment areas vary from 464.0 km^2 to 3048.7 km^2. The DEM data are from 250 m mesh digital elevation data sets made by Japan Geophysical Research Institute. River networks are generated using flow accumulation method provided by Arc/Info software.

2.2 Extraction of width function and area function

When the river network is generated from digital elevation data, the flow direction and accumulative area along the river network are calculated at same time. The flow distance from catchment outlet along the network can also be calculated. In practice, the extraction of width function is much easier than area function without powerful software.

Mathematically, width function W(x) is defined as

$$W(x) = \sum_{j=1}^{N} n_j(x, d_{\min_j}, d_{\max_j}) \tag{2}$$

where, x is the distance from catchment outlet (m); $d_{\min_j}$ and $d_{\max_j}$ are the distances of downstream end and upstream end of link j from catchment outlet respectively.

$$n(x, d_{\min_j}, d_{\max_j}) = \{ \begin{matrix} 1, & d_{\min_j} < x \le d_{\max_j} \\ & 0 \end{matrix} \tag{3}$$

w(x) is the number of streams at distance x. N is the total number of links. By the definition, area function can be written as

$$A(x) = \frac{\sum_{j=1}^{W(x)} A_{C_j}(x) - \sum_{k=1}^{W(x+\Delta x)} A_{C_k}(x+\Delta x)}{\Delta x} \tag{4}$$

where, A(x) is the cumulative area at the distance x (m); W(x) and W(x+Δx) are the number of streams at the distance x and x+Δx respectively; A_C is the cumulative areas from upstream at a given point on the river network; Δx is the distance interval (m).

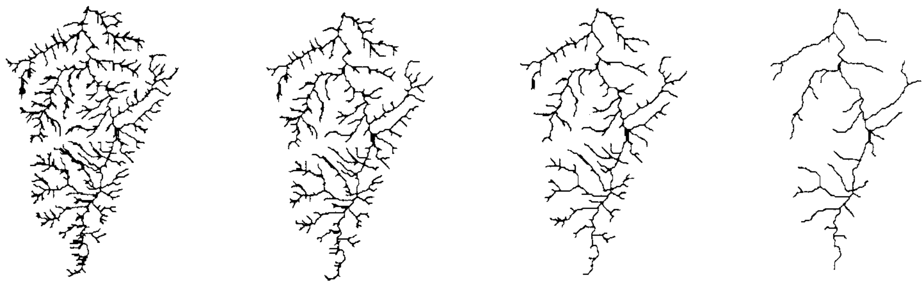
Here, the normalized width function and area function are used. The distance is normalized by the maximum stream length. Width function is normalized by the total number of links. Area function is normalized by total area of the catchment.

3. Effects of threshold Area on Width Function and Area Function

Before studying the geomorphologic properties, an example is given in Fig. 1 to show the river networks change with different threshold areas. Threshold areas are chosen arbitrarily, but include a large range.

3.1 Effect on width function

The width functions are calculated for all the study catchments at eight threshold areas. It is found that width functions extracted from different threshold areas are very similar. This indicates that width function is a stable parameter defining catchment river network structure. Fig. 2 shows an example of width functions extracted by different threshold areas.



(a) (b) (c) (d)

Fig. 1 Maps of river network defined using different threshold areas:
(a) -- 0.3125 km^2, (b) -- 0.625 km^2, (c) -- 1.25 km^2, (d) -- 5.0 km^2 (Hime River, Hokuriku Region)

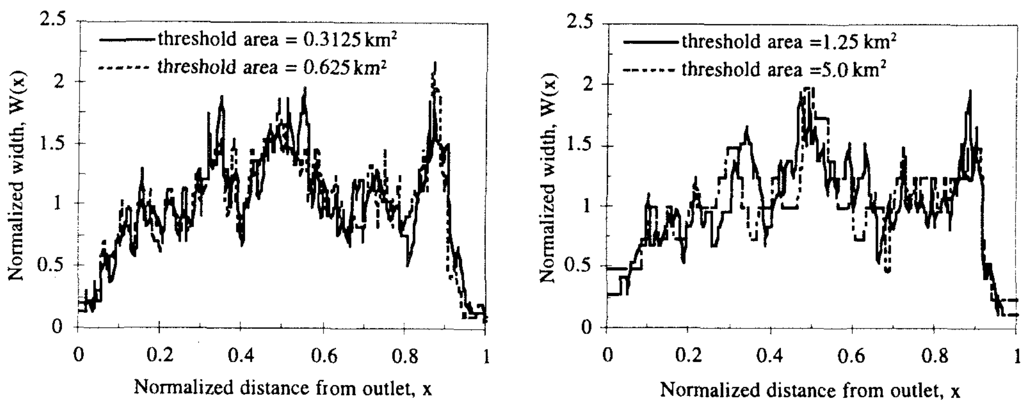


Fig. 2 Width function extracted by different threshold areas (Hime River, Hokuriku Region)

The long-term behavior of chaotic, nonlinear dynamical systems can often be characterized by fractal or multifractal measures. Fractal measure is described using a single fractal dimension. This only gives the bulk or average shape of the structure of the dynamical system. The multifractual case is described by a spectrum $f(\alpha)$ of fractual dimensions called singularity spectrum, (*Halsey* and *Jensen* et al [1986][5]), α is the singularity strength, whose Hausdorff dimension is $f(\alpha)$. *Chhabra* and *Jensen* [1989][6] proposed an algorithm for direct determination of $f(\alpha)$. If a width function is measured using an interval length of δ, the aggregation of its values over each interval of length δ is obtained as

$$\overline{W}_i(\delta) = \int_{(i-1)\delta}^{i\delta} W(x)\,dx \tag{5}$$

A single parameter family of normalized measures $m(q)$ is constructed as

$$m_i(q,\delta) = [\overline{W}_i(\delta)]^q / \sum_j [\overline{W}_j(\delta)]^q \tag{6}$$

The parameter q helps to enhance different regions of the singular measure. For $q > 1$, $m(q)$ amplifies the more singular regions, while for $q < 1$ it accentuates the less singular regions. The Hausdorff dimension of the measure is given by

$$f(q) = \lim_{\delta \to 0} \frac{\sum_i m_i(q,\delta)\log[m_i(q,\delta)]}{\log \delta} \tag{7}$$

The average value of the singularity strength α is calculated by

$$\alpha(q) = \lim_{\delta \to 0} \frac{\sum_i m_i(q,\delta)\log[\overline{W}_i(\delta)]}{\log \delta} \tag{8}$$

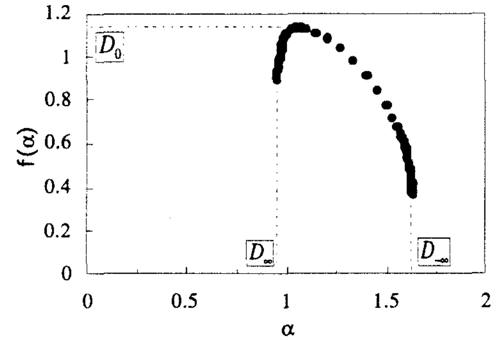


Fig. 3 Singularity spectrum

As shown in Fig. 3, certain features of the curve $f(\alpha)$ are quite general: $\partial f / \partial \alpha = q$, the curve has a single maximum at $q = 0$, and with infinite slope at the ends. Three important parameters characterize this curve, they are D_∞, D_0, and $D_{-\infty}$. D_∞ corresponds to that where the distribution is most concentrated, $D_{-\infty}$ corresponds to that where the distribution is most rarefied.

Fig. 4 shows an example of singularity spectrum derived by different threshold areas, and the parameters characterizing the spectrum are given in Table 1. From the results, we can find that threshold areas mainly change the less singular regions of the width function. So it's easy to understand why the general shape of width functions keeps same (Fig. 2). This regularity is common for all the catchments studied.

3.2 Effect on area function

In the same way, the spectrum of area functions are calculated. Fig. 5 shows an example, and Table 2 gives the characteristic parameters. From the results, we can find that both the more singular region and less singular region change with varying threshold areas. A critical threshold area can be specified for each catchment (Table 3). When the threshold area is smaller than the critical area, the scaling structure of area function can be viewed as same.

Table 1 Parameters characterizing the spectrum of width function(Hime River)

Threshold (pixels)	Area (km^2)	Basin Order	D_∞	D_0	$D_{-\infty}$
2	0.125	5	0.958	1.131	1.633
5	0.3125	5	0.954	1.131	1.494
8	0.5	5	0.947	1.132	1.551
10	0.625	5	0.93	1.131	1.531
20	1.25	4	0.959	1.125	1.468
40	2.5	4	0.956	1.121	1.429
80	5.0	3	0.922	1.125	1.336
160	10.0	3	0.926	1.112	1.300

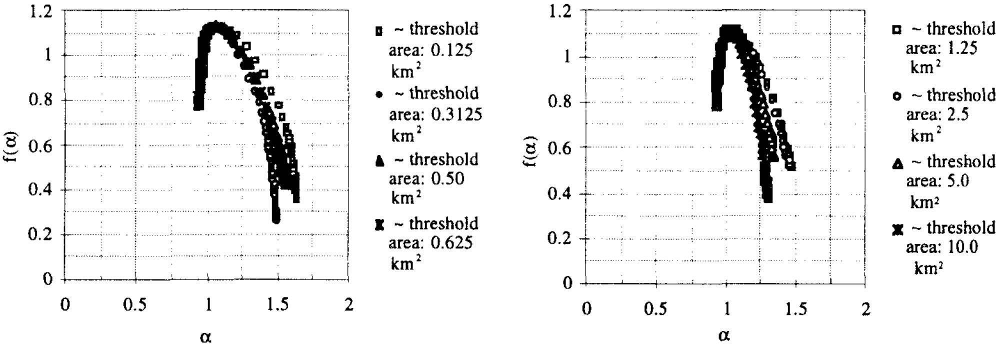


Fig. 4 Singularity spectrum extracted by different threshold areas (Hime River)

Table 2 Parameters characterizing the spectrum of area function (Hime River)

Threshold (pixels)	Area (km^2)	Basin Order	D_{∞}	D_0	$D_{-\infty}$
2	0.125	5	0.937	1.131	1.527
5	0.3125	5	0.9	1.131	1.604
8	0.5	5	0.895	1.132	1.928
10	0.625	5	0.89	1.131	1.859
20	1.25	4	0.775	1.125	1.82
40	2.5	4	0.755	1.121	1.811
80	5	3	0.687	1.125	1.758
160	10	3	0.641	1.112	1.818

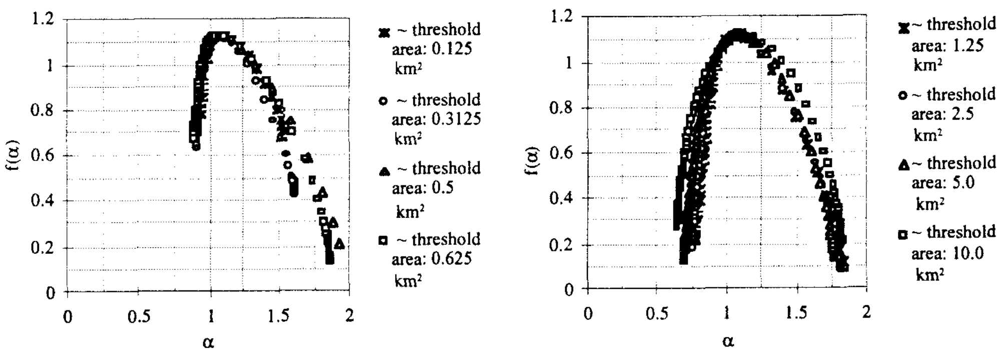


Fig. 5 Singularity spectra of area function (Hime River)

Table 3 Critical threshold area

Catchment	Area (km^2)	Catchment Average Slope (%)	Critical Threshold Area (km^2)
Hime River	708.5	28.2	0.625
Kurobe River	637.0	38.1	1.25
Seki River	1133.6	16.0	2.5
Agatsuma River	1232.8	18.3	0.3125
Watarase River	1236.6	11.9	0.625
Hiji River	1022.8	20.3	2.5
Naka River	776.1	25.8	1.25
Niyodo River	1466.3	24.8	1.25
Yoshino River	3048.7	24.3	2.5
Chikugo River	2276.2	11.8	1.25
Oita River	632.6	13.9	1.25
Onga River	939.7	13.7	2.5
Oono River	1381.9	11.0	2.5
Yamakuni River	464.0	15.7	0.625

3.3 Comparison of width function and area function

As we have seen above, the spectra of width function and area function are quite similar at small threshold areas. Fig. 7 shows the comparison of width function and area function. As a description of geomorphologic property width function is stable, and it can represent area function at small threshold area. Using the flow accumulation procedure to extract river network, only the streams (or parts of the streams) of first order or second order are omitted by increasing the threshold areas, but the accumulative areas at the stream sources change drastically. So the area function can not be extracted by arbitrary threshold area. As shown in Table 3, the critical threshold areas are related to catchment slope and area, but can not simply specify a critical threshold area according to the slope and area. Obviously, river network evolution is related to not only topography but geology and climate. Using specific threshold area (= threshold area A_{th} / catchment area A), the plot of limiting specific threshold areas versus catchment main river length is shown in Fig. 6. The result indicates the limiting specific threshold areas decrease with increasing of catchment main river length. This linear relation is not quite clear, but it gives us a reference to select the map scale for deriving area function from width function.

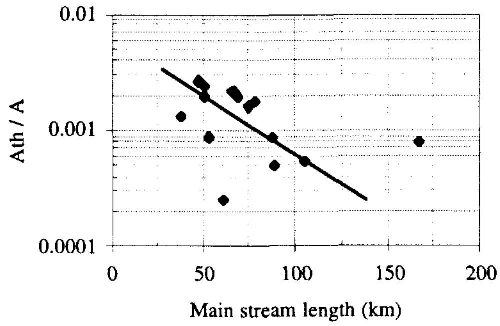


Fig. 6 Plot of A_{th}/A versus main stream length

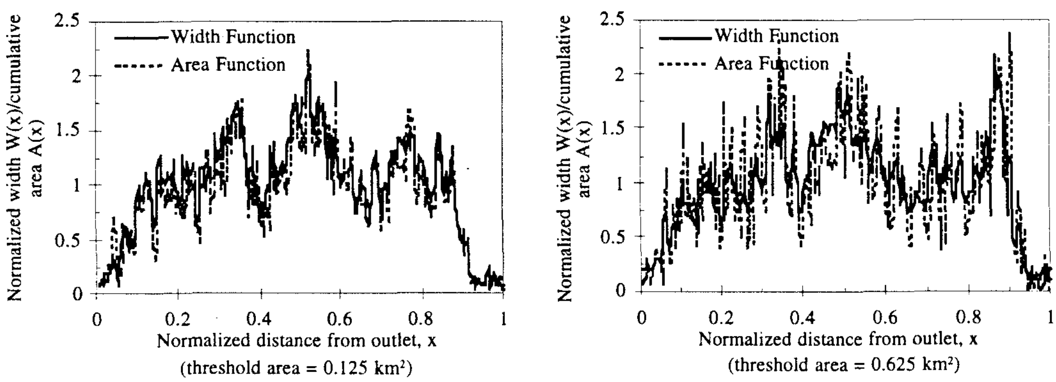


Fig. 7 Comparison of width function and area function (Hime River, Hokuriku Region)

4. Conclusion

In this paper, we discussed the width function and area function extracted from DEMs. The stability of width function and area function were analyzed. Width function as the geomorphologic property of the catchment is very stable, and very close to area function at small threshold areas. This makes it easy to obtain and use width function in hydrologic modeling. Area function can not be extracted by using arbitrary threshold area, because its scaling structure changes very much at large threshold area. Coupling with area function, the catchment heterogeneities can be described mathematically. Width function provides new possibilities of developing simple network routing models to simulate catchment hydrology. This concept is useful in surface hydrology, especially for ungauged large catchments.

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