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Modeling Infiltration into a Multi-layered Soil during an Unsteady Rain

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Summary: The Green-Ampt model is generalized to model infiltration into a multi-layered soil during an unsteady rain. One dimensional numerical simulation scheme of the Richards equation is also developed which is suitable to deal with transfers between two kinds of boundary conditions --- water flux control and water head control. Modeling results of the generalized Green-Ampt model are compared with a numerical simulation for cases of 2 and 3 layered soil as examples. It is shown that the proposed model matches the Richards equation quite well with computation time less than tenth of the latter, thus promising to play a role in watershed modeling.

Keywords: infiltration, multi-layered soil, the Green-Ampt model, the Richards equation

1. Introduction

Infiltration is an important process in hydrological cycles because it is a basis for determination of surface runoff generation, recharge to groundwater and actual evapotranspiration, etc. A lot of infiltration models had been suggested since the beginning of this century, e.g., the Green-Ampt model (1911), the Horton model (1933) and the Philip model (1957), etc.

Combining the Darcy's law with continuous equation, Richards (1931) derived a partial differential equation for moisture movement in unsaturated soil. This Richards equation has a clear physical basis and it may be most scientific to model infiltration. However, there are limitations to apply it to a large-scale watershed modeling because of heavy computation time and memory requirement and a lot of parameters needed to specify.

The Green-Ampt model has drawn attentions of many researchers in recent three decades because of its simplicity, physically-based characteristic and measurable parameters. Though it was originally a infiltration model for dry and uniform soil under surface ponding, it was extended to model infiltration during a steady rain by Mein & Larson (1973), to model infiltration into nonuniform soil by Bouwer (1969), to model infiltration during an unsteady rain by Chu (1978), to model infiltration into two-layered soil profiles during steady rains by Moore & Eigel (1981). It was also used or referred to in many watershed models, e.g., Kite (1991) and Huang et. al.(1996), etc.

In many cases in watershed modeling, vertical heterogeneity of soil can be treated as stratified soil layers, e.g., surface compressing in urbanized areas, surface sealing and cultivation in farmlands, in addition to natural stratification caused by geological processes. However, the general form of the Green-Ampt model for multi-layered soil has not been observed, letting alone that both multi-layered soil and unsteady rain are considered at same time. This research is aimed to modify the Green-Ampt model to make it applicable to actual cases of both soil stratification and rain unsteadiness.

2. Derivation of A General Model for a Multi-layered Soil and an Unsteady Rain

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2.1 The Green-Ampt model

Considering infiltration into a vertical uniform soil column when surface is ponded, Green and Ampt proposed an infiltration model by assuming there is a wetting front which separates saturated soil above from soil below and by using the Darcy's law. If the depth of ponding is negligible, then the Green-Ampt model can be written as:

$$f = k \cdot (1 + A / F) \qquad F = k \cdot t + A \cdot \ln(\frac{A+F}{A}) \qquad with \quad A = SW(\theta_s - \theta_0) \tag{1}$$

in which f = infiltration rate; F = accumulated infiltration; SW = capillary suction at the wetting front; k = hydraulic conductivity in the wetted zone; θs = moisture content in the wetted zone; θ_0 = initial moisture content; and t = time.

For infiltration during an unsteady rain, the modified Green-Ampt model can be written as (Chu, 1978):

$$before\ ponding \qquad f = I, \qquad F = F_{nb-1} + (t - t_{nb-1})I$$

$$after\ ponding \qquad f = k \cdot (1 + A / F), \qquad F - F_p = k(t - t_p) + A \cdot \ln(\frac{A+F}{A+F_p}) \tag{2}$$

$$with \quad F_p = \frac{A}{I_p/k - 1}, \qquad t_p = t_{n-1} + \frac{F_p - F_{n-1}}{I_p}, \qquad A = SW(\theta_s - \theta_0)$$

in which F_p = accumulated infiltration at the instant of surface ponding; t_p = time to surface ponding; I_p = rain intensity during the *nth* time step when surface ponding occurs; I = rain intensity; n & nb=1,2,..., time steps.

2.2 A general model for a multi-layered soil and an unsteady rain

Supposing that the wetting front is in *mth* soil layer, the average conductivity $\bar{k}$ in the transmission zone behind the wetting front can be given by the harmonic mean as follows (Bouwer, 1969; Moore & Eigel, 1981):

$$\bar{k} = (\sum_1^m Li) / (\sum_1^m \frac{Li}{ki}) \tag{3}$$

If surface ponding occurred from the beginning of a rain event and ponding is continuing, the Green-Ampt model (eq.(1)) will be applicable before the wetting front enters into second soil layer. After it enters *mth* layer, based on the Darcy's law we have

$$\frac{dF}{dt} = f = \bar{k}\frac{SW_m + L}{L} = \frac{SW_m + L}{\frac{L}{\bar{k}}} = \frac{SW_m + \sum_1^{m-1} L_i + (L - \sum_1^{m-1} L_i)}{\sum_1^{m-1}\frac{L_i}{k_i} + \frac{L - \sum_1^{m-1} L_i}{k_m}}$$

Because $F = \sum_1^{m-1} L_i\Delta\theta_i + (L - \sum_1^{m-1} L_i)\Delta\theta_m$ which can be rearranged as $L - \sum_1^{m-1} L_i = \frac{F - \sum_1^{m-1} L_i\Delta\theta_i}{\Delta\theta_m}$,

then
$$\frac{dF}{dt} = \frac{SW_m + \sum_1^{m-1} L_i + \frac{F - \sum_1^{m-1} L_i\Delta\theta_i}{\Delta\theta_m}}{\sum_1^{m-1}\frac{Li}{k_i} + \frac{F - \sum_1^{m-1} L_i\Delta\theta_i}{k_m\Delta\theta_m}} = k_m \cdot \frac{\sum_1^{m-1}\frac{L_i}{k_i} + \frac{F - \sum_1^{m-1} L_i\Delta\theta_i}{k_m\Delta\theta_m} + (\frac{\sum_1^{m-1} L_i + SW_m}{k_m} - \sum_1^{m-1}\frac{L_i}{k_i})}{\sum_1^{m-1}\frac{L_i}{k_i} + \frac{F - \sum_1^{m-1} L_i\Delta\theta_i}{k_m\Delta\theta_m}}$$

Let
$$A = \frac{\sum_1^{m-1} Li + SW_m}{k_m} - \sum_1^{m-1}\frac{L_i}{k_i}, \qquad B = \sum_1^{m-1}\frac{L_i}{k_i} - \frac{\sum_1^{m-1} L_i\Delta\theta_i}{k_m\Delta\theta_m}, \qquad C = k_m\Delta\theta_m,$$

integrating the above equation leads to:

$$\int_{F_{m-1}}^{F} dF = \int_{t_{m-1}}^{t} k_m(1 + \frac{A}{B + F/C})dt \qquad \Rightarrow \qquad \int_{F_{m-1}}^{F} (1 - \frac{AC}{AC + BC + F})dF = \int_{t_{m-1}}^{t} k_m dt$$

Let $A_{m-1}=AC$, $B_{m-1}=BC$, we will get:

$$f = k_m \cdot (1 + \frac{A_{m-1}}{B_{m-1} + F})$$
$$F - F_{m-1} = k_m (t - t_{m-1}) + A_{m-1} \cdot \ln(\frac{A_{m-1} + B_{m-1} + F}{A_{m-1} + B_{m-1} + F_{m-1}}) \quad (4)$$

with
$$A_{m-1} = (\sum_1^{m-1} L_i - \sum_1^{m-1} L_i k_m / k_i + SW_m)\Delta\theta_m$$
$$B_{m-1} = (\sum_1^{m-1} L_i k_m / k_i)\Delta\theta_m - \sum_1^{m-1} L_i \Delta\theta_i$$
$$F_{m-1} = \sum_1^{m-1} L_i \Delta\theta_i$$

where t_{m-1} = time when the wetting front reached interface of *mth* and *(m-1)th* layers and it can be solved successively from t_1, t_2, ..., to t_{m-1} with t1 by equation (1). L = depth of wetting front; L_i = thickness of *ith* soil layer; $\Delta\theta = \theta_s - \theta_0$. Subscript *i* and *m* represent numbers of soil layers.

If surface ponding firstly occurs in *mth* layer, we will have:

$$I_p = f = k_m(1 + \frac{A_{m-1}}{B_{m-1} + F_p}) \quad \Rightarrow \quad F_p = \frac{A_{m-1}}{I_p/k_m - 1} - B_{m-1}$$

$$F_p - F_{n-1} = I_p(t_p - t_{n-1}) \quad \Rightarrow \quad t_p = t_{n-1} + \frac{F_p - F_{n-1}}{I_p}$$

According to (4), if ponding had occurred from beginning and had been continuing, the pseudo-time t'_p to infiltrate F_p could be solved from following equation:

$$F_p - F_{m-1} = k_m(t_p' - t_{m-1}) + A_{m-1} \cdot \ln(\frac{A_{m-1} + B_{m-1} + F_p}{A_{m-1} + B_{m-1} + F_{m-1}})$$

Considering the difference of time coordinate in real situation with the assumed one, (4) should be modified to :

$$F - F_{m-1} = k_m\left[t - t_{m-1} - (t_p - t_p')\right] + A_{m-1} \cdot \ln(\frac{A_{m-1} + B_{m-1} + F}{A_{m-1} + B_{m-1} + F_{m-1}})$$

Combining above two equations, we get

$$f = k_m \cdot (1 + \frac{A_{m-1}}{B_{m-1} + F})$$
$$F - F_p = k_m(t - t_p) + A_{m-1} \cdot \ln(\frac{A_{m-1} + B_{m-1} + F}{A_{m-1} + B_{m-1} + F_p}) \quad (5)$$

Equations (4) and (5) are the generalized Green-Ampt model for a multi-layered soil and an unsteady rain. It can be proved that equation (4) is also true for the case that the wetting front enters *mth* layer with the ponding firstly occurred at *(m-1)th* layer and continued since then, and equation (5) is also true for the case that the surface is re-ponded after no ponding at last time step with the wetting front at *mth* layer.

3. Numerical Model of the Richards Equation

For the stratified soil layers, it is convenient to choose water pressure head or matrix head ψ as a variable, then one-dimensional Richards equation has the form as follows:

$$c(\psi)\frac{\partial\psi}{\partial t} = \frac{\partial}{\partial z}(k(\psi)\frac{\partial\psi}{\partial z}) - \frac{\partial k(\psi)}{\partial z} \quad (6)$$

in which $c(\psi) = \partial\theta / \partial\psi$, specific soil moisture capacity; $k(\psi)$ = soil hydraulic conductivity at pressure head ψ; θ = volumetric soil moisture content; z = distance below the surface; t = time.

When infiltration during rain is simulated by using it, two kinds of boundary conditions will be encountered. One is pressure head control when surface is ponded,

$$\begin{aligned} \psi &= \psi_0 & & t = 0, \ z \geq 0 \\ \psi &= \psi_0 & \text{when} \quad & t > 0, \ z \to \infty \\ \psi &= 0 & & t > 0, \ z = 0 \end{aligned} \quad (7)$$

another is flux control, when surface is not ponded,

$$\begin{aligned} &\psi = \psi_0 & & t = 0, \quad z \geq 0 \\ &\psi = \psi_0 & when \quad & t > 0, \quad z \rightarrow \infty \\ &-k(\psi)\frac{\partial \psi}{\partial z} + k(\psi) = I(t) & & t > 0, \quad z = 0 \end{aligned} \tag{8}$$

in which ψ_0 = the initial water pressure head in soil; I(t) = rain intensity at time t.

Equation (6) is a second-order non-linear partial differential equation. The implicit finite difference scheme is chosen to convert it into algebraic equations and Tri-Diagonal Matrix Algorithm (TDMA) method is used to solve them. To deal with non-linear parameters $c(\psi)$ and $k(\psi)$, iteration is conducted and under-relaxation factor is adopted as 0.5. For the boundary condition of flux control (eq.(8)), the water balance in the first half grid is used to convert it into the first algebraic equation. Because $c(\psi)$ and $k(\psi)$ are changed greatly with the variation of ψ, and two kinds of boundary conditions switch from one to another, the implicit finite difference scheme and iteration method are used to ensure the convergence and steadiness of computation.

4. Verification of the General Model

To evaluate the generalized Green-Ampt model, it is applied to infiltrations into uniform soil profile, two-layered soil profile and three-layered soil profile during an unsteady rain, and compared with numerical simulation of the Richards equation which is assumed to give a correct result.

For simplicity, unsteady rain intensity I is expressed by a sine function of time t, i.e., $I(t) = I_m \sin(\pi t/T)$ where I_m is the maximum rain intensity, T is the rain duration. I_m =0.3cm/min and T=300min are assumed in this study.

Four kinds of soil, sand, loam, Kantoloam and clay are considered. The main parameters of these soils are referred to Herath (1987), Chung and Horton (1987). Two relations are needed by the Richards equation. One is the relation of soil moisture content θ and suction S (negative pressure head), expressed as (Haverkamp et al.,1977):

$$\frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{\alpha}{\alpha + (\ln S)^{\beta}} \tag{9}$$

in which θs = saturated moisture content; θr = residual moisture content; α, β = constants. The other is the relation of unsaturated hydraulic conductivity $k(\theta)$ and soil moisture content θ , expressed as (Mualem,1978):

$$k(\theta) = k_s\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^n \qquad with \qquad n = 0.015 \int_{S=15atm}^{S=0} \gamma_w S \cdot d\theta + 3.00 \tag{10}$$

in which k_s = saturated hydraulic conductivity; γ_w = specific weight of water.

Many researchers studied parameters of the Green-Ampt model, e.g., Bouer (1969), Mein & Larson (1973), Neuman (1976) and Rawls et al.(1983). Here, the moisture content in wetted zone is considered to be saturated soil moisture θ_s. The capillary suction SW at wetting front is related to initial soil suction and is computed according to the Neuman method:

$$SW = \int_0^{S_0} k_r(\theta) \cdot ds \tag{11}$$

in which $k_r(\theta)=k(\theta)/k_s$, the relative hydraulic conductivity; S_0 = the initial suction in soil.

Soil parameters are shown in Table 1.

All of soil layers are assumed to have a same initial suction S_0 so as to ensure modeling results comparable. The initial suction is correspondent to 30% saturation of sand having a value of 68.5cm. Though capillary suction SW can be computed from equation (11), it can also be fitted. The difference of computed SW and fitted SW is believed to be caused by the saturation assumption in the wetted zone. For multi-layered soil profiles, soil parameters can be used directly in the case of coarse over fine. However, in the case of fine over coarse , the coarse lower layers can not reach saturation because the surface layer restricts infiltration, the parameters of k_s, θ_s and SW in lower layers are different with original ones but can be obtained by trial and error (Moore, 1981).

Infiltration into uniform soil layers of the four soil types during the assumed unsteady rain is shown in Fig. 1., whereas infiltration into two-layered soil during the unsteady rain is shown in Fig. 2., and infiltration into three-layered soil during the unsteady rain is shown in Fig. 3. From these figures we can see that the modeling results generally match quite well with the numerical simulation of the Richards equation. The computation time by the generalized Green-Ampt model is less than tenth of that by the Richards equation.

Table 1. Soil parameters

	soil types	θs	θr	α	β	n	k_s (cm/ min)	computed SW (cm)	fitted SW (cm)	S_0 (cm)	θ_0	L_i (cm)
(1)	sand	0.400	0.077	1.75E10	16.95	3.37	0.150	39.5	22.0	68.5	0.174	500
uniform	loam	0.422	0.104	6451	5.56	3.97	0.042	38.5	26.0	68.5	0.321	500
soil	Kantoloam	0.707	0.598	72.8	3.92	3.11	0.060	10.1	3.5	68.5	0.620	500
layers	clay	0.394	0.120	6.579E7	9.00	4.38	0.012	66.9	67.0	68.5	0.392	500
(2)												
fine over	loam	0.422					0.042		26.0			10
coarse	sand	0.310					0.125		30.0			490
(3)												
fine/	clay	0.394					0.012		67.0			10
coarse/	loam	0.340					0.025		45.0			10
coarser	sand	0.320					0.050		38.0			490

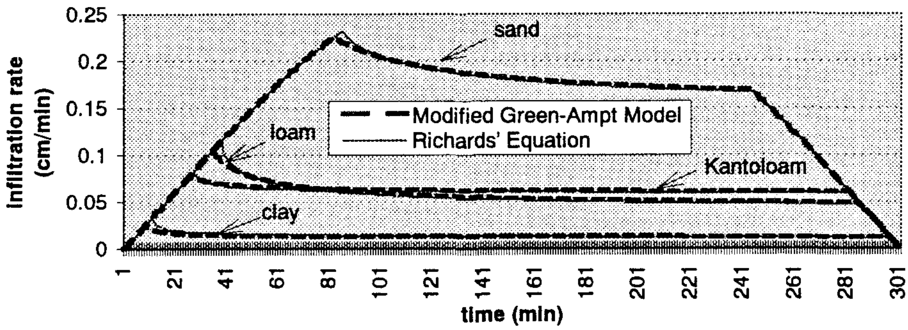

Fig. 1 Infiltration into uniform soil

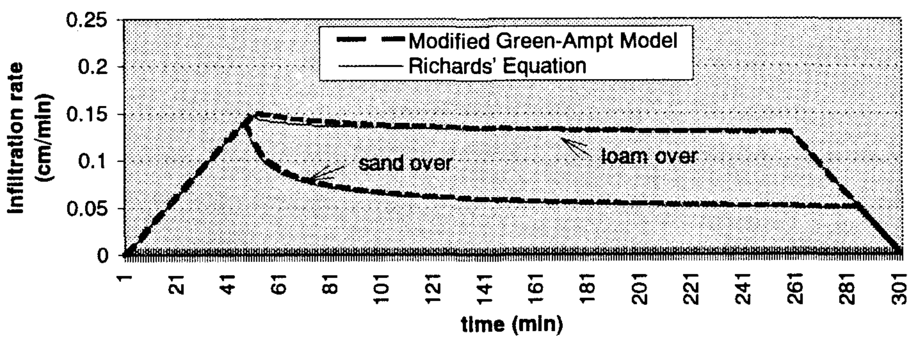

Fig. 2 Infiltration into two-layered soil

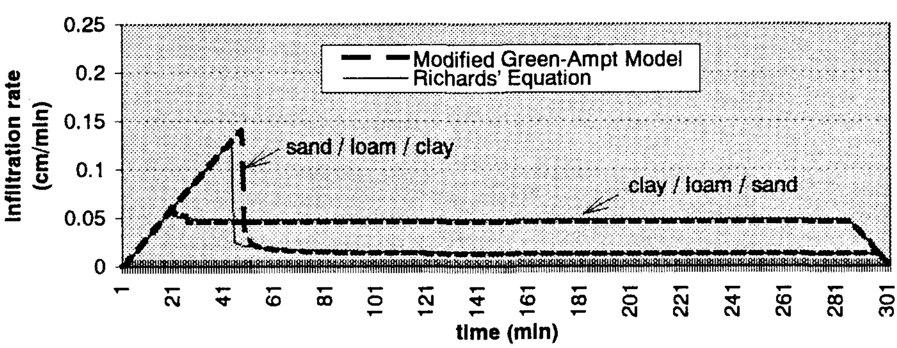

Fig. 3 Infiltration into three-layered soil

5. Concluding Remarks

From above analysis, it can be concluded that:

(1)The Green-Ampt Model can be extended to model infiltration into a multi-layered soil during an unsteady rain. A general form has been proposed for the first time.

(2) It is promising to apply the general form of the Green-Ampt model to watershed modeling for its higher computation efficiency than the Richards equation.

(3)However, when it is applied, it should be mentioned that there may be a limitation for numbers, thickness and property differences of soil layers, because the assumption of wetting front may not be applied in complicated cases.

(4)Though the proposed model has been favorably compared with the Richards equation, it needs further verification by experiments and observations.

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