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DEPOSITION OF THE HIGH CONCENTRATED FLOW ON THE FLOOD PLAIN

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A study has been done on the determination of the direction of bed movement in high concentrated flow. Criteria are derived to determine direction of bed movement. Oscillation of bed is discussed by using different numerical schemes. The criteria are used to suggest a new numerical scheme for determination of deposition and erosion. Propagation of high concentrated flood flow and bed levels are simulated. The experimental results are compared with simulated results to verify the simulation.
Keywords: High concentrated flow, Two-dimensional simulation, Deposition and erosion, Simulation of flood flow.

1-Introduction

Propagation of high concentrated flood flow in flood plain is a complicated phenomenon. The flow condition in steep channel is usually supercritical but the flow condition can be changed when torrent flow leaves steep channel and enters into a flood plain with gentle slope (Authors 1995a and 1995b). A flood flow may have smaller sediment transport capacity on the flatter slope. Also wider channel of flood plain causes smaller depth and smaller velocity. The smaller depth and smaller velocity will decrease sediment transport capacity. Therefore, more sediment deposits on the flood plain. Different flow conditions can be taken into account in flow simulation as presented by authors (1995b). Also, here we are going to discuss the importance of different flow conditions in the simulation of bed levels of two-dimensional propagation of high concentrated flow.

2-Required Equations

Required Equations are derived by conservation law of fluid, solid mass, and momentum. Energy conservation is equivalent to momentum conservation, but it is not applicable for a flow with steep front. Therefore, Conservation form of momentum equations should be used for simulation of the high concentrated flood.

The governing equations of the flow can be derived by integrating of the Navier Stokes equations for an incompressible fluid over the flow depth. Two-dimensional mass and momentum conservation equations may be written as follows:

- The Continuity Equation of flow :

$$\frac{\partial h}{\partial t}+\frac{\partial (uh)}{\partial x}+\frac{\partial (vh)}{\partial y}=-\frac{\partial z_b}{\partial t} \qquad (1)$$

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- The Momentum Equation of flow in x direction :

$$\frac{\partial(uh)}{\partial t}+\frac{\partial(u^2h)}{\partial x}+\frac{\partial(uvh)}{\partial y}+gh\frac{\partial(z_b+h)}{\partial x}+\frac{\tau_{bx}}{\rho_t}=0 \quad (2)$$

- The Momentum Equation of flow in y direction :

$$\frac{\partial(vh)}{\partial t}+\frac{\partial(uvh)}{\partial x}+\frac{\partial(v^2h)}{\partial y}+gh\frac{\partial(z_b+h)}{\partial y}+\frac{\tau_{by}}{\rho_t}=0 \quad (3)$$

where h = flow depth; u = flow velocity in the x-direction; v = flow velocity in the y-direction; g = acceleration due to gravity; z_b = bed level; τ_{bx} = x-component of resistance to flow; τ_{by} = y-component of resistance to flow; ρ_t = specific or bulk density of debris flow.

The continuity equation of the solid phase can be derived by conservation law. Since concentration has considerable variation by time and space, It is necessary to include all possible terms of the differential equation. The equation can be defined as follows:

$$\frac{\partial(ch)}{\partial t}+c_*\frac{\partial z_b}{\partial t}+\frac{\partial(uhc)}{\partial x}+\frac{\partial(vhc)}{\partial y}=0 \quad (4)$$

where c = mean volume concentration of solids in entire flow depth; c_* = solid concentration in bed. The mean volume concentration of solids can be determined by following equation:

$$c=\frac{q_s}{q}$$

where q is flow discharge per unit width. The symbol q_s is solid phase discharge per unit width. It may be calculated by following equation for two layers high concentrated flow (Hashimoto, H. and Hirano M. 1995).

$$\frac{q_s}{\sqrt{sgd^3}}=\frac{14}{3}\tau_*^{3/2}\left(1-\frac{\tau_{*c}}{\tau_*}\right)\frac{1}{(\alpha-tan\theta_0)cos\theta_0} \quad (5)$$

where d = grain size; τ_*= nondimensional tractive stress; τ_{*c}= nondimensional critical shear stress; θ_0 = bed slope in the flow direction; α = coefficient; s = nondimensional submerged density of the particle.

Since the governing equations are nonlinear and first order hyperbolic partial differential equations, no analytical solution is available except for very simplified one-dimensional cases. Therefore, they are solved by suggested numerical method.

3-Direction of Bed Movement

Flow condition changes by space in the flood plain. It will influence direction of bed movement. Therefore, direction of bed movement should be studied in different flow conditions. Equation (1) can be rewritten by using equation (4) as follows:

$$\frac{c_*-c}{h}\frac{\partial z_b}{\partial t}+\frac{\partial c}{\partial t}+u\frac{\partial(c)}{\partial x}+v\frac{\partial(c)}{\partial y}=0 \quad (6)$$

Since concentration and depth are function of x, y, and t, equation (6) can be simplified by using chain rule as follows:

$$c_1\frac{\partial z_b}{\partial t}+\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial x}+v\frac{\partial h}{\partial y}=0 \quad (7)$$

where c_1 is defined by:

$$c_1=\frac{c_*-c}{h\frac{\partial c}{\partial h}} \quad (8)$$

Cunge, et al (1980) showed that it is possible to study bed perturbations by quasi-steady assumption in one dimensional flow. Here for two dimensional flow, we rewrite equations (2) and (3) by using equation (1) and quasi-steady assumption as:

$$uh\frac{\partial u}{\partial x}+vh\frac{\partial(u)}{\partial y}+gh\frac{\partial(z_b+h)}{\partial x}+\frac{\tau_{bx}}{\rho}=0 \quad (9)$$

$$uh\frac{\partial v}{\partial x} + vh\frac{\partial (v)}{\partial y} + gh\frac{\partial (z_b+h)}{\partial y} + \frac{\tau_{by}}{\rho} = 0 \qquad (10)$$

These equations can be expressed in following form:

$$(1\text{-}Fr^2\cos\phi)\frac{\partial h}{\partial x} - \frac{Fr^2}{2}\sin(2\phi)\frac{\partial h}{\partial y} + \frac{\partial z_b}{\partial x} = -\frac{\tau_{bx}}{\rho g h} \qquad (11)$$

$$(1\text{-}Fr^2\sin\phi)\frac{\partial h}{\partial y} - \frac{Fr^2}{2}\sin(2\phi)\frac{\partial h}{\partial x} + \frac{\partial z_b}{\partial y} = -\frac{\tau_{by}}{\rho g h} \qquad (12)$$

where ϕ is angle between x-direction and flow direction. Replacing equations (11) and (12) in equation (7) leads:

$$\frac{\partial z_b}{\partial t} + \frac{1}{c_1}\frac{u}{Fr^2-1}\frac{\partial z_b}{\partial x} + \frac{1}{c_1}\frac{v}{Fr^2-1}\frac{\partial z_b}{\partial y} = \frac{\tau_{bx}}{\rho g h}\cos\phi + \frac{\tau_{by}}{\rho g h}\sin\phi \qquad (13)$$

This equation can be compared with exact differential of z_b and following criteria may be deriven:

$$\left(\frac{dx}{dt}\right)_{z_b} = \frac{1}{c_1}\frac{u}{Fr^2-1} \qquad (14)$$

$$\left(\frac{dy}{dt}\right)_{z_b} = \frac{1}{c_1}\frac{v}{Fr^2-1} \qquad (15)$$

Since c_1 is positive, the direction of movement of bed is same to the direction of u and v for supercritical flow. It has opposite direction to the direction of u and v for subcritical flow. Therefore, bed moves in different direction in different flow conditions.

4-Numerical Scheme

Calculations have been done in two stages. In first stage equations (1) to (3) have been integrated to determine flow variables by implicit scheme. In second stage, the bed levels have been calculated.

Determination of Flow Variables

Equations (1) though (3) can be rewritten in matrix form as:

$$\mathbf{U}_t + \mathbf{E}_x + \mathbf{F}_y + \mathbf{S} = \mathbf{0} \qquad (16)$$

Each vector of the **U**, **E**, **F** and **S** can be spilt into two components (Authors 1995). Therefore equation (16) may be written as follows:

$$\mathbf{U}_{1_t} + \mathbf{E}_{1_x} + \mathbf{F}_{1_y} + \mathbf{S}_1 = 0 \qquad (17)$$

$$\mathbf{U}_{2_t} + \mathbf{E}_{2_x} + \mathbf{F}_{2_y} + \mathbf{S}_2 = 0 \qquad (18)$$

Components of the vector of flow variables (**U**) can be determined at the discrete points of the independent variables (x, y and t) by numerical method as presented by authors (1995a).

Determination of Bed Level

Bed level can be determined by numerical integration of equation (6). Following example is used to examine different schemes. The configuration of flume and flood plain are shown in figure 1.

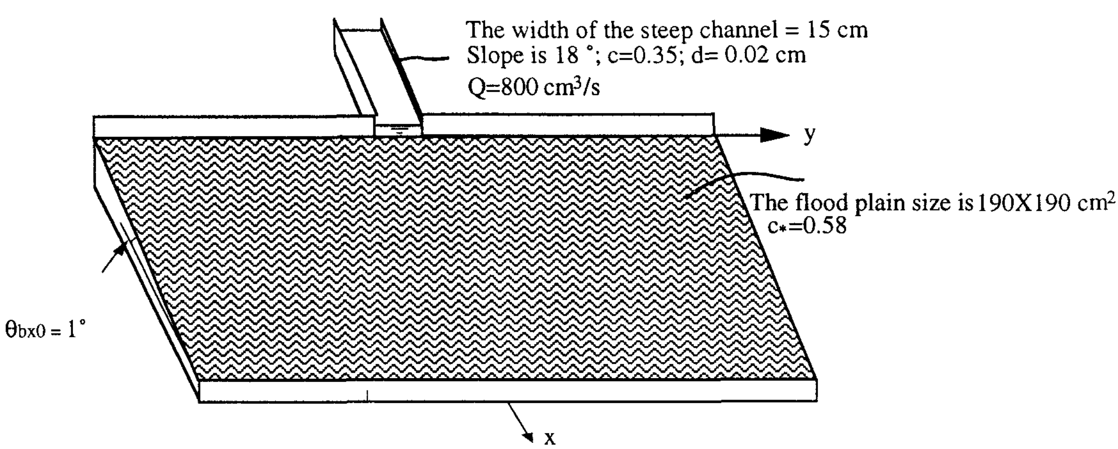

Fig. 1 Experimental Setup

Backward difference, central difference, and forward difference are used to integrated equation (6). The calculated bed levels on the axis of flood plain are shown by figures 2 through 4. Figure 2 shows that using backward difference may cause oscillation of bed level for supercritical flow. However, it has smooth result for subcritical flow. Figure 3 shows using central difference may produce oscillation of the bed level for supercritical flow too. Figure 4 shows that using forward difference guides to smooth bed level for supercritical flow. Consequently, these numerical experiments also show that different flow condition should be considered in using suitable scheme.

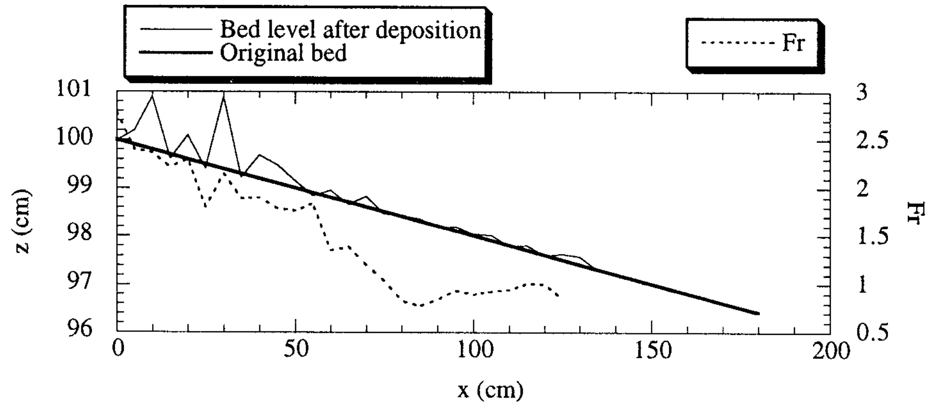

Fig. 2 Froude number and bed level by using backward differences at y=0

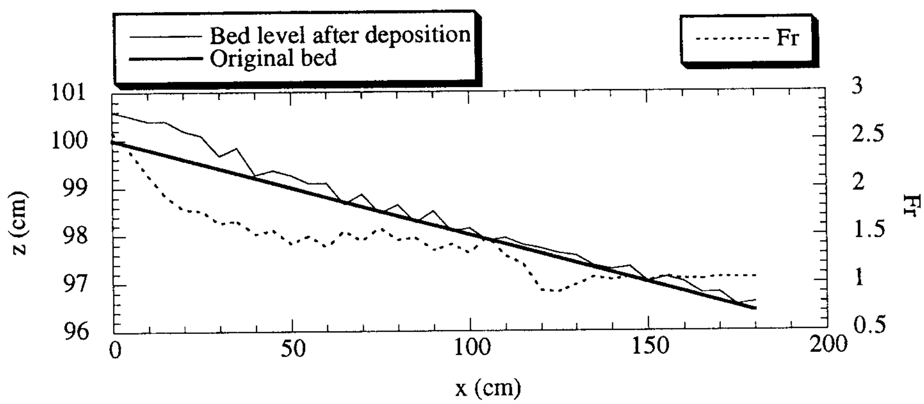

Fig. 3 Froude number and bed level by using central differences at y=0

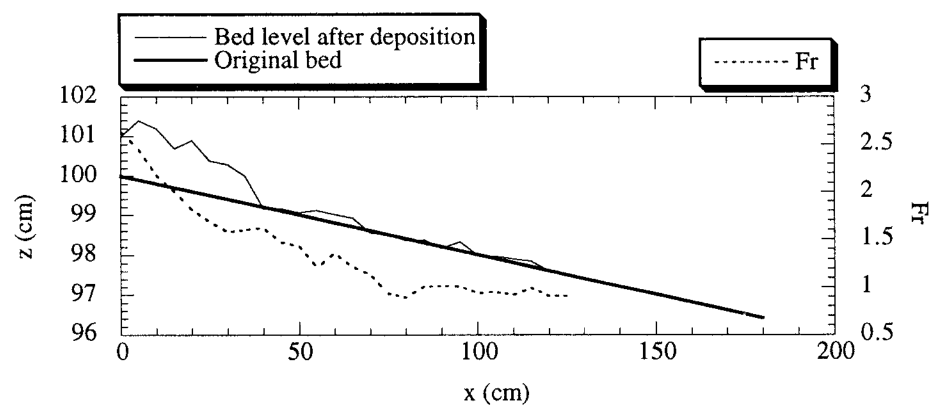

Fig. 4 Froude number and bed level by using forward differences at y=0

Suggested Numerical Scheme

Here a numerical scheme is suggested for determination of bed level by taking the direction of bed movement into account. By using equations (14) and (15), we used forward difference for supercritical flow and backward difference for subcritical flow. Bed levels can be determined by using the numerical scheme as follows:

$$z^{n+1} = z^{n} + \frac{\Delta t\, h_{j,k}}{2(c_{j,k}^{n+1} - c_*)}\left[\frac{c_{j,k}^{n+1} - c_{j,k}^{n}}{1/2\Delta t} + AFx\, u_{j,k}^{n+1}\frac{c_{j+1,k}^{n+1} - c_{j,k}^{n+1}}{\Delta x} + (1 - AFx)\, u_{j,k}^{n+1}\frac{c_{j,k}^{n+1} - c_{j-1,k}^{n+1}}{\Delta x} + AFy\, v_{j,k}^{n+1}\frac{c_{j,k+1}^{n+1} - c_{j,k}^{n+1}}{\Delta y} + (1 - AFy)\, v_{j,k}^{n+1}\frac{c_{j,k}^{n+1} - c_{j,k-1}^{n+1}}{\Delta y}\right] \quad (19)$$

where AFx and AFy are as follows:

$$AFx = \begin{Bmatrix} 0 \text{ for } \frac{u}{Fr^2 - 1} \le 0 \\ 1 \text{ for } \frac{u}{Fr^2 - 1} > 1 \end{Bmatrix}, \quad AFy = \begin{Bmatrix} 0 \text{ for } \frac{v}{Fr^2 - 1} \le 0 \\ 1 \text{ for } \frac{v}{Fr^2 - 1} > 1 \end{Bmatrix} \quad (20)$$

Calculated bed levels and Froude numbers on the axis of flood plain are shown by figure 5. This figure shows that oscillations of bed are controlled for both supercritical and subcritical flows by suggested scheme.

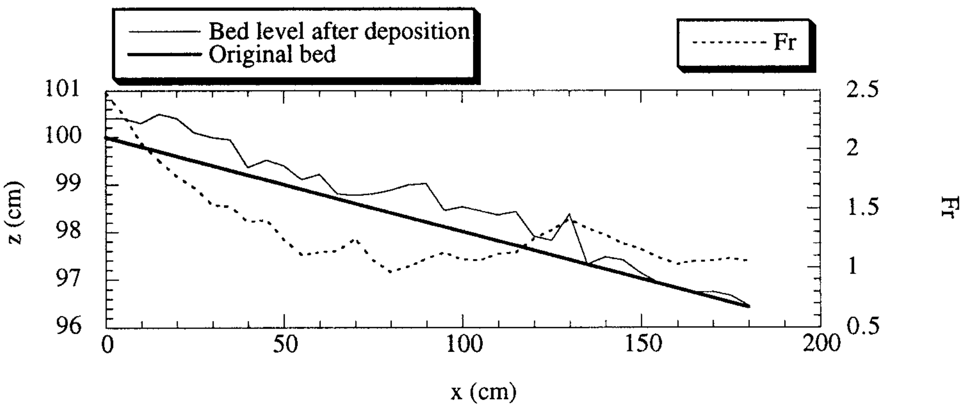

Fig. 5 Froude Number and bed level by using suggested scheme at y=0

5-Verification of Simulation

Experimental result of mud flow is used to verify the result of simulation (Arai, M. 1991). Experimental setup is shown by figure 1. Figure 6 shows comparison between calculated bed profile and experimental bed levels on the axis of flood plain. Figure 7 shows comparison between calculated contour lines of bed and experimental results. The numbers on the contour lines show bed levels. Both figures demonstrate good agreement between simulation and experiment. Consequently, suggested simulation method is verified by experimental result.

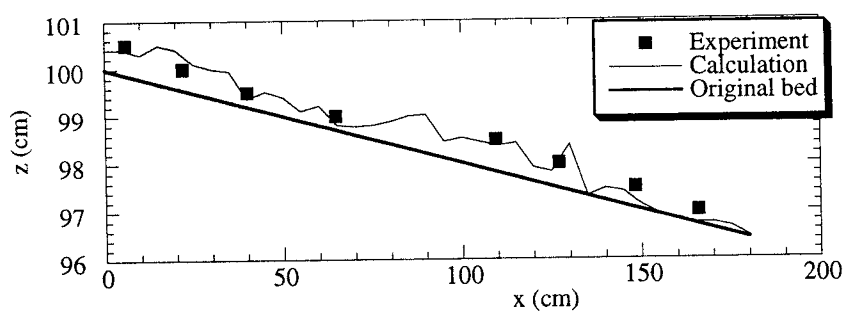

Fig. 6 Comparison between calculated bed level and experimental ones at y=0

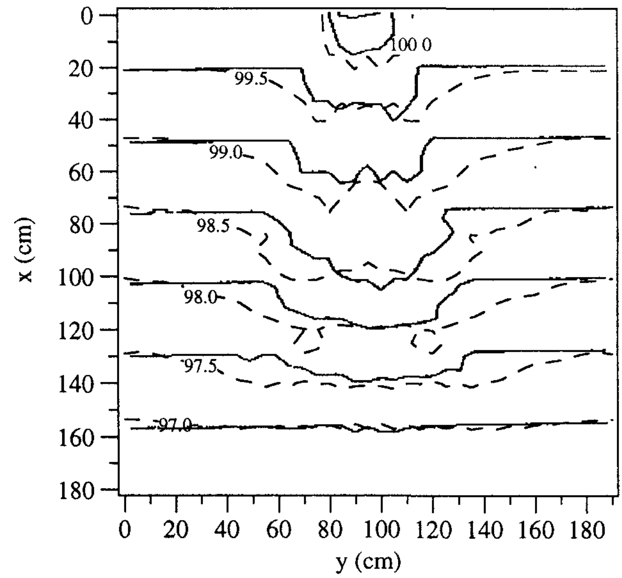

Fig. 7 Contour lines of bed in the experiment (rigid lines) and calculation (dashed lines)

6-Conclusions

This study shows that bed movement may have different directions due different flow conditions. Discussions on using different numerical scheme show that using backward and central differences may produce oscillations for supercritical flow. However, backward difference has smooth results for subcritical flow. Using forward scheme shows smooth result for calculation of bed levels in supercritical flow. Here, a new scheme is suggested by taking the directions of bed into account. The oscillation of bed controlled in different flow condition by suggested scheme. The results of simulation by the suggested scheme have good agreement with experimental ones.

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