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Characteristics of Mixing Length in Dispersion Through Unsaturated Glass Beads Media

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In this research, the effect of grain sizes on dispersion phenomena in unsaturated flow conditions were investigated. Miscible displacement experiments were carried out using three different but uniform sizes of glass beads as porous media. Mixing length theory concept was employed to analyze the experimental data. Mixing lengths were found almost constant to each media having uniform grain size distribution, in the lower range of moisture contents. They were however found increasing with increasing grain size of the media. The ratio of the mixing length to the grain size of the media was obtained almost constant. This constant number, named "Mixing Length Number" was found almost equal to 8. Applicability of the proposed model for dispersion coefficient based on mixing length theory was checked by experimental data.
Keywords: dispersion, unsaturated porous media, mixing length theory

INTRODUCTION

Miscible displacement studies have been usually carried out to comprehend the phenomena of solute transport through the porous media. Several researchers such as Nielsen and Biggar (1961), Gupta et al. (1973a), van Genuchten and Wierenga (1977), De Smedt et al. (1986) have done various studies in the past few decades taking different aspects of the phenomena of the solute transport, e.g. presence of mobile and immobile water, reaction of solute with the medium etc. In all cases, the dispersion coefficient is a universally important parameter.

Dispersion coefficient is empirically expressed as a linear function of pore water velocity as:

$$D = D_o + \alpha \bar{v} \quad (1)$$

where D_O is the effective molecular diffusion in a porous medium (cm^2/s); α is the dispersivity (cm) and $\bar{v}$ is the mean pore water velocity (cm/s). Originally, equation (1) was used in solute transport study under saturated conditions. Kirda et al.(1973), Yule and Gardner(1978), and De Smedt et al. (1986) showed that dispersion coefficients could be expressed as in equation (1) under unsaturated conditions also. Their experimental conditions were however in the lower range of moisture content. Gupta et al. (1973a,b) experimented by controlling flux and moisture content independently and found dispersion coefficient increasing with increasing pore water velocity but in most cases, decreasing with increasing moisture content(θ). These results were obtained by analyzing D-$\bar{v}$ and D-θ relations separately. Careful observation however shows that D-$\bar{v}$ relationship is linear for each θ and α is changing with θ. Recently, Matsubayashi et al. (1995) carried out the miscible displacement experiments to see the characteristics of the dispersion coefficient up to the higher moisture content range. They discussed the dispersion phenomena through a porous media, applying mixing length theory concept using soil characteristic curves, i.e., suction-moisture content (ψ–θ) and hydraulic conductivity-moisture content (K–θ) relationships. They observed dispersion coefficient linearly increasing with pore water velocity until the moisture content in the porous media reaches a certain value (degree of saturation < 75%) and then decreasing beyond that value. They discussed these results theoretically and also elucidated why D-$\bar{v}$ relation is linear in the lower range of moisture content based on the mixing length concept. This concept states that dispersion coefficient is the product of the mixing length of the flow paths and the standard deviation of the pore water velocities inside the porous medium.

The aim of this research is to see the characteristics of the mixing length, which is one of the important parameters in the model, in relation to grain size distribution. Especially we focus to the effect of mean grain size using different uniform glass beads media. We discuss about the mixing length in the

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lower range of moisture content. Electrical conductivity was used as a tracer to monitor the change in concentration of the solution with space and time inside the porous media to see the dispersion. A model for the dispersion coefficient is also proposed.

THEORY

Flow pattern inside a porous media depends on the shape and size of the grains, as well as the amount of moisture in it. When the moisture content inside the media is small, only the smaller pores are filled with water. With increasing moisture content, larger pores will be gradually filled with water. The pore water velocities in these larger pores are faster than in smaller ones. This phenomena causes a larger variation in pore water velocities with the increase in moisture content. In addition, variations of the flow velocities across each pore channel is also more in larger pores.

Water paths inside the media are not separated like in "capillary bundle model". Instead they meet within a certain distance and the solute will get mixed to attain a local equilibrium. Fig. 1 shows a schematic illustration of pore water velocities (v) relative to mean pore water velocity ($\bar{v}$) in a porous media. If the phenomena is observed in a moving coordinate system with velocity $\bar{v}$, we can see that the water particles are moving upwards and downwards with velocity v' $(=v-\bar{v})$ of each particle and mix with each other after moving a certain distance. If their is a gradient of solute concentration, solute will be transferred from the portion of higher concentration to lower one. This phenomena is similar to the mixing of particles in turbulent flow. Hence, we apply the concept of "Mixing Length Theory" in this study. The distance stated above is called a "Mixing Length". In such cases,

$$q_c = D\partial C/\partial x = l\sqrt{\overline{v'^2}}\,\partial C/\partial x \tag{2}$$

where q_c is the flux of solute (g/cm^2/s), C is the concentration of the solute (g/cm^3) and l, the mixing length (cm). The variation of v' is defined within the mixing length.

Based on these concepts, the dispersion phenomena in a porous media can be analyzed as follows:

When the moisture content profile is homogeneous inside the porous media, velocity distribution can be expected to be similar in each section of the media. We also assume that pores are filled with water from the smaller pores to the larger pores, up to that pore whose pore diameter corresponds to the diameter of a pore which can support the capillary interface for the given capillary suction. We use "capillary bundle model" at each section of the media, but actually pores are connected to each other in the mixing length. Within these assumptions, the pore size and velocity distributions can be estimated from $\psi-\theta$ and $K-\theta$ relationships.

(i) The pore diameter, d, and the suction head, ψ, corresponding to moisture content, θ, can be related as:

$$d = 4\gamma / \rho g \psi \tag{3}$$

where γ and ρ are surface tension (dyne/cm) and density of water (g/cm^3) respectively and g is the acceleration due to gravity (cm/s^2).

(ii) Let's assume a unit cross sectional area of the medium. The number of pores, dN, of size d, for each increment of $d\theta$, becomes

$$dN = d\theta / (\pi d^2/4) \tag{4}$$

(iii) The incremental discharge, dQ, through pipes of diameter d can be written from Poiseuille's equation as:

$$dQ = dN\, C(\theta) d^4 \tag{5}$$

where $C(\theta)$ is a coefficient depending on θ.

Darcy flux under unsaturated condition can be given by,

$$q(\theta) = \int_0^\theta C(\theta)\, d^4 / (\pi d^2/4)\, d\theta \tag{6}$$

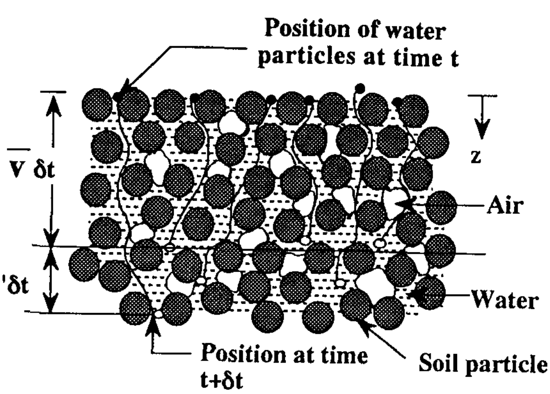

Fig. 1 Schematic diagram of the flow inside the unsaturated porous media

(iv) When the moisture content profile inside the media is homogeneous in a vertical column experiment, the hydraulic conductivity, K, equals Darcy flux, since hydraulic gradient inside the media under that condition is unity. In such case, Darcy flux can be written by Kozeny's relation as:

$$q(\theta) = K(\theta) = K_S(\theta/\theta_S)^{\beta} \tag{7}$$

where θ_S is the saturated moisture content. Coefficients Ks and β can be determined from $K-\theta$ relationship.

(v) Using equations (6) and (7), the coefficient C in equation (6) can be determined for each θ. Using these estimated C values for each pipe of size, d, the pore water velocity v was determined as:

$$v = dQ/dN(\pi d^2/4) = 4C(\theta)d^2/\pi \tag{8}$$

(vi) The mean pore water velocity at moisture content, θ, i.e., when pores of diameter $\leq d$ are filled with water, is given by,

$$\bar{v}_{(\theta)} = \frac{1}{\theta}\int_0^{\theta} v\, d\theta \tag{9}$$

The variance of the velocity is then estimated by,

$$\sigma_{(\theta)}^2 = \frac{1}{\theta}\int_0^{\theta}(v-\bar{v})^2\, d\theta \tag{10}$$

If we relate σ with $\bar{v}$, we will have $\sigma(\theta) = f(\bar{v}(\theta))$. It will be shown later (in result and discussion section) that this function can be expressed linearly as $\sigma = \lambda\bar{v}$ where λ is a constant.

Finally, dispersion coefficient can be expressed as:

$$D = l\,\sigma = \lambda l \bar{v} \tag{11}$$

where, λl corresponds to dispersivity, α, of equation (1). The mixing length, l, which is the main objective of this paper, can be estimated from D (determined by experiments), λ (determined from σ and $\bar{v}$ relationship) and $\bar{v}$ from equation (9).

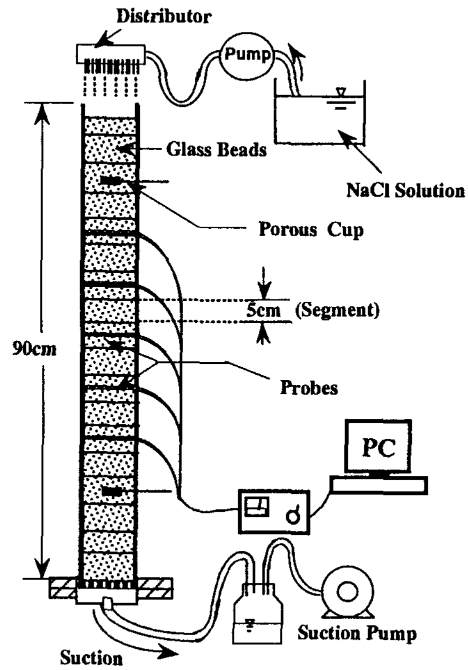

Fig. 2 Experimental Set-up

EXPERIMENTS AND ANALYSIS

A schematic illustration of the experimental apparatus is shown in Fig. 2. The column is of 10 cm inside diameter and 90 cm in height. The column was made up of 18 pieces of 5 cm high inter connectable PVC rings. Glass beads were used as the porous media and NaCl as the conducting solution.

Glass beads were first washed with pure water. Probes and PVC rings were stacked as shown in Fig. 2 and the rings were filled one by one with these glass beads. Packing of the beads was done carefully by tapping the column uniformly to get same average bulk density. In each experiment a suction was applied at the bottom of the media to keep the moisture content inside the column uniform.

The NaCl solution of a given conductivity (ranging from 400 to 800 μS/cm) was applied over the media using a distributor. Steady state condition was established by applying a constant rainfall over the media until the input rate from the top of the media was equal to the outflow rate from the bottom. Porous cups connected to the pressure gauges were used to measure the suction heads inside the media to check the uniformity and steadiness of the flow condition during the experiment. After attaining the steady state flow condition, another NaCl solution with higher electrical conductivity (ranging from 1500 to 2000 μS/cm) was applied as a displacing solution at the same rate using another distributor.

Specific electrical conductance was used as the tracer to determine the dispersion inside the porous media. Electrical conductance probes installed in the column at 20, 30, 40, 50 and 60 cm from the top media surface were used to monitor the output voltage of the beads and solution. A probe was constructed using two cylindrical stainless steel tubes having outside diameter 2.1 mm, and a PVC ring with 10 cm inside diameter and 5 cm in height. The two stainless steel tubes were inserted at the middle of the ring, parallely, keeping them 1.1 cm apart. Output voltages monitored by the probes were recorded in the computer and converted into actual solution conductivities using calibration curves. Probes were calibrated previously to find the relationship between output voltage measured by probes and the actual solution electrical conductivity for a given moisture content. Each 5 cm ring was dismantled at the end of the experiment and the dry bulk density and the moisture content were determined by gravimetric method. We found these variables homogeneous inside the media.

Experiments were conducted at a room temperature of 20±1 °C. Three sets of experiments under various degrees of saturation were carried out, using three types of media. Grain size distribution of each media is sharp. We thus consider these media as having uniform grain size of 0.25 mm, 0.50 mm and 0.75 mm respectively.

Figure 3 shows breakthrough curves used to determine the mean pore water velocity and the dispersion coefficient, as an example. In Fig. 3, the observed values of the concentrations at different locations of the media are denoted by open circles. The mean pore water velocity was determined by calculating the mean travel time taken by the solution to reach a certain location, using the breakthrough curve for each probe; as:

$$\bar{v} = L / \Delta t \qquad (12)$$

where, L is the distance (cm) of a certain probe from the top of the media, Δt is the time (s) taken by that probe to indicate the mean value of the solution concentration.

Dispersion coefficient was estimated by fitting the observed data to the analytical solution of one dimensional advection dispersion equation (see Rose and Passioura, 1971) using mean pore water velocity calculated in equation (12). Fitting was obtained to minimize the sum of the square of the difference between observed and analytical solution of all probes. In Fig. 3 estimated BTCs are shown by dotted lines. Good fittings were obtained.

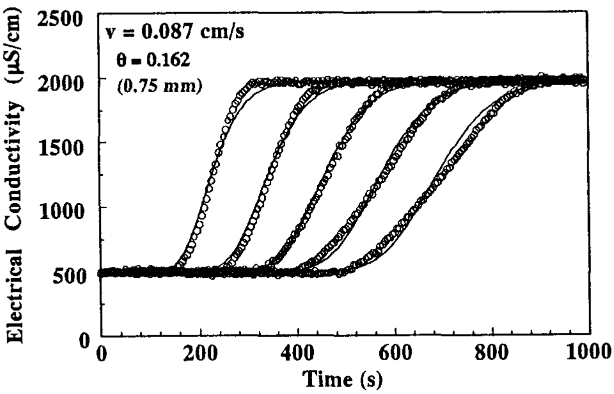

Fig. 3 Breakthrough curves

RESULTS AND DISCUSSION

Dispersion coefficients obtained from a series of experiments in each set are shown in Fig. 4. D-$\bar{v}$ relationships are linear in all three cases with increasing gradient with increasing grain size of the media.

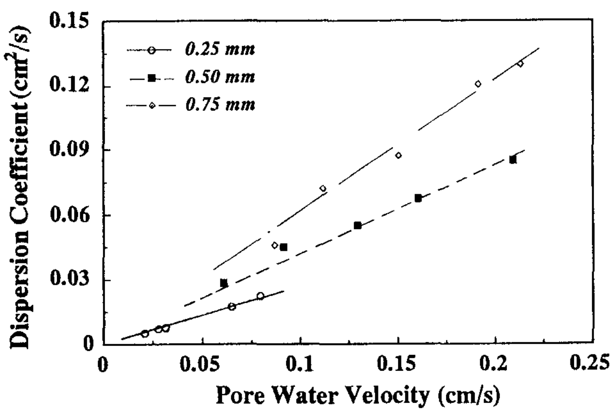

Fig. 4 Relationships between dispersion coefficients and pore water velocities of three media

Fig. 5 shows the ψ–θ relationships of three different sizes of beads. It shows that the shapes of the ψ–θ curves are similar but bubbling pressure is decreasing with increasing grain size, viz., 20 cm, 12 cm and 8 cm for 0.25 mm, 0.50 mm and 0.75 mm sized beads respectively. This is due to the presence of smaller sizes pores in smaller grain size media and larger in larger grain size media.

In Fig. 6 hydraulic conductivities are plotted against moisture content in a log-log scale. K-θ relationships are linear in log - log scale in all three cases as described by Kozeny's relationship given in

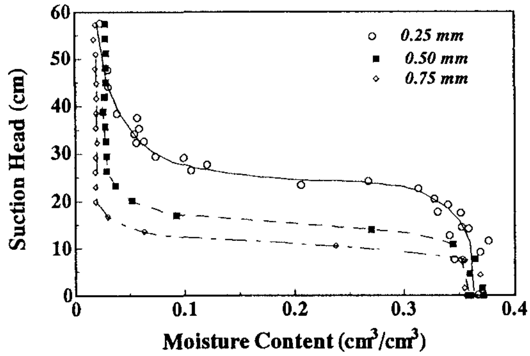

Fig. 5 ψ–θ relationships of three media

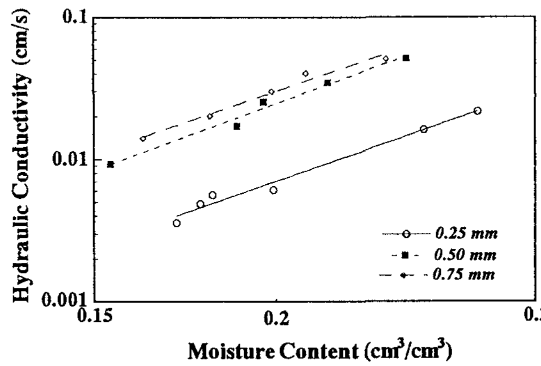

Fig. 6 K–θ relationships of three media

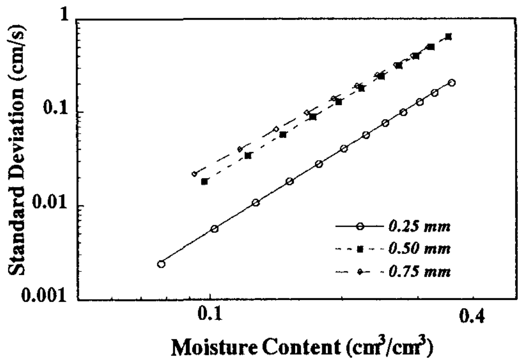

Fig. 7 Relationships between standard deviation and moisture content of three media

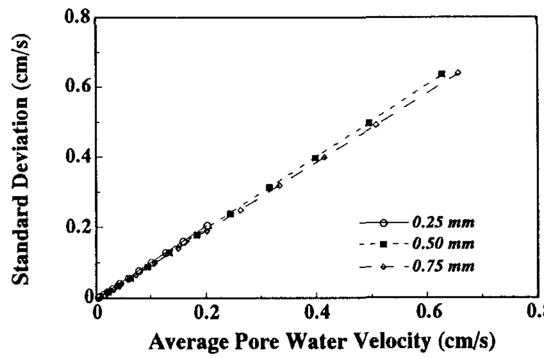

Fig. 8 Relationships between standard deviation and pore water velocity of three media

equation (7). Hydraulic conductivity depends on many factors mainly on moisture content and existing pore sizes. For the same moisture content, larger pores are filled with water in larger grain size media than in smaller grain size one. This leads to larger hydraulic conductivities in larger grain size media. This is evident from $\psi-\theta$ and $K-\theta$ relationship shown in Figs. 5 and 6. $K-\theta$ relationship of 0.50 mm grain is closer to that of 0.75 mm grain than 0.25 mm grain size. This is similar in $\psi-\theta$ relation also.

Standard deviations of the pore water velocities obtained by employing $\psi-\theta$ and $K-\theta$ relationships, i.e., square root of equation (10), are plotted as a function of moisture content in Fig. 7. With the increase in moisture content, standard deviation is also increasing. For the same moisture content, it is more in larger grain size media than in smaller size one. It also shows that the gradient of lines are smaller in larger grain size media. The characteristic of standard deviation is similar to $\psi-\theta$ and $K-\theta$ relationships in terms of grain size. The plots of σ against $\bar{v}$ for three sizes of media are shown in Fig. 8. They show that σ is linearly related to $\bar{v}$ in all cases. The gradient λ obtained from $\sigma-\bar{v}$ relationships are 1.028, 1.022 and 0.981 respectively for 0.25 mm, 0.50 mm and 0.75 mm grain sizes. The value of λ can be taken as 1.0, in average. σ can then be expressed as:

$$\sigma = 1.0\bar{v} \tag{13}$$

Mixing lengths obtained using equation (11) for three uniform size glass beads media are plotted in Fig. 9. It shows that mixing lengths are almost constant for a given grain size media in this considered lower range of moisture contents (<0.27). The average mixing length in this range is about 0.23 cm, 0.40 cm and 0.59 cm for the grain size of 0.25 mm, 0.5 mm and 0.75 mm respectively. This result shows that mixing length is a function of grain size of the porous media. This is because solute

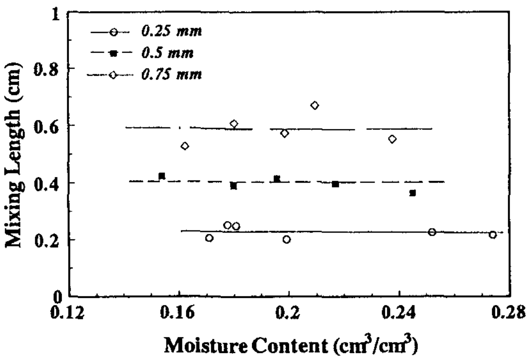

Fig. 9 Relationships between mixing length and moisture content of three media

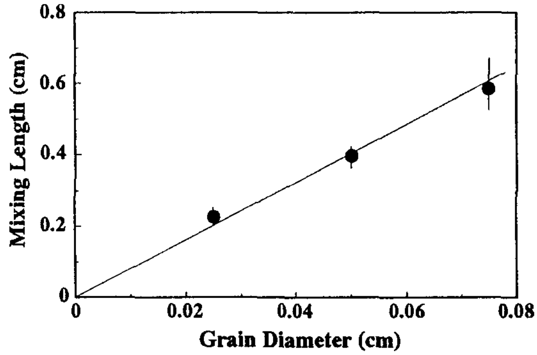

Fig. 10 Relationships between mixing length and grain size of three media

has to travel a larger distance before meeting one water path with another one in the case of larger grain size.

In Fig. 10 average mixing length is plotted against average grain diameter. Vertical lines in the plot show the range of mixing length for each group size. Fig. 10 shows that mixing length can be taken as linearly related to grain size for medium having sharp grain size distribution. We take the ratio of the "Mixing Length" to "Grain Size" and arrive to a constant number which is about 8. We give the name for this number "Mixing Length Number" for the porous media. It tells the similitude of the unsaturated flow through uniform grain size porous materials.

The applicability of the model was checked by using equation (11) with estimated parameters, λ and l. Estimated and observed dispersion coefficients are plotted in Fig. 11. It shows that equation (11) is quite satisfactory to estimate the dispersion coefficient in a porous media having sharp grain size distribution.

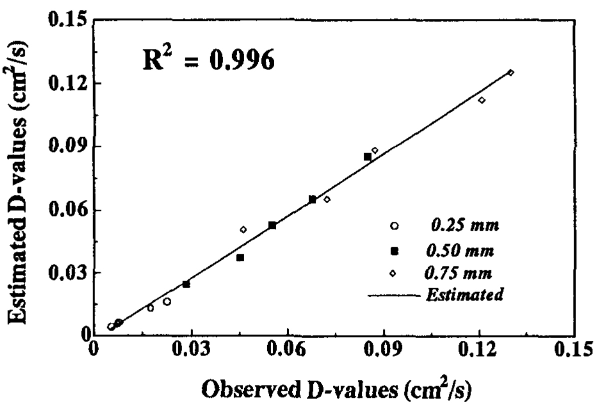

Fig. 11 Estimated dispersion coefficient vs observed dispersion coefficient

CONCLUSION

The results obtained from the miscible displacement experiments through uniform grain size media in the lower range of moisture content can be summarized as follows:

1. Mixing length was found almost constant for a certain grain size porous media.
2. Mixing length was found a function of grain size. The ratio of mixing length to the grain size, called 'Mixing Length Number', was found to be 8.
3. The proposed model for dispersion based on mixing length theory concept was found quite adequate to estimate the dispersion coefficient in the porous media.

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