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PROPOSAL OF TWO DIMENSIONAL FLOOD ROUTING METHOD

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This study describes two dimensional and one dimensional hydrodynamic models, to simulate watershed flow by treating surface runoff on a watershed of simple geometry. This model is formulated by a hydrodynamic system with the assumption of unsteady flow as steady flow which can reduce computation time. The numerical computation for this model is executed by use of the eight point implicit scheme. The Incremental Dynamic Programming(IDP) technique is adopted for the optimization of grid size during the computation. The model is applied to compute the routing of the floods in the Wichon IHP experimental basin located in southern part of Korea. Comparison of the results between the proposed model and the general type of two dimensional and one dimensional models indicate that the proposed model is nearly as good as, and computationally much faster than, the general two dimensional and one dimensional models.

Keywords : two dimensional flow, channel routing, grid optimization

1. INTRODUCTION

A number of one dimensional flood routing model have been developed and these model can route flood in a channel accurately [1]. A one dimensional model is possible for channel routing but is impossible to simulate the spreading of flood in a basin area because of two dimensional characteristics of flood propagation [2]. The two dimensional flood routing is simulated with numerical approximation and is needed to be improved as the fast computation time with easy equation. For this objective, the assumption of unsteady flow as steady flow is suggested as the deleting the unsteady term in the governing equation. And to make accurate and capable a numerical simulation, it is necessary to pay attention to both the computational scheme for calculation and computational grid size. The computationally useful method for grid optimization is developed, based on a measure of the interpolation error associated with numerical simulation model. The proposed IDP optimization technique [3] is used for the optimization of grid size and adjustment of parameters. The result is intended to be used to improve the quality of numerical simulation solutions by altering the location of computational nodes. Finally the proposed two dimensional model and one dimensional model are compared with general type of model and observed data.

2. THEORETICAL FORMULATION

1) Basic Theory of Proposed Model

By the assumption of incompressibility, uniform velocity distribution in vertical direction and small bottom slope, the general governing equations for two dimensional unsteady flow are expressed in conservation form [4] as

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$$\frac{\partial U}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+S=0$$

$$U=\begin{bmatrix} h \\ uh \\ vh \end{bmatrix}, E=\begin{bmatrix} uh \\ u^2+0.5gh^2 \\ uvh \end{bmatrix}, F=\begin{bmatrix} vh \\ uvh \\ v^2+0.5gh^2 \end{bmatrix}, S=\begin{bmatrix} 0 \\ -gh(S_{ox}-S_{fx}) \\ -gh(S_{oy}-S_{fy}) \end{bmatrix} \quad (1)$$

where h is flow depth, u is velocity component in X direction, v is velocity component in Y direction, g is acceleration due to gravity, S_{ox}, S_{oy} are bed slopes in X and Y directions, respectively, and S_{fx}, S_{fy} are friction slopes in X and Y directions, respectively.

In general case, the friction slopes (S_{fx} and S_{fy}) in X and Y directions are expressed as

$$S_{fx}=\frac{n^2u\sqrt{u^2+v^2}}{h^{1.33}}, S_{fy}=\frac{n^2v\sqrt{u^2+v^2}}{h^{1.33}} \quad (2)$$

where n is Manning's roughness coefficient. When assuming that the flow is steady, the governing equations for two dimensional steady flow are expressed from eq.(1) as follows:

$$\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+S=0$$

$$E=\begin{bmatrix} uh \\ u^2+0.5gh^2 \\ uvh \end{bmatrix}, F=\begin{bmatrix} vh \\ uvh \\ v^2+0.5gh^2 \end{bmatrix}, S=\begin{bmatrix} 0 \\ -gh(S_{ox}-S_{fx}) \\ -gh(S_{oy}-S_{fy}) \end{bmatrix} \quad (3)$$

2) Numerical Approximation Scheme

In order to use the numerical approximation, the vector term of **U**, **E**, **F**, **S** is divided by two components and eq (1) is written as

$$\frac{\partial U_1}{\partial t}+\frac{\partial E_1}{\partial x}+\frac{\partial F_1}{\partial y}+S_1=0$$

$$U_1=\begin{bmatrix} 0.5h \\ uh \\ 0 \end{bmatrix}, E_1=\begin{bmatrix} 0.5uh \\ u^2+0.5gh^2 \\ 0 \end{bmatrix}, F_1=\begin{bmatrix} 0.5vh \\ uvh \\ 0 \end{bmatrix}, S_1=\begin{bmatrix} 0 \\ -gh(S_{ox}-S_{fx}) \\ 0 \end{bmatrix} \quad (4)$$

$$\frac{\partial U_2}{\partial t}+\frac{\partial E_2}{\partial x}+\frac{\partial F_2}{\partial y}+S_2=0$$

$$U_2=\begin{bmatrix} 0.5h \\ 0 \\ vh \end{bmatrix}, E_2=\begin{bmatrix} 0.5uh \\ 0 \\ uvh \end{bmatrix}, F_2=\begin{bmatrix} 0.5vh \\ 0 \\ v^2+0.5gh^2 \end{bmatrix}, S_2=\begin{bmatrix} 0 \\ 0 \\ -gh(S_{oy}-S_{fy}) \end{bmatrix} \quad (5)$$

The vector components of flow **U** is determined by the discrete points of variables(x,y,t) through the eqs. (4) and (5). To make accurate and effective a two dimensional simulation model, the computational scheme for calculating the computational grid size is carefully treated. In this study the proposed 8 point implicit scheme was applied to compute the accurate result. The time derivatives are approximated by a forward different quotient centered between the i_{th} or $i+1_{th}$

points along X axis and j_{th} or $j+1_{th}$ points along Y axis, i.e.,

$$\frac{\partial U_n}{\partial t}+\frac{\partial E_n}{\partial x}+\frac{\partial F_n}{\partial y}=\frac{U^{j+1}_{n,i+1}+U^{j+1}_{n,i}-U^{j}_{n,i+1}-U^{j}_{n,i}}{2\Delta t}+\frac{E^{j+1}_{n,i+1}-E^{j+1}_{n,i}}{\Delta x}+\frac{F^{j+1}_{n,i+1}-F^{j+1}_{n,i}}{\Delta y} \tag{6}$$

The proposed eight point scheme[1]. is shown in equation (7) where P is the variables

$$\begin{aligned} K_{x_i}^{j+1}&=P_1K_{x_{i-4}}^{j}+P_2K_{x_{i-3}}^{j}+P_3K_{x_{i-2}}^{j}+P_4K_{x_{i-1}}^{j}+P_5K_{x_i}^{j}+P_6K_{x_{i+1}}^{j}+P_7K_{x_{i+2}}^{j}+P_8K_{x_{i+3}}^{j}\\ K_{y_i}^{j+1}&=P_1K_{y_{i-4}}^{j}+P_2K_{y_{i-3}}^{j}+P_3K_{y_{i-2}}^{j}+P_4K_{y_{i-1}}^{j}+P_5K_{y_i}^{j}+P_6K_{y_{i+1}}^{j}+P_7K_{y_{i+2}}^{j}+P_8K_{y_{i+3}}^{j}\end{aligned} \tag{7}$$

$$\begin{aligned} P_1&=0.01438a^3-0.08173a^2+0.06934a\\ P_2&=-0.01678a^3-0.03899a^2+0.05577a\\ P_3&=0.25057a^3+0.05914a^2-0.30971a\\ P_4&=-0.66780a^3+0.63087a^2+1.03693a\\ P_5&=0.66780a^3-1.37254a^2-0.29527a+1\\ P_6&=-0.25057a^3+0.81085a^2-0.56028a\\ P_7&=0.01678a^3-0.08934a^2+0.07256a\\ P_8&=0.01438a^3-0.08173a^2+0.06934a\end{aligned} \tag{8}$$

The quality of finite element solution is improved by optimizing the disposition of the nodes. The conventional study relied on their experience to restrict grids that make an efficient use of available technique. It is also possible to improve the quality of existing meshes by iteratively using predefined guidelines for the distribution of nodes.

3) Fundamental Theory of Optimization Model

The proposed IDP method considers the development of such a criterion and the fundamental theory of equation is given as followings.

If permit the policy to vary by, $\delta u(t), \delta u(t)_1, t\in\langle t_0, t_f\rangle$ equation of new enhanced policy given by

$$U(t)=\overline{U}(t)+\delta u(t)+\delta u(t)_1 \tag{9}$$

will influence the trajectory of multi unit system. The new trajectory will given by

$$S(t)=\overline{S}(t)+\delta s(t)+\delta s(t)_1 \tag{10}$$

where $U(t)$ is decision vector of each reservoirs, $S(t)$ is state vector of each reservoirs, $\delta u(t)$ and $\delta s(t)$ are changes in trial policy , respectively $\delta u(t)_1$ and $\delta s(t)_1$ are adjustable values for enhanced IDP programming which are used in local calculation in each domain, for $t\in\langle t_0, t_f\rangle$. In the following, in the place of $\delta u(t)$ and $\delta s(t)$ the variables $\Delta U(t)=\delta u(t)+\delta u(t)_1$, $\Delta S(t)=\delta s(t)+\delta s(t)_1$(which include the adjustable vectors $\delta u(t)_1$ and $\delta s(t)_1$) will used. Introduction of eq.(14) and (15) into fundamental DP(Dynamic Programming) will produce a set of equations such as

$$\frac{\partial(\overline{s}+ds)}{\partial t}=\phi\ \overline{s}+\Delta s,\ \overline{U}+\Delta U \tag{11}$$

$$\text{Maximize}\overline{F}\langle S_f, t_f\rangle=\int_{t_0}^{t_f} R\langle\overline{s}+\Delta s,\ \overline{U}+\Delta U,\ t\rangle dt \tag{12}$$

$$\frac{\partial F^*(\overline{s}+\Delta s,\ t)}{\partial t}=\max\left\{R\left[\overline{s}+\Delta s,\ \overline{U}+\Delta U,\ t\right]+\left\langle F_s^*\left[\overline{s}+\Delta s,\ t\right],\ \phi\left[\overline{s}+\Delta s,\ \overline{U}+\Delta U,\ t\right]\right\rangle\right\}\Delta U \tag{13}$$

It must be noted that as yet no restrictions have been put on the magnitudes of ΔS and ΔU. $F_s^*\left[\overline{s}+\Delta s,\ t\right]$ may be expressed by a power series expansion with respect to $\overline{s}$ as follows

$$F^*[\bar{s}+\Delta s,\ t]=F^*[\bar{s}\ t]+\langle f_s^*[\bar{s}+\Delta s,\ t],\ \Delta s\rangle+0.49\langle \Delta s, f_{ss}^*[\bar{s},\ t],\ \Delta s\rangle+h.t \tag{14}$$

$$F^*[\bar{s}\ t]=\bar{F}[\bar{s}\ t]+C^*[\bar{s}\ t] \tag{15}$$

where $F^*[\bar{s}\ t]$ is the maximum return due to the optimal trajcetory measured with respect to the trial trajcetory from time t_0 to t_f. $F^*[\bar{s}\ t]$ is the return due to the trial trajectory $\bar{s}$ from time t_0 to t_f. $C^*[\bar{s}\ t]$ is the difference between the maximum return due to the optimal trajectory $s^*(t)$ and trial trajectory. $F^*[\bar{s}\ t]$ is an m-dimensional vector equal to $(\frac{\partial F^*}{\partial s_i}, i=1,2,\ldots,m)$ evaluated at $\bar{s}(t)$. $F_{ss}^*[\bar{s}\ t]$ is an m times m-dimensional matrix equal to $(\frac{\partial^2 F^*}{\partial s_i \partial s_j}, i, j=1,2,\ldots,m)$ evaluated at $\bar{s}(t)$. $F_{ss}^*[\bar{s},\ t]\Delta s$ is a m-dimensional vector and h.t stands for higher order terms. $F_{ss}^*[\bar{s},\ t]\Delta s$ is expressed

$$F_s^*[\bar{s}+\Delta s,\ t]=F_s^*[\bar{s},\ t]+F_{ss}^*[\bar{s},\ t]\Delta s+h.t \tag{16}$$

Substituting eqs.(14),(15) and (16) into eq.(13) for the sake of convenience, dropping $[\bar{s},\ t]$ wherever possible

$$\begin{aligned}&-\frac{\partial \bar{F}}{\partial t}-\frac{\partial c^*}{\partial t}-\left\langle\frac{\partial F_s^*}{\partial t},\Delta s\right\rangle-0.49\left\langle\Delta s,\ \frac{\partial F_{ss}^*}{\partial t},\Delta s\right\rangle-h.t\\ &=\max\left\{R[\bar{s}+\Delta s,\ \bar{U}+\Delta U,\ t]+\left\langle\left(F_s^*+F_{ss}^*\Delta s\right)+h,t,\ \phi[\bar{s}+\Delta s,\ \bar{U}+\Delta U,\ t]\right\rangle\right\}\end{aligned} \tag{17}$$

The solution of eq.(15) will provide $\Delta u^*(t), t\in\langle t_0, t_f\rangle$, which is the amount that the trial policy must be incremented at time t to obtain the optimum policy, thus

$$u^*(t)=\bar{u}(t)+\Delta u^*(t),\ t\in\langle t_0, t_f\rangle \tag{18}$$

where $u^*(t)$ is the optimum policy. But the solution of eq.(17) requires possibly infinite computing time and storage requirements for the parameters of the power series expansion. In order to make the solution of eq.(16) possible, truncation of higher order terms is needed. This can only be justified if Δs is small enough to make these terms negligible. Assuming that Δs is kept small enough so that the highest order terms retained in eq.(17) are quadric

$$\begin{aligned}&-\frac{\partial \bar{F}}{\partial t}-\frac{\partial c}{\partial t}-\left\langle\frac{\partial F_s}{\partial t},\Delta s\right\rangle-0.49\left\langle\Delta s,\ \frac{\partial F_{ss}}{\partial t},\Delta s\right\rangle-h.t\\ &=\max\left\{R[\bar{s}+\Delta s,\ \bar{U}+\Delta U,\ t]+\left\langle\left(F_s+F_{ss}\Delta s\right)+h,t,\ \phi[\bar{s}+\Delta s,\ \bar{U}+\Delta U,\ t]\right\rangle\right\}\end{aligned} \tag{19}$$

$$F_s=\frac{\partial F}{\partial s}[\bar{s}+\Delta s,\ t]=F_s[\bar{s}\ t]+\bar{F}_{ss}\Delta s \tag{20}$$

$$F[\bar{s}+\Delta s,\ t]=\bar{F}[\bar{s}\ t]+c+\langle f_s\Delta s\rangle+0.49\langle\Delta s, f_{ss}\ \Delta s\rangle \tag{21}$$

This process reduces the global optimization of eq.(17) to a local optimization, i.e., optimization takes place in the neighborhood of the trial trajectory. Therefore, solution of eq.(19) is an improvement over that of trial trajectory and not an optimum one. It is because of this F has replaced F^* in eq.(19). However if the improved trajectory is optimized again in its neighborhood, it may prove a still better trajectory. By use of local optimization with an adjustable vector, the trajectory gradually converges with the optimal trajectory.

3. APPLICATION TO THE REAL BASIN

The study area in is located at the southern part of Korea as shown in Fig.1. The geometrical data of grids are 10 x 10 m and the initial conditions at the upstream and downstream boundaries are specified by the observed data[6].

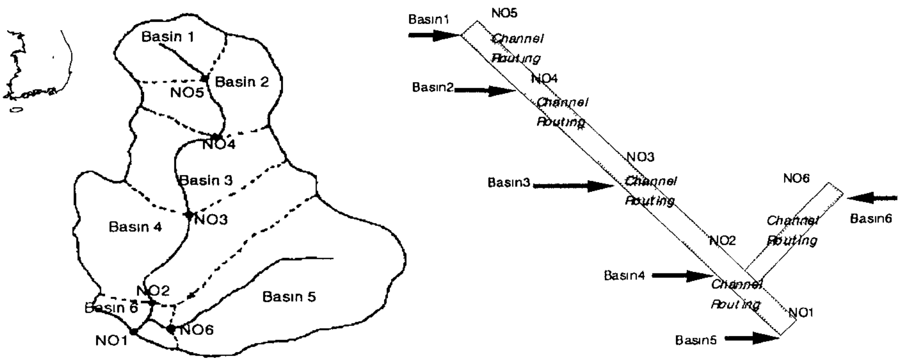

Fig. 1 The schematic configuration of study basin

The simulation was executed according to station points (from 1 to 6) as shown in Fig. 2. The comparison among observed, approximated and general type model represents good agreement. The computational examples presented here were performed on an IBM compatible personal computer with pentium processor. For each of the examples, the execution time was approximately 31 min., 57 min., respectively, for the proposed two dimensional approximation model, general two dimensional models. The different execution times for the approximation and general models are partly due to optimized smaller computation grid adopted with IDP technique, and partly due to faster convergence of numerical results.

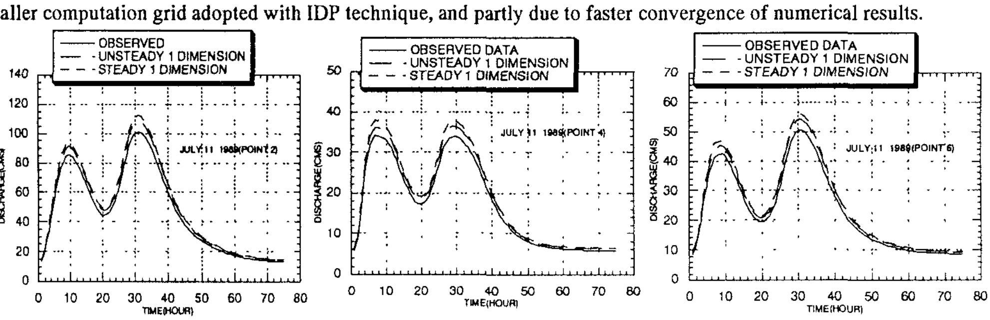

Fig.2 The comparison of hydrographs among observed, unsteady 1 dimensional model and steady 1 dimensional model

Through the two dimensional approximation model, velocities and depths were computed as shown in Fig.3 and Fig.4.

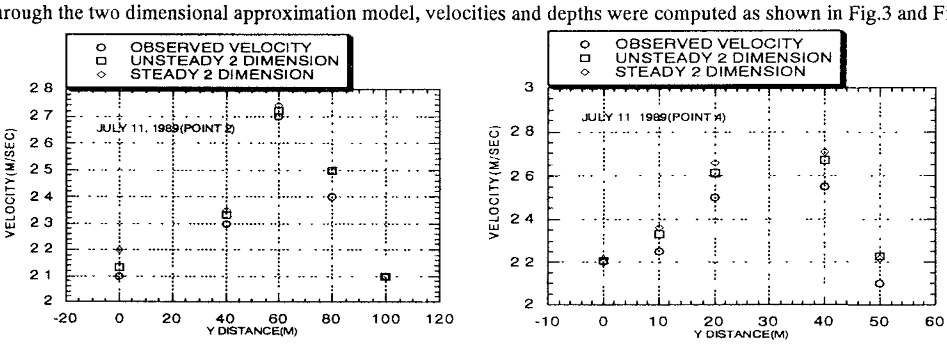

Fig.3 The comparison of velocities among observed, unsteady 2 dimensional model and steady 2 dimensional model

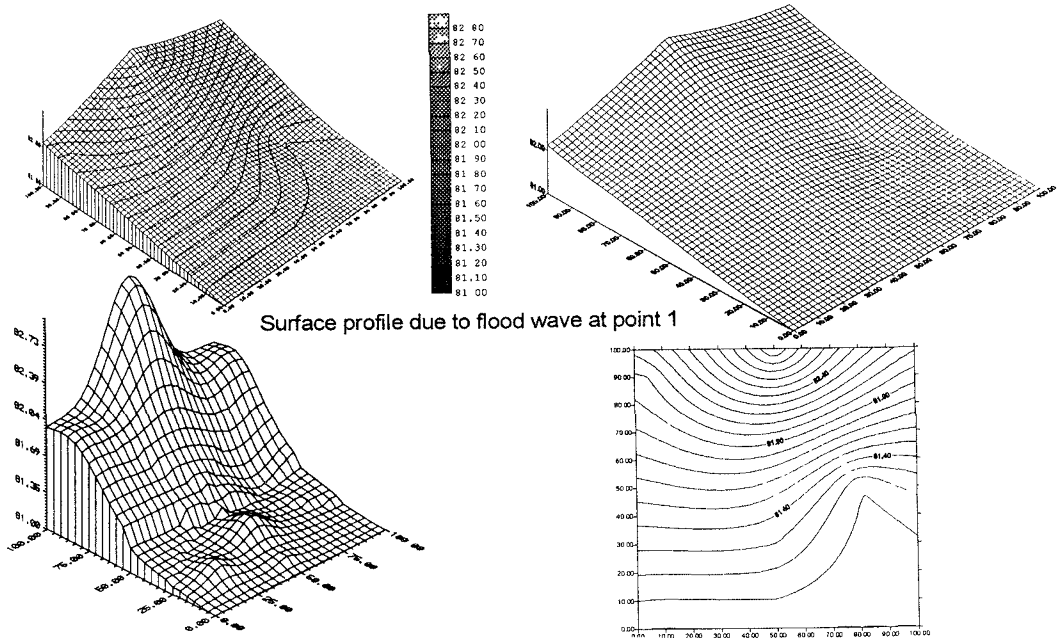

Fig.4 The two dimensional distribution at point 1

4. CONCLUSION

The two dimensional and one dimensional hydrodynamic models are employed to simulate the watershed flow in this study. For reducing the computational time and simplifying formulation, the assumption of unsteady flow as steady flow is suggested. To make accurate and capable a numerical simulation, the parameter calibration and grid size are executed by the use of IDP optimizing technique. The proposed eight point scheme can also solve the numerical approximation well.

The suggested approximated one dimensional model can simulate the channel flow in the real basin as accurately as general type of one dimensional model.

The proposed approximated two dimensional model is nearly as accurate as the general type of two dimensional model and is faster in computation and easier in numerical converge due to grid size optimization and simple form of equation.

The suggested approximated two dimensional model simulates the propagation of flood wave front and shows a good agreement between observed velocity and computed one.

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