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FLOW RESISTANCE IN ALLUVIAL CHANNELS

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Equivalent roughness and flow resistance of erodible beds in open channel flow in the presence of sand waves are theoretically investigated. Based on boundary layer theory, the geometrical properties of sand waves are introduced as dominant factors affecting on flow resistance. Using linear stability analysis, dominant length of sand waves is obtained. Applying one dimensional wave equation and continuity equation for sediments, the height and steepness of sand waves are obtained. Suspended sediment effect is taken within the scope of study. The results of the presented model are significantly better than previous studies.
Key Words *: flow resistance, alluvial channels, relative roughness, sand waves, wave steepness* .

1. INTRODUCTION

The interrelation between geometrical characteristics of sand waves and resistance to flow has been studied by many investigators, such as Shinohara-Tsubaki [18], Fredsoe [5], van Rijn [20] and Yalin [25], among others. Their formulae are valid in a limited range of hydraulic conditions and data.

In this study, based on boundary layer theory, the relative roughness height is obtained as a function of sand waves dimensions. The effects of sand wave dimensions and suspended sediments on relative roughness are clarified. The obtained values of friction factor are compared with a wide range of observed data collected by others from field and experimental flumes. Finally, error analysis is used for the comparison between the presented model for flow resistance and the previous approaches.

2. EQUIVALENT ROUGHNESS OF SAND WAVES

Following the boundary layer theory, the general semi-logarithmic function of flow resistance, according to Prandtl's mixing length theory, stands as

$$\frac{u}{u_*} = 8.5 + \frac{1}{\kappa} \ln \frac{y}{k_s} \tag{1}$$

in which, y is the distance above the virtual flat boundary, κ is the universal von Karman constant, k_s is the equivalent roughness height, u is the velocity of the flow at a distance y from the boundary, u_* is the mean shear velocity equal to $\sqrt{g\ h\ S}$, g is the gravity acceleration, h is the mean depth of the flow, and S is the energy slope.

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When the surface roughness stands out into the fully turbulent zone, such as the existence of two dimensional triangular elements, the laminar sublayer is destroyed. Moreover, the velocity distribution is affected by k_s which represents, in this case, a virtual hydrodynamic roughness parameter describing size, space and shape of roughness elements. Thus, the semi-logarithmic velocity distribution is assumed to apply :

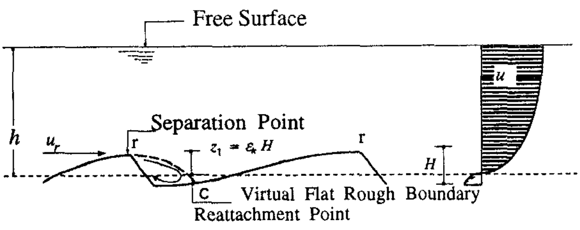

Fig. 1 Definition sketch for sand waves

$$\frac{u}{u_*} = 8.5 + \frac{1}{\kappa} ln \frac{y \; exp \, [\, \kappa \, (\frac{u_r}{u_*} - 8.5) \,]}{z_1} \tag{2}$$

The parameter z_1 is the height of element over the virtual boundary. At this boundary the velocity of flow u is equal to zero, and u_r is the velocity at the top of roughness element.

Comparing Eqs. 1 and 2 to determine the equivalent roughness height k_s as a function of flow characteristics;

$$\frac{k_s}{H} = \varepsilon_* \; exp \; [\, \kappa \, (8.5 - \frac{u_r}{u_*}) \,] \tag{3}$$

where H is the roughness element height, and $\varepsilon_* = z_1 / H$.

Resistance to flow in open channels with flat bed is a frictional drag. When the bed is covered with a discrete roughness elements, additional resistance is imposed on the flow representing the form drag. Thus,

$$\tau_0 = \tau' + \tau'' \tag{4}$$

where τ_0 is total shear stress, τ' is the skin friction over a unit area of the bed, and τ'' is the equivalent shear stress due to form drag force which stands as

$$\tau'' = C_D \, \rho \, \frac{H}{L} \frac{u_r^2}{2} \tag{5}$$

where C_D is the drag coefficient of roughness element, ρ is the fluid density, and L is the length of the triangular two dimensional roughness element. Substituting from Eqs. 4 and 5 into Eq. 3 , the parameter k_s can be represented as follows

$$\frac{k_s}{H} = \varepsilon_* \; exp \; \{ \, \kappa \, [\, 8.5 - \sqrt{\frac{2}{C_D} \frac{L}{H} \frac{\psi''}{\psi}} \;\,] \, \} \tag{6}$$

where $\psi = \tau_0 / s \gamma d$ is the dimensionless tractive shear stress, $\psi'' = \tau'' / s \gamma d$ is shear stress due to form pressure, s is the specific weight of sediment in the water, γ is the unit weight of water, and d is the mean diameter of particles.

3. DEVELOPMENT OF THE NEW MODEL

3.1 Determination of drag coefficient " C_D "

Based on dimensional analysis, the drag coefficient C_D for fixed roughness triangular element is a function of the geometrical characteristics of flow and the roughness element, $C_D = \phi \, (h/H \;, H/L \,)$.

Even though, drag coefficient C_D has been studied by Vittal et al. [21] and Raudkive [15] among others, there is no accurate formula to predict its value.

Based on others's experimental data, a multiple regression technique is used to derive an empirical expression for C_D as a function of bed forms height and steepness . The obtained expression is in the form,

$$C_D = 1.25 \; (H/h)^{0.5} \; (H/L)^{0.25} \tag{7}$$

Figure 2 shows the agreement between measured and calculated values of C_D , and Fig. 3 shows the effect of H/h on the values of C_D with parameter H/L , and verified with collected data.

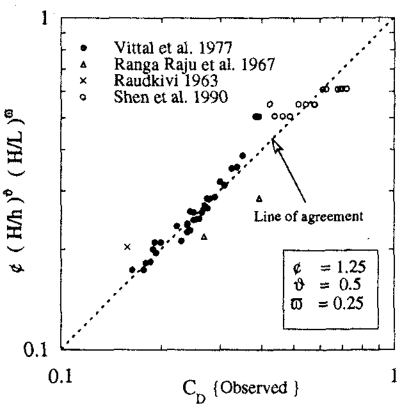

Fig. 2 Relation between C_D and effective parameters

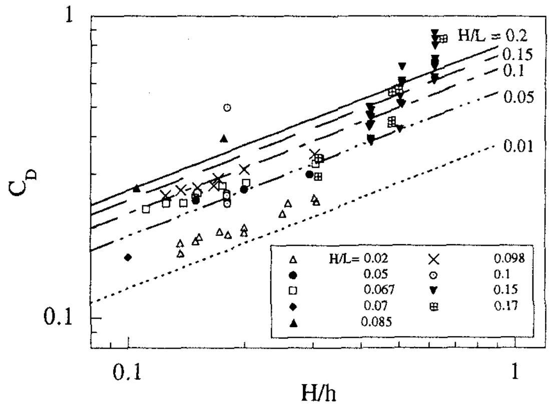

Fig. 3 Relation between H/h and C_D with parameter H/L

3.2 Determination of the parameter "ε_*"

For triangular elements, the acceptable empirical formula known to the authors which reflects steepness effect is the equation suggested by Pillai [13] according to his experimental study on two dimensional triangular roughness.

$$\varepsilon_* = z_1/H = 0.333\ (H/L)^{-0.3} \tag{8}$$

3.3 Evaluation of form drag tractive shear stress ratio ψ''/ψ

In this study, based on conventional concepts, the flow is considered uniform and sand waves are treated as a series of expansions, Rouse [16]. Thus,

$$\frac{\psi''}{\psi} = \frac{\alpha_e}{2}\ \frac{H^2}{L h}\ (\frac{u_m}{u_*})^2 \tag{9}$$

in which α_e is the loss coefficient for gradual expansion $\approx$ 0.8 according to Gibson graph, Rouse [16], and u_m is the mean velocity of flow. Substituting Eqs. 7, 8 and 9 into Eq. 6 yields

$$\frac{k_s}{H} = \frac{1}{3}\ (\frac{H}{L})^{-0.3} \exp\ \{\ \kappa\ [\ 8.5\ -\ 0.8\ \ (\frac{H}{h})^{0.25}\ (\frac{H}{L})^{-0.125}\ (\frac{u_m}{u_*})\ ]\ \} \tag{10}$$

In order to examine the obtained expression with the available data, and to show the significance of steepness and height on relative roughness coefficient, Eq. 10 is presented in Fig. 4 through a family of curves relating between k_s/H and steepness H/L with parameter H/h for one case of $u_m/u_* = 12$. As shown in the figure, roughness increases monotonously with the increase of steepness, and decreases with the increase of the ratio H/h . The curves are generally in agreement with the ploted data.

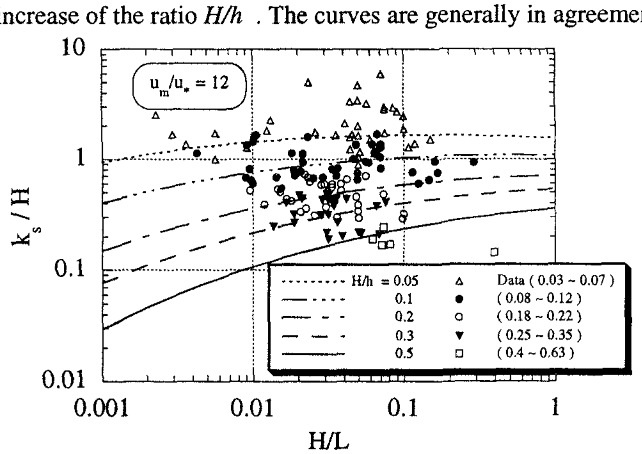

Fig. 4 Relation between k_s/H and H/L with parameter H/h

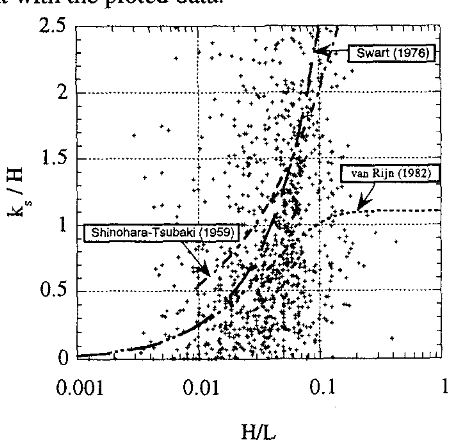

Fig. 5 Evaluation of previous studies

The empirical equations, presented by Shinohara-Tsubaki [18], Swart [19], and van Rijn [20], are the only presented materials, known to the authors, for equivalent roughness height. Fig. 5 shows that, the three previous expressions have a limited range of application . In Fig. 6, a group of 1000 data is used to verify the new developed equation, Eq. 10. The presented figure shows a good agreement, except a few points which ascribed by low values of u_m/u_* . In summary, Eq. 10 gives better results than the others, since it has two more parameters .

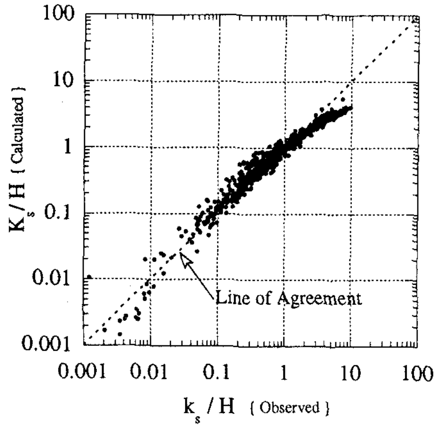

Fig. 6 Verification with observed data from flumes and natural rivers

4. HEIGHT AND STEEPNESS OF SAND WAVES

The geometrical characteristics of fully developed sand waves in alluvial channels were analytically investigated in two consecutive papers by the authors [12 and 23]. The effect of suspended sediment was investigated among other flow parameters. The theoretical expression for wave length was presented as :

$$\frac{L}{h} = 2\pi / \beta \tag{11}$$

where β is the dominant wave number, stands for $\beta = k_c \{(1/F^2) - 1\}^{1/2}$, in which F is the Froude number equal to $u_m/\sqrt{g\,h}$, and k_c is constant equal to 0.45 and 1.35 for dunes and ripples, respectively. The theoretical expression for height was presented as

$$\frac{H}{h} = \frac{1}{\omega} \cdot \frac{1 - e^{-\alpha_1 (2\pi/\beta)W} + (2\pi/\beta)(1-\alpha_1)W}{1 - \dfrac{1}{\alpha_1 (2\pi/\beta)W}\left(1 - e^{-\alpha_1 (2\pi\ \beta)W}\right)} \tag{12}$$

where $\alpha_1 = L_1/L$ is the measure of dimensionless length of the upstream face (0.5 ~ 1.0), L_1 is the distance from the trough to the crest of the wave, and W denotes the effect of suspended sediment, which is related to the parameter w_0/u_* as follows :

$$W = \frac{k_0\, w_0}{\xi_*\, u_*}\{\phi(\sigma) - F(\sigma)\} \quad \text{and} \quad k_0 = \frac{1}{k\ (1 - u_{*c}/u_*)} \tag{13}$$

in which w_0 is the fall velocity of sediment particles, $k = 8.5$, $\xi_* = 0.05$ and u_{*c} is the critical shear velocity. The functions $\phi(\sigma)$ and $F(\sigma)$, stand for

$$\phi(\sigma) = \frac{1}{\sqrt{2\pi}\ \sigma} exp\left(-\frac{\sigma^2}{2}\right) \quad and \quad F(\sigma) = \int_\sigma^\infty \frac{1}{\sqrt{2\pi}} exp\left(-\frac{t^2}{2}\right) dt \tag{14}$$

in which $\sigma = w_0/\sqrt{w'^2}$, and w' is the vertical turbulence velocity.

The dimensionless propagation velocity ω is represented as

$$\omega = \frac{M\ (1 - \lambda_1 \beta^2 F^2)}{(\beta^2 E^2 + (1 + E\,W)^2)} \frac{\left[\{1 - (1 + \frac{q\,\alpha}{\varphi^2} + \lambda_2 \beta^2) F^2\}(\beta^2 E_* q\,\alpha + 2) + \frac{3F^2}{\varphi^2}(q\,\alpha - 2E_*)\right]}{\{1 - (1 + \frac{q\,\alpha}{\varphi^2} + \lambda_2 \beta^2) F^2\}^2 + \frac{9F^4}{\beta^2 \varphi^4}} \tag{15}$$

in which $M = \dfrac{m}{1 - \Psi_c/\Psi}$, $q = \dfrac{4/3}{(1 - \Delta_0)(1 - \Delta_0/3)}$, $\Delta_0 = 6/(\varphi + 2)$,

$$E = \frac{\lambda_d d}{h}(1+A_* \bar{\Phi}_B) \ , \quad \bar{\Phi}_B = 8(\psi_e - \psi_c)^m \ , \quad \text{and} \quad E_* = E + \frac{1}{\beta^2}(1+E\,W)W \tag{16}$$

where ψ_e is the effective tractive stress for sand waves, $\psi_c = u_{*c}^2 / s\,g\,d$ is the dimensionless critical tractive stress, $a = 5$, $\lambda_d = 100$, $A_* = 10$, $m = 3/2$, φ is the velocity coefficient equal to u_m / u_* , α is a parameter denoting the asymmetric distribution of bed shear stress = 5, and λ_1 and λ_2 represent the curvilinearity centrifugal effects of both bed and water surfaces, and have the values of 0.5 and 0.33, respectively. The steepness of bed forms, representing dunes and ripples, is derived as

$$\frac{H}{L} = 0.16 \ \frac{\beta}{\omega} \cdot \frac{1 - e^{-\alpha_1 (2\pi/\beta)W} + (2\pi/\beta)(1-\alpha_1)W}{1 - \frac{1}{\alpha_1 (2\pi/\beta)W}\left(1 - e^{-\alpha_1 (2\pi/\beta)W}\right)} \tag{17}$$

5. FLOW RESISTANCE ESTIMATION

The final step in this study is to introduce a model for estimating the friction factor in alluvial streams in the presence of sand waves. In the calculation of the friction factor f, 3 variables , the unit discharge q /or water depth h, particle diameter d , and slope S , are used as input data. A reasonable value of f, which is defined by Darcy-Weisbach law $u_m/u_* = \sqrt{8/f}$, is assumed. By using Eqs. 1, 10, 12 and 17, a new value friction factor f is obtained. By applying the iteration technique, the calculated friction factor f_{calc} is derived. In the calculation procedure, side-wall correction method, derived by Vanoni-Brooks, is used to calculate bed hydraulic radius R_b . The effect of suspended sediment on von Karman parameter κ is presented by using Hino's equation [8]. The available data, collected from 40 reliable sources, are used to calculate f . The obtained results are compared with the observed ones f_{meas} . Discrepancy ratio f_{calc}/f_{meas} is calculated for the data. Figure 7, shows a relation between discrepancy ratio f_{calc}/f_{meas} and H/L . Figure 8, shows a relation between f_{calc}/f_{meas} and H/h . From both graphs, it is noticed that discrepancy ratio varied symmetrically in both sides of average line $f_{calc}/f_{meas} = 1$ except a few points at the extreme values of H/h .

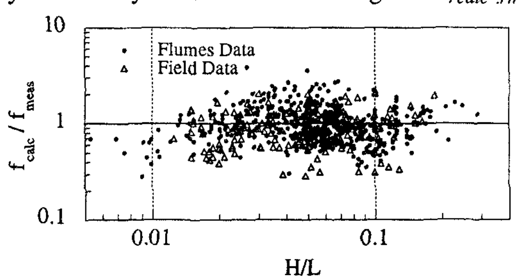

Fig. 7 Comparison between calculated and observed values for different H/L

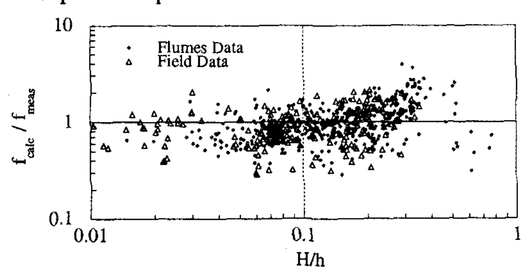

Fig. 8 Comparison between calculated and observed values for different H/h

6. VERIFICATION AND COMPARISON WITH PREVIOUS STUDIES

The approach explained above is applied to 958 data consists of 625 flumes data and 333 rivers and small channels data. In Table 1, an error analysis is carried out on natural rivers data to calculate mean error, standard deviation, and percent scores of errors in the given intervals.

Also, an error analysis is pursued to compare between the presented method and the recent previous models which added more contributions than the earlier ones. Part of the methods used sand wave dimensions as parameters, such as van Rijn's approach [20]. The others not, such as White et al. method [24]. Table 2 shows that the new model has more advantage than others in estimating the friction factor of movable bed with sand waves because of its good results, and because of its ability of using wide range of variables and flow parameters, such as suspended sediment parameter.

Table 1 Error analysis of presented resistance equation

Rivers &Channels	No. of data	Mean error	Standard deviation	Scores of Errors in Intervals, as a percentage									
				- 5,5	- 10,10	- 15,15	-20,20	-25,25	-30,30	-35,35	-40,40	-45,45	-50,50
1) Small Streams { Ackers 1966 }	30	- 0.055	0.452	13.33	20	23.33	30	46.67	50	53.33	66.67	73.33	86.67
2) Luznice River { Martinec 1967 }	6	- 0.005	0.163	0	16.67	50	100	100	100	100	100	100	100
3) Missouri River { Sayre et.al. 1972 }	6	0.141	0.303	16.67	50	50	66.67	66.67	66.67	66.67	66.67	66.67	83.33
4) Hii River {M. C. Japan, C.E.lab. 1977}	55	0.060	0.638	10.91	32.73	40	40.09	56.36	61.82	70.91	76.36	85.45	85.45
5) Oyodo River {M. C. Japan, C.E.lab. 1977}	18	0.178	0.33	16.67	38.89	44.44	61.11	61.11	66.67	66.67	66.67	77.78	77.78
6) Zaire River { Peters 1978- (9) }	21	0.325	0.92	23.81	38.1	47.62	66.67	76.19	76.19	6.19	76.19	80.95	85.71
7) ACOP Canals { Haque et al. 1983 }	31	- 0.109	0.2	32.26	48.39	61.29	70.97	77.42	77.42	83.87	87.1	90.32	90.32
8) Bergsche Mass { Adriaanse 1986- (9) }	26	0.034	0.343	0	19.23	46.15	57.69	65.38	69.23	76.92	84.62	84.62	88.46
9) Meuse River { Brilbois 1988- (9) }	60	- 0.042	0.307	21.67	35	50	58.33	73.33	81.67	81.67	83.33	83.33	83.33
10) Jamuna River { Klaassen 1991- (9) }	33	- 0.073	0.357	6.06	24.24	27.27	33.33	36.36	51.52	57.58	63.64	72.73	79.11
11) Parana River { van Rijn 1991- (9) }	13	- 0.318	0.130	0	0	7.69	15.38	30.77	38.46	69.23	84.62	84.62	84.62

Table 2 Comparison with previous approaches {using laboratory data }

Authors	Consider S.Waves dimensions (L & H)	Mean error	Standard deviation	Scores of errors in intervals, as a percentage									
				- 5,5	- 10,10	- 15,15	-20,20	-25,25	-30,30	-35,35	-40,40	-45,45	-50,50
1) The presented model	yes	0.045	0.489	10.58	20.31	32.25	43.86	55.29	61.95	67.06	71.84	75.09	79.69
2) Brownlie [1983]	no	0.313	0.893	8.94	17.26	26.37	33.91	41.92	51.81	56.36	62.48	68.13	71.12
3) Engelund et al.[1967]	no	- 0.08	0.44	8.16	18.05	27.78	33.91	46.31	55.57	61.53	67.97	73.78	80.53
4) Fredsoe [1975]	yes	0.069	1.52	7.85	18.21	25.75	32.02	37.36	43.33	48.82	55.26	62.79	68.60
5) Grade et al. [1966]	no	2.07	2.76	5.34	8	10.83	15.69	18.21	20.25	22.61	24.17	26.37	29.67
6) van Rijn [1984]	yes	0.47	1.23	10.12	21.13	30.55	35.52	41.38	47.42	52.93	57.55	63.23	67.14
7) White et al. [1980]	no	- 0.228	0.371	8.49	16.51	25.47	35.85	41.67	49.84	55.19	65.09	71.06	79.25
8) Yalin [1993]	yes	- 0.105	1.49	5.68	11.72	18.82	26.64	34.99	42.27	49.20	55.77	64.12	71.04

7. CONCLUSIONS

The boundary layer theory has been generalized to account for obtaining roughness and resistance over movable undulated bed. Accordingly, roughness coefficient is obtained analytically as a function of dimensionless parameters denoting height, steepness of sand waves and shear stress. Since sand waves size and steepness equations are available, it is easy to estimate the roughness and friction parameters from the fundamental properties of flow and bed material. The model gives much better results than the previous ones.

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