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A TWO-DIMENSIONAL SIMULATION FOR PROPAGATION OF FLOOD WAVES

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A two-dimensional model is introduced for simulation of wave propagation of flood flows. The model has implicit finite difference scheme for integration of conservative form of equations of depth-averaged two-dimensional flow. Artificial viscosity is suggested as dissipative interface device and it can control nonlinear instability of two-dimensional model. Advantages of using smaller ratio of tail water to upstream head have been illustrated by examples. The propagation of flood waves in flood plain, which comes from steep channel, has been simulated and the change of flow condition has been illustrated.

Keywords: Two-dimensional simulation, Implicit finite difference scheme, Wave propagation of flood flows.

1-Introduction

A model for the simulation of flood is required for the analysis of flood propagation generated by storms or failure of dams and dykes. One-dimensional model has been used for these analyses widely and still research is done to modify them (Kutija 1993). However propagation of flood flow can be simulated better by two-dimensional models especially in flood plains. The computation of two-dimensional shock is more complicated than the computation of one-dimensional flood due to the need for suitable simulation of boundary condition and instability problems in two directions.

The simulation of flows by numerical models are suffering from different numerical instability problems. Explicit schemes depend on the Courant-Friedrichs-Lewy condition for numerical instability and they suffer from the requirement of small computation time steps (Zhao et al. 1994). Larger time steps become more important in modeling discontinuous flow, in which it is required longer simulation's time. Another instability problem, from which numerical schemes suffer, is nonlinear instability. Implicit schemes are unconditionally stable due to Courant criteria, but still nonlinear instability develops slowly until it spoils calculations. Simulation of discontinuous flows such as shocks have more instability problems than continuous flows (Abbott, 1989). Artificial viscosity is suggested to be used for dissipation of nonlinear instability. Illustrated examples present shock capturing ability of the model in simulation of the propagation of flood waves.

2-Governing Equations

Propagation of flood on flood plain is a kind of the discontinuous flow which includes steep front. For case like this, it has been proven that the weak solutions of homogeneous differential equations in the

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conservation form tend to the discontinuous solutions of the integral relationships (Lax, 1954; Abbott 1979). Therefore, it is necessary to use conservation form of the governing equations for the simulation of the propagation of the flood waves.

Equations of two-dimensional unsteady free-surface flow may be derived by depth averaged integration of three dimensional equations of conservation of mass and momentum by following assumptions: (1).The pressure distribution is hydrostatic. (2) The velocity distribution is uniform over the flow depth. (3) The channel bottom is rigid. (4) Bottom shear stress is dominant and all other shear stresses are neglected. (5) Friction losses are computed using steady state formulas. (6)The channel bottom slope is small. These assumptions are usually valid except that the pressure distribution may not be hydrostatic, where water surface has sharp curvatures. The matrix form of two dimensional momentum conservation equations can be written in the Cartesian orthogonal coordinate system with the x-y plane parallel to the channel bottom as follows:

$$\mathbf{U}_t + \mathbf{E}_x + \mathbf{F}_y + \mathbf{S} = \mathbf{0} \quad \text{.......(1)}$$

in which:

$$\mathbf{U} = \begin{bmatrix} h \\ uh \\ vh \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} uh \\ u^2h + (1/2)gh^2\cos\theta_{bx0} \\ uvh \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} vh \\ uvh \\ v^2h + (1/2)gh^2\cos\theta_{by0} \end{bmatrix},$$

$$\mathbf{S} = \begin{bmatrix} 0 \\ -gh(S_{0x}\cos\theta_{bx0} - S_{fx} - \sin\theta_{bx0}) \\ -gh(S_{0y}\cos\theta_{by0} - S_{fy} - \sin\theta_{by0}) \end{bmatrix}$$

where h = flow depth; u = flow velocity in the x-direction; v = flow velocity in the y-direction; g = acceleration due to gravity; S_{0x}= channel bottom slope in the x-direction; S_{0y}= channel bottom slope in the y-direction; θ_{bx0}= inclination of x axes; θ_{by0}= inclination of y axis; and S_{fx} and S_{fy} are the slopes of the energy grade lines in the x and y-directions, respectively. According to above assumption S_{fx} and S_{fy} are computed by using the steady state friction formulas like Manning formula. These factors can be overestimated and unrealistic when the depth is too small. A small depth h_1 may be used to avoid from overestimation (Richtmyer and Morton 1957) as follows:

$$S_{f_x} = \frac{n^2 u \sqrt{u^2+v^2}}{(h+h_1)^{4/3}} \quad \text{.......(2a)} \qquad S_{f_y} = \frac{n^2 v \sqrt{u^2+v^2}}{(h+h_1)^{4/3}} \quad \text{.......(2b)}$$

in which n = Manning's roughness coefficient; h_1 = required minimum depth for energy loss calculation. Because equation (1) has the source term **S** and the term acts as source or sink, it is not in full conservation form. Since the contribution of this term is usually small the conservative properties are not significantly impaired (Chaudhry 1993). Since the governing equation, equation (1), is nonlinear and first order hyperbolic partial differential equations, no analytical solution are available except for very simplified one-dimensional cases. Therefore, they are solved numerically.

3-Numerical Scheme

Equation (1) has been integrated by Implicit Scheme of finite difference method. The Alternating Direction Implicit (ADI) algorithm has also been employed to decrease the number of equations that should be solved simultaneously. In order to use ADI algorithm, each vector of the **U**, **E**, **F** and **S** can be spilt into two components. Therefore equation (1) may be written as follows:

$$\mathbf{U}_{1_t} + \mathbf{E}_{1_x} + \mathbf{F}_{1_y} + \mathbf{S}_1 = 0 \qquad (3)$$

$$\mathbf{U}_{2_t} + \mathbf{E}_{2_x} + \mathbf{F}_{2_y} + \mathbf{S}_2 = 0 \qquad (4)$$

where:

$$\mathbf{U}_1 = \begin{bmatrix} 0.5\,h \\ uh \\ 0 \end{bmatrix}, \quad \mathbf{U}_2 = \begin{bmatrix} 0.5\,h \\ 0 \\ vh \end{bmatrix}, \quad \mathbf{E}_1 = \begin{bmatrix} 0.5\,uh \\ u^2h + (1/2)gh^2 \cos\theta_{bx0} \\ 0 \end{bmatrix}, \quad \mathbf{E}_2 = \begin{bmatrix} 0.5\,uh \\ 0 \\ uvh \end{bmatrix}$$

$$\mathbf{F}_1 = \begin{bmatrix} 0.5\,vh \\ uvh \\ 0 \end{bmatrix}, \quad \mathbf{F}_2 = \begin{bmatrix} 0.5\,vh \\ 0 \\ v^2h + (1/2)gh^2 \cos\theta_{by0} \end{bmatrix}, \quad \mathbf{S}_1 = \begin{bmatrix} 0 \\ -gh(S_{0x}\cos\theta_{bx0} - S_{fx} - \sin\theta_{bx0}) \\ 0 \end{bmatrix},$$

$$\mathbf{S}_2 = \begin{bmatrix} 0 \\ 0 \\ -gh(S_{0y}\cos\theta_{by0} - S_{fy} - \sin\theta_{by0}) \end{bmatrix}$$

Components of the vector of flow variables (**U**) can be determined at the discrete points of the independent variables (x, y and t) by numerical integrating equations (3) and (4). The notation, which are used here for the finite-difference mesh in x, y and t space, are as follows. The number of grid in x-direction is counted by the subscript i, the y-direction by the subscript j and the t-direction by superscript k. The vector of flow variables (**U**) is known at k time level and its components are to be determined at $k+1$ time level. Then for equations (3) and (4), the following finite difference equations can be derived for grid point i and j.

$$\mathbf{U}_{1_{ij}}^{k+1} = \mathbf{U}_{1_{ij}}^{k} - 1/2\,\Delta t\left(\frac{\nabla_x \mathbf{E}_{1_{i,j}}^{k+1}}{\Delta x} + \frac{\nabla_y \mathbf{F}_{1_{ij}}^{*}}{\Delta y} + \mathbf{S}_{1_{i,j}}^{k+1}\right) \qquad (5)$$

$$\mathbf{U}_{2_{ij}}^{k+1} = \mathbf{U}_{2_{ij}}^{k} - 1/2\,\Delta t\left(\frac{\nabla_x \mathbf{E}_{2_{ij}}^{*}}{\Delta x} + \frac{\nabla_y \mathbf{F}_{2_{ij}}^{k+1}}{\Delta y} + \mathbf{S}_{2_{i,j}}^{k+1}\right) \qquad (6)$$

Backward space differences have been used in x and y-directions and forward space difference has been used for time. Since the scheme is implicit, it uses $k+1$ time level for space derivatives excepted for cross terms in $\mathbf{F}_{1_y}$ and $\mathbf{E}_{2_x}$, in which we use mix time level as following. In equation (5), v and in equation (6), u are in time level k and all other components of vectors are in time level $k+1$. The scheme has four half time steps. In first and third times' steps, equation (5) is used to determine the components of $\mathbf{U}_1$. In second and forth times' steps, equation (6) is used to determine the component of $\mathbf{U}_2$. Since original differential equations are nonlinear partial differential equations, the result of these finite equations is a system of nonlinear equations which have to be solved simultaneously.

4-Nonlinear instability

In propagation of flood waves, the governing partial differential equations of flow, equation (1), become highly nonlinear because of existence of the advective terms. In such cases even appropriate numerical scheme may have instability problems. Even though Courant instability problem is solved by using implicit scheme, these schemes still are suffering by nonlinear instability growing slowly until it spoils calculation. The dissipative interface can be useful to suppress nonlinear instabilities. Here, the procedure of Jameson et al. (1981) is used. Since non linearity of governing equations is high in front area, it is important to employ more dissipative interface in front area than other areas. This procedure adds an additional dissipation to large gradients while it leaves smooth area relatively undisturbed. Numerical experiments show this artificial viscosity can suppress oscillation of nonlinear instability in different Courant numbers.

5-Model Applications

Two examples have been analyzed to demonstrate the application of presented scheme on the simulation of propagation of flood waves. These problems have been studied to illustrate the ability of the model to simulate the two-dimensional propagation of flood waves on the flood plain located in the end of steep channel. This exists when steep mountainous channel ends to flood plain like debris fan. Because of discontinuous values of both velocity and depth, as a numerical problem it has more discontinuity in initial conditions. In these examples, a steep channel with 20cm-wide ends to expansion of 190cm-wide and 170cm-long flood plain (figure 1). The flood plain has one degree inclination in x-axis direction and -0.5 degree inclination from middle of plat in both sides of y-direction. The flow at the end of the steep channel has 3.25cm-depth. It enters flood plain with 62.17 cm/s velocity. The propagation of flood waves has been simulated in the flood plain. The mesh size of 5cm×5cm has been used for simulation. The perspectives of two dimensional propagated waves of flood in the flood plain have been illustrated after 2.5 seconds in figure (2). The flow condition is supercritical when it leaves steep channel and enters flood plain. In the flood plain flow condition changes from supercritical to subcritical in x direction. In y direction flow condition stays subcritical because of inverse slope.

Figure (1). Configuration of the steep channel and flood plain

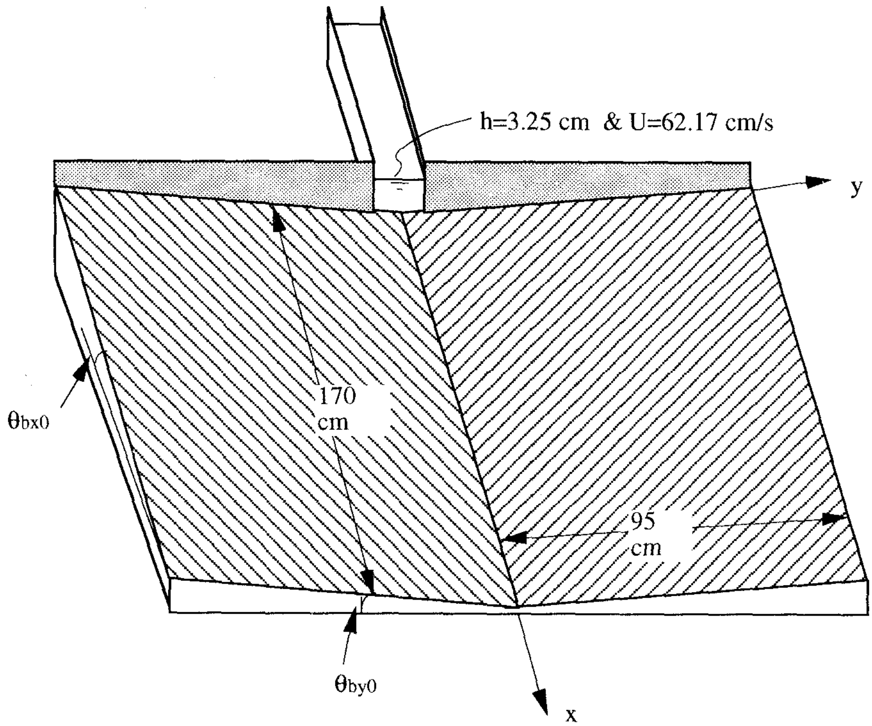


Figure (2). Propagation of flood waves after 2.5 seconds for example (1)

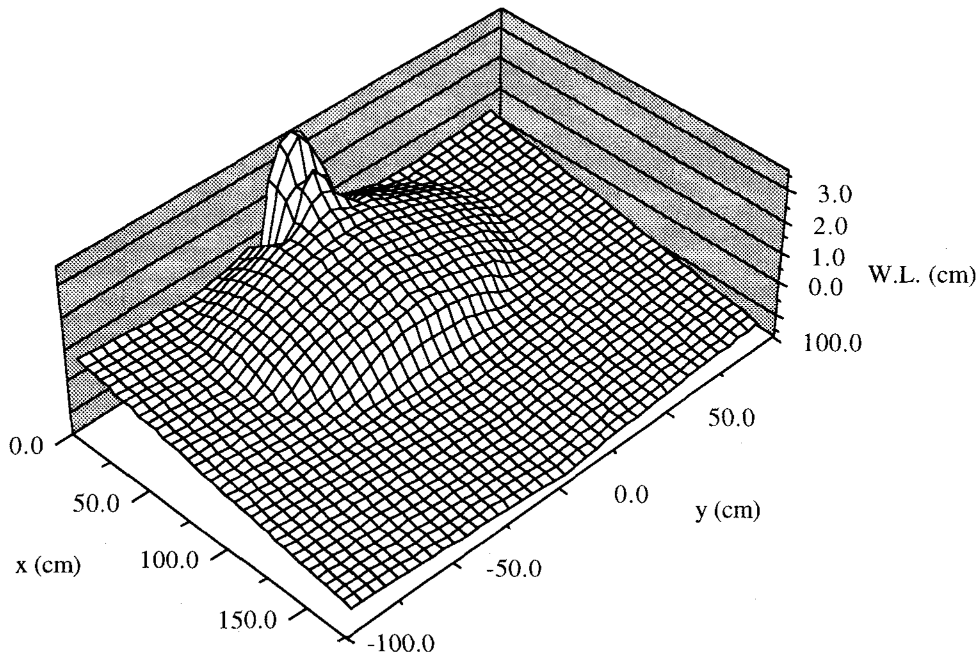
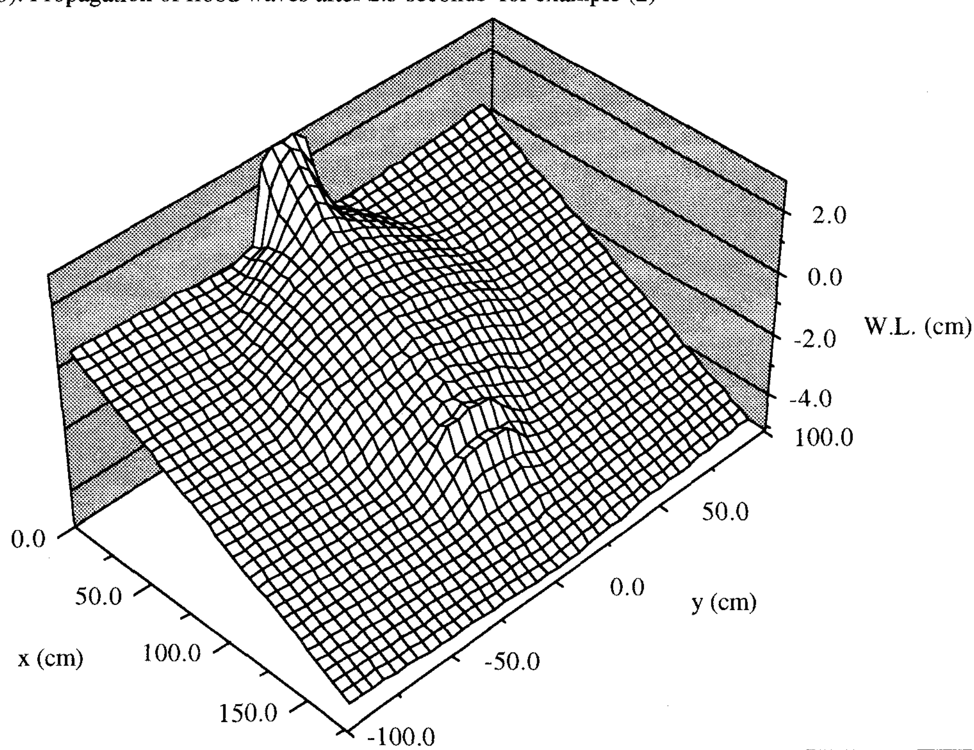


Figure (3). Propagation of flood waves after 2.5 seconds for example (2)



In the second example, slope of flood plain in x direction has been increased to 3 degrees. The perspective of flood has been showed after 2.5 seconds in figure (3). In this example flow condition stays supercritical in x direction while in y direction due to inverse slope it is subcritical.

The oscillation of flow variables by nonlinear instability has been dissipated by the artificial viscosity successfully. The ratio of the depth in the flood plain to the depth of the steep channel reduced to 0.01 which the MacCormack scheme failed for this ratio less than 0.25 and the Gabutti Scheme for less than 0.2 (Fennema and Chaudhry 1990). In the examples nonlinear instability of discontinuous flow increases by using discontinuous initial values of velocity and depth instead of the only discontinuous depth in typical dam breach. Comparison between figures 2 and 3 shows effect of steeper flood plain on shape of flood waves. In figure (2), hydraulic jump exists in the end of steep channel. Since flow condition stays supercritical in figure (3), there is no hydraulic jump. These examples demonstrate ability of the scheme in the simulation of propagation of flood waves.

6-Conclusions

A two-dimensional model is introduced for simulation of wave propagation of flood flows. The model has implicit finite difference scheme and uses ADI algorithm to reduce number of equations in system of equations. The artificial viscosity is suggested to dissipate the oscillation of flow variables due to nonlinear instability. It can control the oscillation of flow variables in shock area successfully without disturbing other parts. The scheme has ability of shock capturing of two-dimensional discontinuous flow as demonstrated by the examples. The Model has been applied to the simulation of wave propagation of flood flow in the flood plain with inclination in x and y directions. The ratio of tail water to upstream head reduced to 0.01 while the MacCormack scheme can not work for the ratio less than 0.25 and the Gabutti Scheme for less than 0.2. Ability of shock capturing of the model has been examined when both initial velocity and initial depth are discontinuous. In the examples, instability has been controlled successfully. The flow conditions and the propagated waves of flood have been illustrated.

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