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A New Numerical Model for Dam-Break Problem

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A newly developed numerical model, based on the fully conservative flux splitting technique, by Jha et al. is further investigated. Dam-break flood waves in rectangular, trapezoidal and suddenly expanding channels are simulated to demonstrate model's ability to handle natural channel geometry. The model's response to the variations in the bed roughness and the time weighting factor is also examined.

Keywords: Finite difference, Conservative splitflux, Dam-break

1. INTRODUCTION

The governing equations for dam-break flood wave propagation in a channel are generally solved by a mathematical model and, therefore, many mathematical models[1)] exist for this purpose. However, there still does not exist a model which can be claimed as giving the best result in all possible situations.

Jha et al.[2)] developed a new model based on a one-parameter finite difference scheme. This model utilizes the concept of approximate Jacobian[3)] which facilitates fully conservative splitting of the flux vector. The superiority of the Jha et al's model[2)] over the MacCormack[4)] and the Gabutti[5)] schemes for dam-break flood wave and reflected shock wave propagation in horizontal and frictionless rectangular channels was established.

This paper continues the further investigation of the Jha et al's[2)] model and its application to the dam-break problem in rectangular and trapezoidal channels as well as in a expanding channel. The behaviour of the new model with varying Manning's coefficient and the time weighting factor is also investigated.

2. GOVERNING EQUATIONS

The governing equations for unsteady open channel flow can be expressed in vector form as

$$\frac{\partial U}{\partial t} + M\frac{\partial U}{\partial x} + S = 0 \quad \text{.......} (1)$$

where

$$U = \begin{bmatrix} A \\ Q \end{bmatrix} \; ; \; M = \begin{bmatrix} 0 & 1 \\ \frac{-Q^2}{A^2} + \frac{gA}{B} & \frac{2Q}{A} \end{bmatrix} \; ; \; S = \begin{bmatrix} 0 \\ -gI - gA(S_o - S_f) \end{bmatrix} \quad \text{.........}(2)$$

wherein A = cross-sectional area; Q = discharge; B = top width of flow at height h from channel bottom; g = acceleration due to gravity; S_0= bed slope; S_f= friction slope; x = distance along the channel; and t = time.

The friction slope, S_f, is computed by the Manning's formula. I is the force exerted by channel walls due to expansions and contractions and is given by

$$I = \int_0^h (h - \eta)\frac{\partial W(\eta)}{\partial x} d\eta \quad \text{..........} (3); \quad W(\eta) = \frac{\partial A}{\partial \eta} \quad \text{..........} (4)$$

where $W(\eta)$ is the channel width at distance η from the channel bottom. The basic assumptions behind the governing equations are: (1) water is incompressible, (2) pressure is hydrostatic, (3) bottom slope of the channel is sufficiently small, and (4) geostrophic effects and wind stresses are negligible. Since the governing

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equations are hyperbolic, **M** can be diagonalized and split into positive and negative components. Eq.1 can, thus, be written as

$$\frac{\partial \mathbf{U}}{\partial t} + \mathbf{M}^{+}.\frac{\partial \mathbf{U}}{\partial x} + \mathbf{M}^{-}.\frac{\partial \mathbf{U}}{\partial x} + \mathbf{S} = 0 \quad \text{...} (5)$$

The space derivative associated with the positive component of the Jacobian matrix represents the information carried along the positive characteristic coming from upstream of the flow and it may be approximated by a backward space difference. Likewise, the space derivative associated with the negative component of **M** may be evaluated by a forward space difference. Therefore, the scheme based on Eq.5 will appropriately handle the directional property of signal propagation. However, Eq.5 is not in the conservation form. The correction necessary to retain the conservative properties while using Eq.5 is explained next.

3. CONSERVATIVE SPLITTING

For Euler equations Roe[3] developed a technique for constructing approximate Jacobian ensuring conservation. The idea was subsequently applied to shallow water equations by Glaister[6], Alcrudo et al.[7] and Jha et al[2]. Roe's technique uses mean value theorem. Following Roe's approach an approximate Jacobian of flux is constructed for every pair of adjacent nodes which satisfies conservative properties and is consistent with the governing equations. The details of the approximate Jacobian may be referred to the paper by Roe[3] or by Jha et al.[2]. We utilise this concept of approximate Jacobian and re-define Jacobian **M** as

$$\mathbf{M}_i^{+} = \widetilde{\mathbf{M}}_{i-1/2}^{+}\ ,\ \mathbf{M}_i^{-} = \widetilde{\mathbf{M}}_{i+1/2}^{-}\ ,\ \text{and}\ \widetilde{\mathbf{M}}_{i\pm1/2} = \widetilde{\mathbf{M}}(\mathbf{U}_{i\pm1/2}) = \mathbf{M}(\mathbf{U}_i, \mathbf{U}_{i\pm1/2}) \quad \text{...........} (6)$$

where a tilde over **M** indicates the approximate Jacobian. The problem has now reduced to defining the arguments of approximate Jacobian i.e. $\mathbf{U}_{i+1/2}$ and $\mathbf{U}_{i-1/2}$, in Eq.6. These arguments of approximate Jacobian are fully defined if depths and velocities at points (i-1/2) and (i+1/2) are known. The velocities and depths at points i+1/2 and i-1/2 are expressed as

$$u_{i\pm1/2} = \frac{\sqrt{Q_i u_i} + \sqrt{Q_{i\pm1} u_{i\pm1}}}{\sqrt{A_i} + \sqrt{A_{i\pm1}}} \quad \text{...........} (7)\ ; \quad h_{i\pm1/2} = \sqrt{h_i h_{i\pm1/2}} \quad \text{................} (8)$$

4. FINITE DIFFERENCE SCHEME

The following one-parameter scheme is used to advance the solution in time.

$$\mathbf{U}_i^{t+1} + \Delta t\theta\left[\mathbf{M}_{i-1/2}^{+t}\frac{\partial \mathbf{U}^{t+1}}{\partial x} + \mathbf{M}_{i+1/2}^{-t}\frac{\partial \mathbf{U}^{t+1}}{\partial x}\right] = \mathbf{U}_i^t + \Delta t(1-\theta)\left[\mathbf{M}_{i-1/2}^{+t}\frac{\partial \mathbf{U}^t}{\partial x} + \mathbf{M}_{i+1/2}^{-t}\frac{\partial \mathbf{U}^t}{\partial x}\right] + \Delta t\mathbf{S}_i^t .. (9)$$

where superscripts t and t+1 = known and higher time level respectively; subscript i = the grid location; Δt = the time step; and θ = time weighting factor. We make use of the following operators

$$\Delta_x f_i = f_{i+1} - f_i\ ;\quad \nabla_x f_i = f_i - f_{i-1} \quad \text{...} (10)$$

and define $\alpha = \Delta x/\Delta t$, where Δx = grid interval. Thus, the complete finite difference equation is obtained as

$$\mathbf{U}_i^{t+1} + \alpha\theta\left\{\mathbf{M}_{i-1/2}^{+}\left[\nabla_x \mathbf{U}_i^{t+1}\right] + \mathbf{M}_{i+1/2}^{+}\left[\Delta_x \mathbf{U}_i^{t+1}\right]\right\}$$

$$\mathbf{U}_i^t + \alpha(1-\theta)\left\{\mathbf{M}_{i-1/2}^{+}\nabla_x\left[\mathbf{U}_i^t\right] + \mathbf{M}_{i+1/2}^{-}\Delta_x\left[\mathbf{U}_i^t\right]\right\} + \mathbf{S}_i^t \quad \text{......} (11)$$

Details of the finite difference approximations may be referred to Jha et al[2]. Similar sets of finite difference equations for all nodes along a channel can be arranged in the form of a block tri-diagonal matrix.

5. MODEL APPLICATIONS AND RESULTS

The model described above was applied to dam-break problems under many different conditions. The dam located in the middle of a 2000m long channel is removed instantaneously to simulate sudden dam failure. The results are presented in the form of the water surface profile along the channel. Courant number = 1, Δx = 5m and θ = 0.6 are used unless specified otherwise. The ratio of tailwater depth, h_t, to the water depth in the reservoir, h_r, is hereafter referred to as the depth ratio. The mass balance error is of the order of 10^{-2} to 10^{-4}%.

<u>Example 1. Rectangular, Horizontal and Frictionless Channel;</u> Dam-break flood wave propagation in a rectangular, horizontal and frictionless channel is considered to compare results of the present model with the analytical solution[8]. Initially h_r is 5m and h_t is 0.5m giving a depth ratio of 0.1. The computed and analytical water surface profiles at (40 + Δt) seconds are shown in Fig.1. It is seen that the proposed model accurately simulates the dam-break flood wave propagation in this case.

Example 2. Sloping Rectangular Channel with Bed Friction: This example considers the effect of bed friction and the channel bed slope. The channel has a uniform bed slope of 1/1000. Manning's coefficient is specified as 0.025. The initial condition in the flood plain is given as the uniform flow of 2.49 m^3/s per unit width of the channel and that in the reservoir portion is a horizontal water surface with a depth equal to 15m at the dam site. The upstream and downstream boundaries are specified with an inflow and outflow, respectively, of 2.49 $m^3/s/m$. The water surface profile along the channel at $(30 + \Delta t)$ seconds following the dam collapse is shown in Fig. 2. The model yields appropriate shape of the water surface profile.

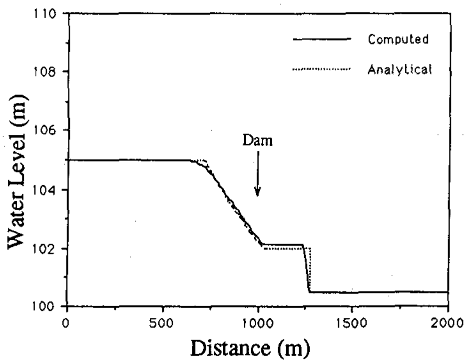

Fig.1 Water Surface Profile Along a Rectangular, Horizontal and Frictionless Channel.

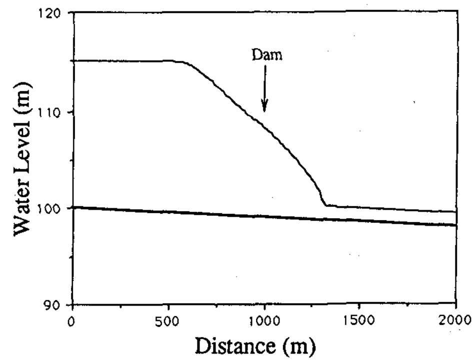

Fig.2 Water Surface Profile Along a Rectangular Channel with Bed Slope and Bed Friction.

Example 3. Trapezoidal Channel: This example considers the propagation of dam-break flood wave in a trapezoidal channel. The channel is horizontal with the Manning's roughness coefficient equal to 0.015. Base width of the channel is 5m and the side slope is 1V:1.5H. The reservoir has 10m deep still water and the stationary tailwater depth is 5m giving the depth ratio equal to 0.5. The water surface profile along the channel at $(50 + \Delta t)$ seconds is shown in Fig.3. The identical initial condition in a horizontal and frictionless rectangular channel would give the front depth (constant depth region) as 7.27m and the location of front at 1468m at time $(50 + \Delta t)$ seconds. When the result shown in Fig.5 is compared with these data, it is obvious that the model gives good result in case of trapezoidal channel.

Example 4. Sudden Expansion in Width: This example considers a sudden expansion downstream of the dam breach section. The channel is rectangular, horizontal and frictionless. The width at 1120m from the upstream end is increased to 1.5 times the width before that section. The initial depth in the reservoir is 15m and the tailwater depth is 1.5m giving a depth ratio of 0.1. The time weighting factor, θ for this example is unity. The water surface profile along the channel at $(40 + \Delta t)$ seconds is shown in Fig.4. When the flow enters the

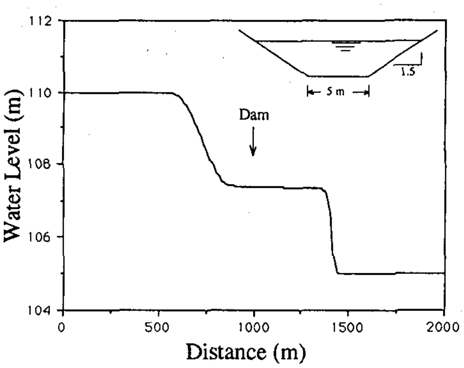

Fig.3 Water Surface Profile Along a Horizontal Trapezoidal Channel With Bed Friction.

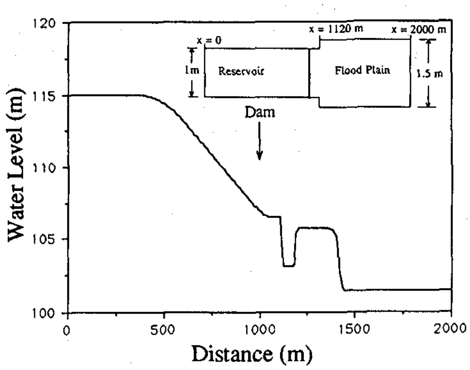

Fig.4 Water Surface Profile Along a Rectangular Channel with Sudden Expansion in Width.

expanded width portion, the depth falls suddenly and the flow becomes supercritical. This supercritical flow continues for some distance before a jump is formed and the flow changes to subcritical. Although the quantitative accuracy could not be verified, this example demonstrates the proposed model's capability in handling the sudden changes in the channel width.

Example 5. The Model's Response to Manning's Coefficient : The response of the model to the changes in the Manning's coefficient is examined for the dam-break problem with a depth ratio of 0.1. The channel is rectangular and horizontal with a reservoir water depth equal to 5m. The water surface profiles for Manning's coefficient equal to 0.0, 0.015 and 0.03 at (40 + Δt) are shown in Fig.5. The negative wave is only slightly affected by the changes in the Manning's coefficient because of the higher depth in this region. The wave front is retarded as the Manning's coefficient is increased and, consequently, depths increase. This figure indicates that the model reasonably responds to the changes in the bed roughness.

Example 6. The Model's Response to Time Weighting Factor : The behaviour of the model with different values of θ is examined next. The channel is rectangular, horizontal and frictionless. The water depth in the reservoir is 5m and the depth ratio is 0.1. The model is run with θ equal to 0.1, 0.6 and 0.9. For θ= 0.1, the model is run at Courant number 0.95 to ensure stability. The computed water surface profiles at (40 + Δt) seconds are presented in Fig.6 along with the analytical solution. The accuracy of the model increases as the time weighting factor is reduced. However, the difference in accuracy is not very significant. It may be preferred to keep the model implicit while maintaining reasonable accuracy. Therefore, the time weighting factor equal to 0.6 is recommended.

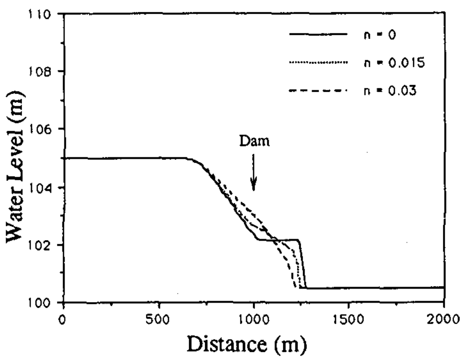

Fig.5 Effect of Manning's n on the Water Surface Profile.

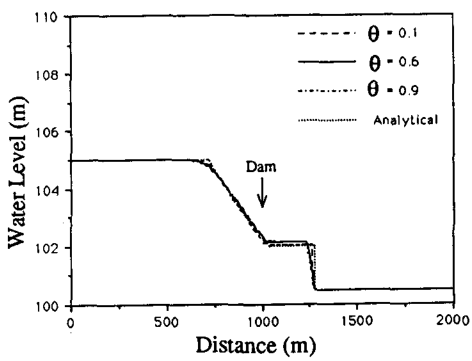

Fig.6 Effect of Time Weighting Factor on the Water Surface Profile.

6. CONCLUSIONS

A new numerical model developed by Jha et al.[2] has been further investigated. The model was applied to a variety of dam-break problems in different channel geometry, bed friction and bed slope conditions. Dam-break flood wave propagation in suddenly expanding channel was also simulated by the model. The proposed model's applicability to all these cases has been demonstrated. The model's response to the changes in the Manning's roughness coefficient was found to be quite reasonable. The effect of the time weighting factor on the model performance was analysed and a value of 0.6 was recommended. The model gives good results for all the cases considered. It can be concluded that the present model can handle the real world problems related to the propagation of dam-break flood wave in an open channel.

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