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Numerical Simulation of Hillslope Infiltration and Discharge into River

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Two-dimensional numerical simulations are carried out to simulate unsaturated and saturated flow in a hillslope and the discharge process from hillslope into river. The soil moisture movement characteristics within hillslope is analyzed in terms of flow pattern and flux distribution. Effects of initial conditions on discharge process are investigated. Based on the numerical simulation results, a simplified hillslope process model is proposed and checked with detailed numerical simulation, which can be used to simulate the water flow within hillslope and interaction with river.

Keywords: infiltration process , hillslope, numerical simulation, Richards' equation, unsaturated-saturated flow, soil moisture, process model

1.INTRODUCTION

In recent decades, much attention has been paid to hillslope hydrology (Philip, 1991). In addition to overland flow, precipitation may also be delivered to river by subsurface flow through the near-surface soil, or deeper ground water flow. Knowledge of the flow process within the hillslope is important not only in its own right, but also because based on the understanding of the flow mechanism, the hillslope hydrology modelling can be simplified so that catchment models can be practical and computable. In most of the previous studies, hillslope was described as a parallel layer of some thickness which overlays on some impermeable bed rock (Ohta, 1983). However in some cases, a hillslope as described in Figure 1 is more realistic where the interaction between unsaturated soil moisture and saturated ground water can take place. Soil moisture movement in a similar hillslope had been investigated by Jackson et al.(1992) which emphasized on near-surface flow. They found parallel flow to surface after the cessation of rainfall, and mainly vertical flow during rainfall. However, they carried out numerical simulation with fixed ground water table and no flow at side boundaries. From a practical point of view, it is important to investigate the soil moisture movement under the interaction of ground water with river flow.

In the present study, flow process within the hillslopes as shown in Figure 1 was simulated, and the discharge from hillslope to river under various initial conditions were investigated. A relative large slope with B being 50.0 meter was adopted to make the simulation more realistic. Simulation results were analyzed in terms of soil moisture movement pattern and flux distribution. The initial condition effect to the simulation results was also studied. Based on the results, a simplified model was proposed, which was used to simulate the soil moisture flux distribution within the hillslope, and discharge into the river. The model computations were compared with detailed numerical simulations.

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2. NUMERICAL SIMULATION DESCRIPTION

To simulate the saturated and unsaturated subsurface flow within a hillslope, the water pressure h-based two-dimensional Richards' equation was adopted and written as,

$$c(h)\frac{\partial h}{\partial t} = k(h)\frac{\partial h}{\partial x} + \frac{\partial}{\partial z}[k(h)\frac{\partial h}{\partial z} + k(h)] \quad (1)$$

where h is water pressure. t is time. x, z are coordinates. $c(h) = \partial\theta/\partial h$ is specific water capacity, k is hydraulic conductivity.

Finite element method was employed to carry out numerical simulation of the equation. The simulation domain and finite element discretization is shown in Figure 1. Only homogeneous soil condition was considered and here mainly Kanto Loam was studied. The soil hydraulic parameters are described using the following functions.

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{[1 + (\alpha|h|)^n]^m} \quad (2)$$

where θ is moisture content, θ_s is saturated moisture and θ_r is residual moisture.

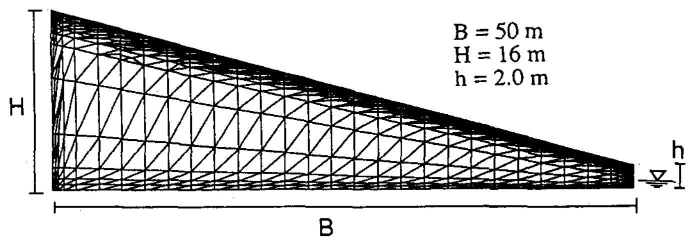

Figure 1: Hillslope flow region; discretization; dimensions

$$k(h) = k_s\frac{\eta}{\eta + |h|^\gamma} \quad (3)$$

where k_s is saturated conductivity. η and γ are parameters. In the two equations parameters α, n, m , η, γ,have the value of 0.09, 1.9, 0.5, 156.0, 2.34 respectively.

The left-side boundary is taken as no-flow. The right-side is also a no-flow boundary except that below the water surface, where the ground water table is fixed to simulate a constant water level river interaction. The bottom is a no-flow boundary simulating an aquiclude. The top boundary receives a constant, nonzero flux as rainfall for a specific period of time followed by zero flux.

The following 4 kinds of initial conditions were used in the numerical simulation. Initial condition (d) was suggested by Wood (1992).

a. Static steady-state condition

Ground water level is kept constant at 0.5 meter above the aquiclude. For each point in the unsaturated zone, the elevation difference between the point and the water table is assigned as the pressure head. This ensures that total hydraulic potential is spatially constant and there is no water flow within the hillslope.

b. Dynamic steady-state condition

With a small flux (0.0000025cm/sec was used in this study) as input at the upper boundary, simulation is carried out for a long period until a relatively stable condition is reached. Then the result is used as initial condition for further simulation.

c. Constant-head condition

By assigning a constant water head (-200.0cm was used in this study) to the slope surface, the calculation is carried out for a long period until a relatively stable condition is reached. Then the result is used as initial condition for further simulation.

d. 1/8 h condition

In this case at each point the water head is set equal to a fraction, here 1/8, of the elevation difference of the point above the datum (water table). There is a vertical flow over the hillslope.

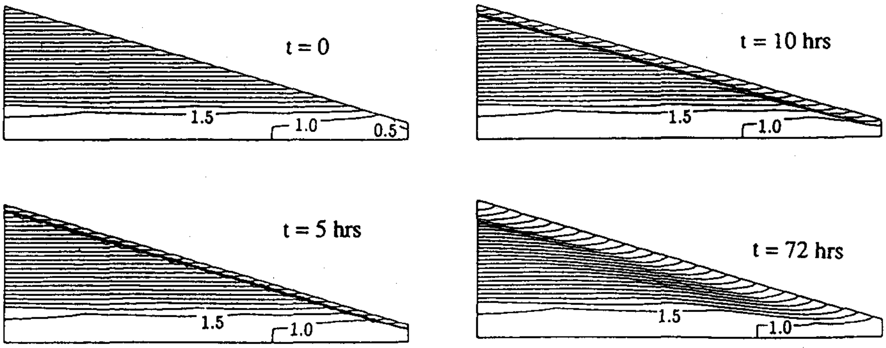

Figure 2: Total Potential Distribution at Different Time (contour interval is 0.5m)

3. RESULTS AND DISCUSSION

In the simulation rainfall intensity was taken as 0.00025cm/sec with a duration of 5 hours which resulted in a 45mm rainfall, followed by about 5 days no rain.

3.1 Soil Moisture Movement

Figure 2 shows the total potential profiles at 0 hour, 5 hour, 10 hour, 72 hour etc., representing soil moisture movement. Here 0 hour denotes the beginning of the uniform intensity rainfall which is of 5 hour duration. For the result shown, dynamic steady state initial condition (b) was used. It can be seen that during the rainfall, infiltration is mainly vertical and the wetting front is almost parallel to the hillslope surface. Later the infiltrated rainfall moves further downwards and reaches ground water. When rain ceases, the total hydraulic potential contours at surface become perpendicular to the hillslope surface. This occurs because the surface has become a no-flow boundary (without considering evaporation), so flow adjacent to this boundary is forced to move nearly parallel to the boundary. Therefore lateral downslope flow in a uniform hillslope happens mainly after rainfall ceases. This observation is true for all the initial conditions and other types of soils used in this study.

3.2 Flux Distribution

Figure 3, originally used by Philip[1991], shows the coordinates systems and the variable definitions that were used in his solution of hillslope infiltration. H, horizontal in slope; V, vertical; P, parallel to slope; N, normal to slope are the components of soil moisture flux, with the positive directions following the arrows in the drawing.

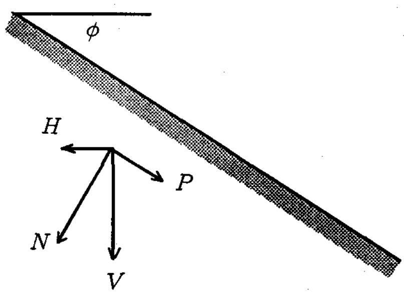

Figure 3: Schematic Figure Illustrating Various Flow Velocity Components

As mentioned by Jackson[1992], a simpler way to determine the occurrence of lateral down slope flow is when H, the horizontal flux component into slope, is negative. In this case the resultant flux will take place towards the lower part of the slope. In the present study, the flux distribution within hillslope was analyzed in terms of vertical component V, horizontal component H, component parallel to hillslope surface P, and component normal to slope N. The flux variation in vertical section of the slope was investigated and one result is shown in Figure 4, which corresponds to flux at section $x = 6.5m$. It can be seen that vertical component is of the same order of magnitude with total flux, while the parallel component and the horizontal one are smaller, especially the horizontal component. Drastic change of flux takes place mainly within the top soil layer, about 2.0 meter below the slope surface, for the cases considered.

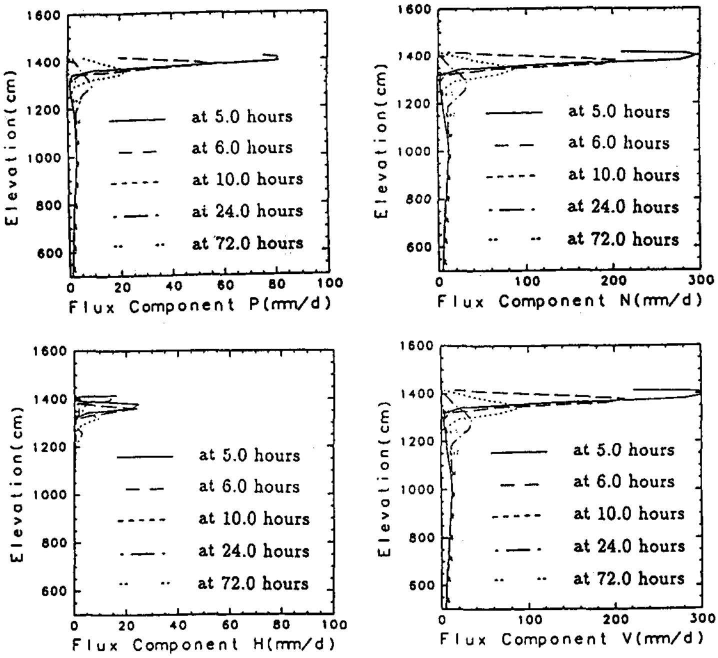

Figure 4: Flux Components Variation in Vertical Section

The relation between the flux along slope P and the conductivity k was examined for each element above ground water level at a few vertical sections. From the results, a linear relation could be written as, $P \propto k \times \sin\phi$, where ϕ is slope angle. Horizontal flux component near surface is negative, meaning that downslope flux takes place there. In the top surface layer, along slope, no great variation of soil moisture movement could be found.

3.3 Effect of Initial Condition

Observed data of hillslope soil water conditions are rarely available while effects of any initial soil water status persist for a considerable period. If the simulation is carried out for a quite long period, the effect of initial condition may be eliminated. However in relation to event-based short time simulation, choosing proper initial conditions is important. To investigate the effect of initial condition, simulations are carried with the 4 different initial conditions . Although the initial conditions do not change the flow pattern of the rainfall infiltration and redistribution, they do affect the discharge from the slope into river. The peak value of discharge into river and its accumulation during simulation period is shown in table 1.

Flow characteristics (slope thickness = 1cm)	initial conditions			
	(a)	(b)	(c)	(d)
peak of discharge(l/sec)	0.606E-3	0.234E-2	0.932E-3	0.769E-2
time to peak (Hours)	22.0	17.0	23.0	6.0
discharge accumulation (l)	149.2	783.7	302.1	1699.0
Note: Rainfall:45mm, 5 hour duration, Simulation period:5days				

Table 1: Discharge characteristics for different initial conditions

It is difficult to say which kind of initial condition is better. It depends on the state of the catchment prior to the event. Initial condition (a), static steady-state condition, is seldom used and hardly occurred in real catchments.

Initial condition (b) may be appropriate for humid areas where an almost uniform input to slope can be assumed. While initial condition (c) is reasonable when simulating a condition after a long time without rainfall. Then a water pressure corresponding to an average top soil pressure can be assigned to surface boundary. With initial condition (d), more water has been stored in slope prior to rainfall, therefore the time to peak is short and the discharge accumulation is large.

3.4 Simplified Model

Based on the simulation results, a simplified process model to simulate the water flow in a hillslope and interaction with river can be constructed considering the dominant flow regimes.

(1). Discretization of the Regime

Considering the different flux components and their relative magnitude and variation of flux within the slope, the flow regime can be divided into three parts. One is near-surface zone. One is deeper-subsurface zone. The third is ground water flow regime.

For numerical simulation, first, slope was divided into a number of vertical columns. Each column consisted of near-surface, deeper-subsurface, and ground water flow component. And then, the near-surface and deeper-subsurface zone were further divided into blocks by a number of horizontal layers.

In near-surface zone, during rainfall only vertical flow was considered. Down slope flow and vertical flow were considered after rainfall ceased.

In deeper-subsurface zone, only vertical flow was considered.

Ground water was recharged from the unsaturated zone. Unconfined ground water flow was simulated by the flow transfer between neighboring block's ground water component.

(2). Equations for Each $Block(i,j)$ at Different Zones

a. Near-surface zone

Parallel and vertical flux components out of block were calculated as,

$$P\_out(i,j) = \beta * (k(S) * \sin\phi) \tag{4}$$

$$V\_out(i,j) = k(S) \tag{5}$$

where $k(S)$ is averaged conductivity for the simulation time step(Δt) and is calculated from block storage S. Parameter β was taken as a averaging parameter for different domain discretizations. Sensitivity analysis of β showed that a value of 1.0 is adequate, and was adopted in this study.

Then storage increase was calculated as,

$$\Delta S(i,j)/\Delta t = (P\_in - P\_out) * \Delta y * \cos\phi + (V\_in - V\_out) * \Delta x \tag{6}$$

b. Deeper-subsurface zone

Vertical flow and storage change were calculated as,

$$V\_out(i,j) = k(S) \tag{7}$$

$$\Delta S(i,j)/\Delta t = (V\_in - V\_out) * \Delta x \tag{8}$$

Since $k(S)$ is nonlinear, an iteration scheme was used to solve equation 4 to 8. The outflow from lowest block becomes the recharge to ground water.

c. Ground water flow

Storage change was calculated as,

$$\Delta S(i,j)/\Delta t = G\_in - G\_out + Rech * \Delta x \tag{9}$$

And then ground water rise (ΔH)was calculated by using the specific yield Sy,

$$\Delta H(i,j) = \Delta S(i,j)/Sy \tag{10}$$

As a validation of the simplified model, comparison of discharge process into river between detail numerical simulation and the result of the simplified model is shown in figure 5. The time step is 1.0 hour and block size is

$0.5m \times 0.5m$. A good agreement can be seen. However with the simplified process model, the computation time takes only about 1 percent of that for detailed numerical simulation.

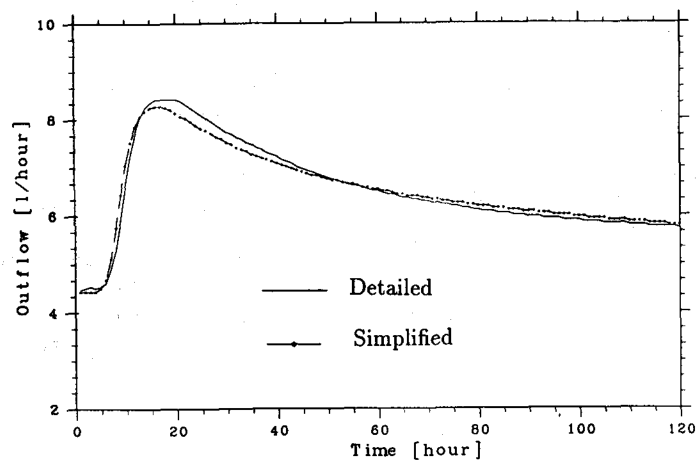

Figure 5: Discharge into River from Slope

4. CONCLUSION

Two-dimensional numerical calculations to simulate the unsaturated-saturated water flow in a hillslope and discharge from hillslope into river were carried out. In the near-surface zone, during rainfall, precipitation seeps into slope in vertical direction and down slope flux occurred after rainfall ceased. This was consistent with the conclusion of previous research (Jackson, 1992). Flux variation in vertical sections along slope shows that down slope flux took place mainly in the near-surface soil layer. Effects of initial conditions on discharge from slope into river were investigated. It is important to choose proper initial condition when discharge process is under consideration, especially for flood forecasting. Based on the simulation results, a simplified process model to simulate the water flow in hillslope and interaction with river was proposed and verified with detailed numerical simulation results. The main advantage of the simplified model is the high computational efficiency, which has significant impact on large-scale simulation. Model provided good results for two types of soils under various initial conditions analyzed in the present study.

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