

## A Modelling of Moisture and Heat Transfers in SPAC

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A dynamic mathematical model is proposed to simulate the moisture and energy fluxes in a plant ecosystem. Hourly and daily processes of five important variables---leaf temperature, leaf moisture potential, air temperature and humidity within the canopy, and temperature on soil surface---can be gained by using the model. Temperature difference between plant leaves and surrounding air is paid attention, and difficultly-measured plant potential or temperature could be derived from normal meteorological data.

Keyword: plant-ecosystem, moisture, energy

### I. INTRODUCTION

Geographers, ecologists, meteorologists and hydrologists are paying a great deal of attention to mathematical modelling of soil-plant-atmosphere systems and its application in many fields such as agriculture, forestry, watershed planning and global climate change(Kienitz, Milly, et al., 1991). Many methods were taken from thermodynamics, aerodynamics, plant physiology and ecology to construct a hydrologic model( Goudrian, 1977; Monteith, 1975; Lange Kappen, Schulze, 1976). The plant and air within the canopy are usually combined into a single layer in most simple models( the large-leaf model), with no or little consideration of the difference between plant temperature and air temperature in the canopy( Stannard, 1993). However, canopy air's temperature and humidity do differ from those of plant leaves, and they need to be distinguished. In recent years much efforts have been made to solve this problem, and some complex models were set up, such as the SIB model (simple biosphere model). Unfortunately these complex models are hardly used to areas with poor data sets, especially in developing countries. So in this study we try to propose a relatively simple model that accounts for the difference between plant and air in canopy and apply it to an agricultural field in China.

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## II. MODEL STRUCTURE

A plant ecosystem is divided into four layers: air just above canopy, leaves and branches, air within canopy, and top soil (Fig. 1). It is supposed that all physical or physiological properties be horizontally homogeneous in each layer. The model structure is largely consisted of five system state equations: energy balances in leaf layer and in surface soil, water flux in plant body, heat and vapour fluxes in canopy air layer. These equations describe energy and moisture transfers between those layers, as the vertical interaction in SPAC (soil-plant-atmosphere continuum) has being mostly emphasized ( Bolle, 1993). Dynamical processes of five state variables (leaf temperature, leaf moisture potential, air temperature and humidity in canopy, and soil temperature) are then resolved out.

This study made an effort to differentiate plant and plant air, and the resulted model may give five key system variables rather than three ones as in other models (Monteith; Goudrian; Yao).

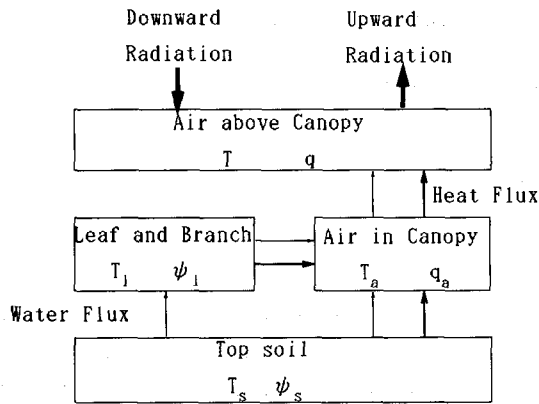


Fig. 1 The system's layer-division in model

### 1. Energy Balance of Leaf Layer

In general, the heat or energy absorbed and stored by crop and soil holds a small part of net radiation energy. For the sake of clarification and simplification, it is assumed that, after reflected by plants and soil, solar and atmospheric radiations that go into the canopy are mainly used by sensible and latent heat transferring, e.g. assuming the system is at its steady state,

$$R_{cs} + R_{cl} = \rho c_p L D_{lat} (T_l - T_a) + \lambda \rho L D_{laq} (q_l - q_a) \quad (1)$$

where  $R_{cs}$  and  $R_{cl}$  ( $J/s \cdot m^2$ ) are short-wave and long-wave net radiation in the canopy;  $\rho$  ( $kg/m^3$ ) and  $c_p$  ( $J/kg \cdot ^\circ C$ ) are air density and specific heat;  $L$  is leaf area index (which is defined as the ratio of total area of leaf surfaces to ground area);  $T_l$ ,  $T_a$  ( $^\circ C$ ) are leaf temperature and canopy air temperature;  $q_l$  and  $q_a$  are specific air humidity on leaf and in the canopy;  $\lambda$  ( $J/kg$ ) is latent heat of water vaporization; and  $D_{lat}$  and  $D_{laq}$  ( $m/s$ ) are heat and vapour transferring coefficients between plant leaves and canopy air layer, respectively.

The net short-wave radiation is expressed as

$$R_{cs}=S_i(1-A_{is}-A_s) \quad (2)$$

in which  $S_i$  is downward solar radiation,  $A_{is}$  is the combined reflection coefficient of canopy and soil, and  $A_s$  is soil's radiation-absorption coefficient.

Let  $T$  be the air temperature above the canopy,  $T_s$  the top soil temperature, then net long-wave radiation can be written as (Yao, 1988)

$$R_{cl}=\epsilon_a\sigma T^4(1-e^{-kL})+\epsilon_s\sigma T_s^4(1-e^{-kL})-2\epsilon_l\sigma T_l^4(1-e^{-kL})/k \quad (3)$$

where  $k$  is plant's radiation-weakening coefficient, and  $\epsilon_a$ ,  $\epsilon_s$ ,  $\epsilon_l$ , and  $\sigma$  are coefficients related to body's emission feature.

## 2. Energy Balance in top Soil

Let  $R_{ss}$  to be net short-wave radiation on soil surface, and  $R_{sl}$  to be net long-wave radiation (from air, plants, and soil), and they are also mainly used to transfer heat and vapour between soil and canopy air layers, with the heat storage of soil neglected. The energy balance is something like

$$R_{ss}+R_{sl}=\rho c_p D_{sat}(T_s-T_a)+\lambda \rho D_{saq}(q_s-q_a) \quad (4)$$

in which  $D_{sat}$  and  $D_{saq}$  are heat and vapour transferring coefficients, and  $q_s$  is specific air humidity in soil pores. Units of these variables are the same as in Eq.(1).

The term  $R_{ss}$  is equal to  $A_s S_i$  and  $R_{sl}$  can be expressed as

$$R_{sl}=\epsilon_a\sigma T^4e^{-kL}+\epsilon_l\sigma T_l^4(1-e^{-kL})/k-\epsilon_s\sigma T_s^4 \quad (5)$$

The specific humidity  $q_l$  or  $q_s$  in equation (1) or (4) is dependent on corresponding temperature and water stress, which may be written as (Yao):

$$q_l=a\exp(b\cdot T_l)\exp(c\psi_l) \quad (6a)$$

$$q_s=a\exp(b\cdot T_s)\exp(c\psi_s) \quad (6b)$$

where  $a=0.00253$ , and  $b=0.0775$ . The specific humidity within leaf stomata and soil pores has proved slowly responding when water stress changes rapidly, if water potential falls in the normal range for crop living. For example, most middle-aged plants alter their relative specific humidity from 0.9978 to 0.9781 when water potential changes from  $-3.0$  to  $-30.0$  bars. That means that  $q_l$  is largely controlled by temperature  $T_l$ , and is close to the saturated one. For the case of study area, soil water is around its field capacity because of frequent irrigation, and  $\psi_s$  keeps a light effect on  $q_s$ . Therefore water stress is negligible here.

Vapour transfer coefficient  $D_{laq}$  is related to leaf water potential  $\psi_l$ , which expresses the potential energy of water in leaves and the potential gradient causes water flow.

$$D_{laq}=D_{lat}(D_0+\chi_l\psi_l)/(D_{lat}+D_0+\chi_l\psi_l) \quad (7)$$

where  $1/D_0$  is the minimum stomatal resistance, and  $-D_0/\chi_l$  represents the leaf water potential when stomata close entirely. Here  $D_{laq}$  is only related to water potential, a most sensitive factor. It might be very difficult to include sunshine and temperature, although they also affect leaf stomata action and vapour transfer.

## 3. Water Flux in Plant's Body

The inner path, through which water runs in plants, may be divided into three parts: soil-root stele, stele-leaf epidermis, and epidermis-air. The water flux in the first part, from soil to root stele, may be described

by following formula (Taylor, 1975):

$$Q_r = A(\psi_s - \psi_r) \quad (8)$$

where  $Q_r$  (m/s) is water uptake rate of roots,  $\psi_s$  and  $\psi_r$  are soil's and root stele's water potential (m), and parameter  $A$  (1/s) accounts for effects of root density and flow resistance caused by soil and root bark.

The flux of water (m/s) in the second part, or in the stem, is written as

$$Q_s = B(\psi_r - \psi_l) \quad (9)$$

in which  $B$  (1/s) is correlated with moisture conductivity in stem and with area of stem's cross-section.

The flux in the third part is just the transpiration rate

$$E_t = \rho L D_{laq} (q_l - q_a) / \rho_w \quad (10)$$

$\rho_w$  being liquid water density ( $\text{kg/m}^3$ ).

Similarly soil evaporation rate is expressed as  $E_s = \rho D_{saq} (q_s - q_a) / \rho_w$ .

It is reasonably assumed that water moves steadily and continuously within whole plant body, which means  $Q_r = Q_s = E_t$ , or rewritten as

$$\psi_l = \psi_s - \rho L D_{laq} (A + B) (q_l - q_a) / (\rho_w A B) \quad (11)$$

#### 4. Heat and Vapour Transform between Canopy Air and Above Air

Neglecting heat convected from outer surrounding, because the study farmland is located at a big plain of some 10,000  $\text{km}^2$  and the horizontal features are rather uniform. The main function of the canopy air is to transform heat or vapour coming from leaves and soil, into the air above canopy. Then we have:

$$\rho c_p D_{aat} (T_a - T) = \rho c_p L D_{lat} (T_l - T_a) + \rho c_p D_{sat} (T_s - T_a) \quad (12)$$

$$\rho D_{aaq} (q_a - q) = \rho L D_{laq} (q_l - q_a) + \rho D_{saq} (q_s - q_a) \quad (13)$$

where  $D_{aat}$  and  $D_{aaq}$  are heat and vapour transferring coefficients between air in canopy and air out of canopy.

### III. ITERATIVE SOLUTION

Five equations (1), (4), (11), (12), (13) have been set up, from which the key state variables ( $T_l$ ,  $\psi_l$ ,  $T_a$ ,  $q_a$ ,  $T_s$ ) can be gained. It is difficult, however, to directly solve these equations, because they are implicit and inter-connected. Here an iteration method is constructed to give approximate solutions.

Rewrite those equations in a simple way

$$\psi_l = f1(T_l, T_a, q_a) \quad (14)$$

$$T_s = f2(T_l, T_a, q_a) \quad (15)$$

$$T_l = f3(T_s, T_a, q_a) \quad (16)$$

$$T_a = f4(T_l, T_s) \quad (17)$$

$$q_a = f5(\psi_l, T_l, T_s) \quad (18)$$

here  $f1$ – $f5$  account for function relationships. Iterative process is illustrated in Fig. 2. The final results  $T_l^{(2)}$ ,  $\psi_l^{(2)}$ ,  $T_a^{(2)}$ ,  $q_a^{(2)}$ ,  $T_s^{(2)}$  are solutions needed. The total evapotranspiration rate is simply the sum of leaf transpiration and soil evaporation:  $E_t = E_l + E_s$ . Convergence of iteration is assured since equations (14)–(18) do not have strong nonlinearity and the iteration is a common used algorithm.

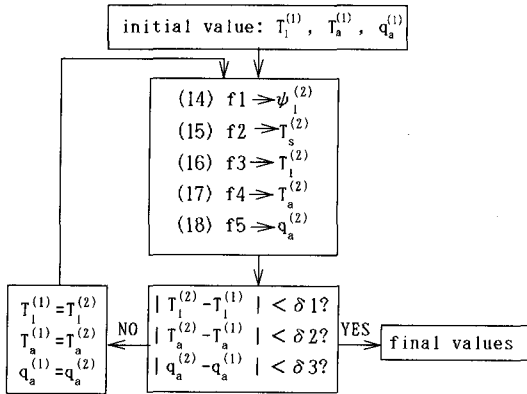


Fig. 2 Illustrative iteration process

Table 1 Simulation results

Item	Unit	Cotton 1984	Cotton 1985	Corn 1984	Corn 1985
T	°C	25.8	25.2	25.8	25.2
q		0.016	0.016	0.016	0.016
T <sub>l</sub>	°C	26.9	26.6	27.4	26.9
T <sub>s</sub>	°C	25.1	25.1	25.6	25.3
ψ <sub>l</sub>	m	-27.1	-19.5	-23.6	-18.7
T <sub>a</sub>	°C	26.3	25.9	26.6	26.1
q <sub>a</sub>		0.017	0.017	0.018	0.018
E <sub>T</sub>	mm	5.13	3.88	5.14	3.98
E <sub>T</sub> <sup>*</sup>	mm	6.15	3.13	4.95	3.63
r	%	16.5	24.2	3.9	9.6

## IV. CASE STUDY

### 1. Site and Data

Cotton and corn communities were selected, located at an irrigation research station, Hebei Province of China. By means of irrigation activity, the water content of soil kept rather steady and always larger than the 60% of field capacity. There were normal measurements about meteorology and crop consumption of water. Simulation period was taken as the August in 1984 and 1985, when the community had its largest leaf area index.

Model inputs included solar radiation, cloud-covering value, sunshine duration, air temperature, air humidity and wind speed, and they were all hourly data obtained from an on-site weather station. Another input item, soil potential(ψ<sub>s</sub>), was given a constant value of -0.404 m(H<sub>2</sub>O).

### 2. Results

The values of model parameters were determined according to site investigation and some laboratory experience. For example, for the cotton case in 1984, most parameters are as following: ρ=1.2047 kg/m<sup>3</sup>, c<sub>p</sub>=1000.0 J/kg·k, λ=2.453x10<sup>6</sup> J/kg, σ=5.67x10<sup>-8</sup> W/m<sup>2</sup>·k, L=3.0, k=0.9, ε<sub>a</sub>=0.847, ε<sub>l</sub>=0.9, ε<sub>s</sub>=1.0, D<sub>0</sub>=0.0033 m/s, χ<sub>a</sub>=1.4x10<sup>-5</sup> s<sup>-1</sup>, ρ<sub>w</sub>=1000.0 kg/m<sup>3</sup>, D<sub>lai</sub>=0.0195 m/s, D<sub>sai</sub>=0.0171 m/s, D<sub>asai</sub>=0.0093 m/s, D<sub>aaq</sub>=0.0093 m/s, A=2.01x10<sup>-7</sup> s<sup>-1</sup>, B=1.767x10<sup>-8</sup> s<sup>-1</sup>. Then hourly dynamics of ψ<sub>l</sub>, T<sub>l</sub>, T<sub>a</sub>, q<sub>a</sub> and T<sub>s</sub> could be calculated with all hourly inputs and above parameters. Evapotranspiration (or water consumption) of the community was computed in turn.

Daily-averaged results are listed in Table 1 for the period August. Hourly dynamics of system state variables is shown in Figure 3 and 4, which are monthly-averaged. Obviously the error (r) of evapotranspiration against measured (E<sub>T</sub><sup>\*</sup>, by using lysimeter and water balance technique) is small.

Direct validation of model performance has not been made because there were no available records of temperature and humidity just within the canopy. Nevertheless this model's validity could be testified to some extent by two facts that the simulated temperature of canopy air is lower than the leaf temperature as seen in most practical circumstances and that the modelled daily evapotranspiration is close to observed.

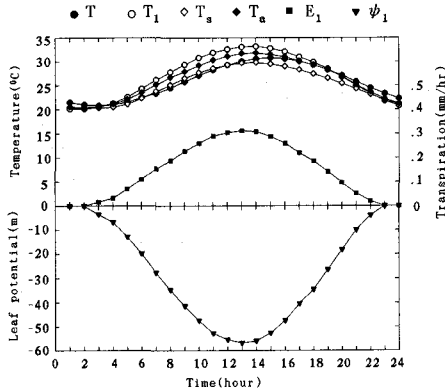


Fig.3 Hourly simulation to cotton in 1984

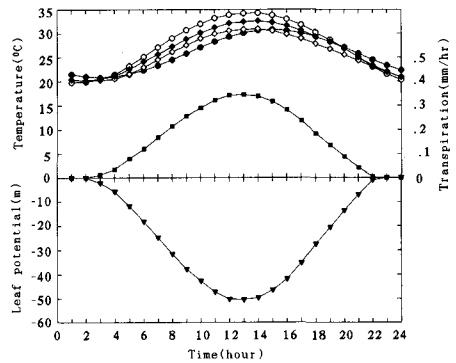


Fig.4 Hourly simulation to corn in 1984

## V. CONCLUSION

The model proposed describes heat and moisture fluxes in a soil-plant-atmosphere system and provides a simulation of hourly process for cotton and corn crops. The temperature difference between leaf, air in canopy and air above canopy is specifically emphasized. According to simulated results, leaf temperature is the highest and soil temperature the lowest. That is because the canopy absorbs most part of net radiation and brings about so-called micrometeorological effect. And a relationship of  $T_l > T_a > T > T_s$  is also frequently observed. The calculated evapotranspiration is comparable to actual water-use. All those six items in Fig.3 and Fig.4 have a reasonable diurnal pattern and a good correspondence.

The proposed model may be applicable to crop ecosystem. There are many problems should be further considered: expanding time and space scale; taking changes of soil moisture; considering heat storage in plant and soil; and connecting with a hydrological model.

## REFERENCES

- Bolle, H.-J.(1993), Scientific goals of the IGBP core project "biospheric aspects of the hydrological cycle", in Exchange Processes at the Land Surface for a Range of Space and Time Scales, IAHS Publ., No.212, pp.3-11.
- Goudrian, J.(1977), Crop micrometeorology: a simulation study, Centre for Agricultural Publishing and Documentation, Netherlands, p.243.
- Kienitz, G., et al.(1991), Hydrological interactions between atmosphere, soil and vegetation, IAHS Publ., No.204, p.494.
- Lange, O.L., et al.(1976), Water and plant life: problems and modern approaches, Springer-Verlag Berlin Heidelberg, pp.1-134.
- Monteith, J.L.(1975), Vegetation and atmosphere, volume I: principles, Academic Press, London, p.342.
- Stannard, D.I.(1993), Comparison of Penman-Monteith, Shuttleworth-Wallace, and modified Priestly-Taylor evapotranspiration models for wildland vegetation in semiarid rangeland, Water Resources Research, Vol.29, No.5, pp.1379-1392.
- Taylor, H.M., and Klepper, B.(1975), Water uptake by cotton root systems: an examination of assumption in the single root model, Soil Science, Vol.120, No.1.
- Yao, H.(1988), Modelling the dynamic heat-moisture-states of soil-plant-atmosphere systems, J. Wuhan University of Hydraulic and Electric Engineering, No.5, pp.19-26.