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Scale Effects of Basin Elements on Coefficients in Runoff-Erosion Modeling

by

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Abstract

Scale effects of basin elements on coefficients in runoff-erosion modeling are discussed by dividing a small test field with the area of 5200 m^2 in Brazil into elements in three ways: 4, 10 and 23 elements. Standardized Powell method is applied for the optimization of parameters in the model. Simulation results of runoff, sediment yield and the coefficients for the three ways of basin division proved that in this particular case the precise division of the river basin into small elements did not always give better result. Although the 10 elements division gave the best results, even simulation results by only 4 elements division were fairly good.
Keywords : scale of basin element, runoff-erosion

1. Introduction

Many types of sediment yield models have been discussed with the mathematical formulation of the pertinent processes [1, 2]. However, there are many parameters to be determined for each river basin in any runoff-erosion model. Runoff and sediment yield data obtained in an experimental basin installed near Sumé in the semi-arid region of Paraíba in the north-east of Brazil are simulated with a numerical model for soil erosion and sediment transport which is summarized by Lopes [3]. The model is based on the kinematic wave assumption for both overland and channel flows, and has four major parameters to be determined. They are the channel erosion parameter, the soil detachability factor, the sediment entrainment parameter by rainfall impact and the initial moisture-tension parameter, in which the former three are constant for the specific place under consideration and the last one depends largely on the preceding rainfall. Conjugate direction method proposed by Powell [4] is used for the determination of these four parameters. In the runoff-erosion model, river basin and channel should be divided into several elements based on soil, slope and surface cover characteristics. Increase of total element number may give more accurate results, but labour in the simulation procedure becomes much bigger. The experimental basin is divided in three ways: 4, 10 and 23 elements, for each of which runoff-erosion analysis is done by the model and the effects of the element scale on the simulation results are revealed.

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2. Method of Simulation by Runoff-Erosion Model with SP Method

Erosion of land surface soil and sediment transport are caused by overland and channel flows. These flows are analyzed based on the kinematic wave approximation. The details of the runoff-erosion model used here are shown in the previous paper [5]. There are many parameters to be given in the model, some of which may be assumed to be universal. However, parameters such as moisture-tension parameter and saturated hydraulic conductivity should be given for the soil conditions of the specific basin under consideration. Major parameters to be determined in the runoff-erosion model (WESP (Watershed Erosion Simulation Program) by Lopes [3]) are N_s, a, K_L and K_I, where N_s is the moisture-tension parameter, a is the channel erosion parameter, K_L is the soil detachability factor and K_I is sediment entrainment parameter by rainfall impact. The parameter N_s depends on the moisture condition at the beginning of rainfall, but the other three parameters can be assumed to be constant for the specific basin. These parameters are determined by conjugate direction method proposed by Powell [4], which is the method to find values of n parameters x_1, x_2, ..., x_n, so that the values of a function of these parameters, J (x_1, x_2, ..., x_n), is a minimum. The four parameters are standardized by their initial values as

$$x_1=\frac{N_s}{N_{s0}},\qquad x_2=\frac{a}{a_0},\qquad x_3=\frac{K_L}{K_{L0}},\qquad x_4=\frac{K_I}{K_{I0}} \qquad (1)$$

where suffix 0 means the initial values. When these four variables are given, runoff discharge L and sediment yield E can be calculated by the runoff-erosion model. The function J to be minimized is defined as

$$J=\left|\frac{L_o-L_c}{L_o}\right|+\left|\frac{E_o-E_c}{E_o}\right| \qquad (2)$$

where suffix o means observed data and suffix c means calculated values. L_c and E_c are functions of x_1, x_2, x_3 and x_4, and therefore so is J. These four parameters are to be optimized by the Standardized Powell method (SP method) [6] so that the evaluation function J becomes a minimum. All the parameters should be positive, and if some of the parameters become negative, a penalty function V is added to J so that the evaluation function becomes excessively large. The following function is introduced here :

$$V=\sum_{i=1}^{4} V_i \qquad (3)$$

where $V_i = 0$ when $x_i \geq 0$, and $V_i = (x_i - e_i)^2$ when $x_i < 0$ and $e_i = 1$. Fig. 1 is the flow chart of calculation by the SP method combined with the runoff-erosion model (WESP).

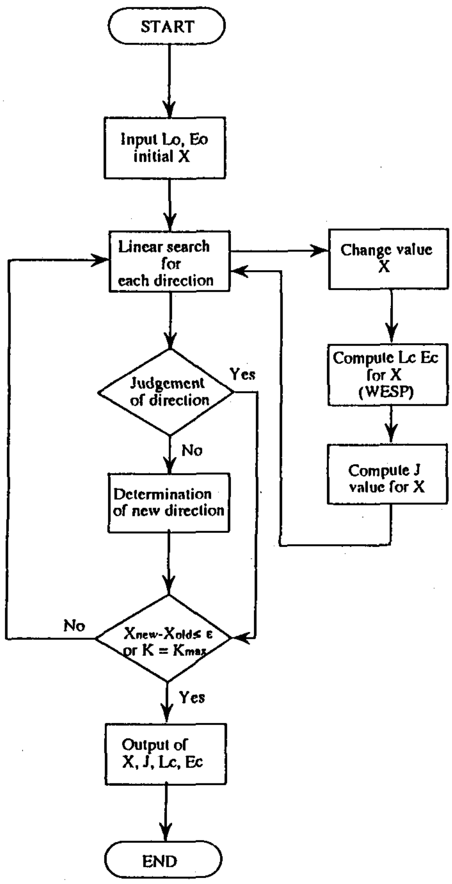

Fig. 1 Flow chart of calculation by SP method.

3. Experimental Basin and Observed Data

An experimental basin as shown in Fig. 2 was installed for the measurement of runoff discharge and sediment yield from the basin due to rainfall in the semi-arid region in the north-east of Brazil. The feature of geometry, soil and vegetation of the basin and the way of measuring runoff and yield sediment are summarized in the previous paper [5]. Observed data at the outlet (point A in Fig. 2) are shown in Table 1, where E_o is the observed sediment yield in kg, L_o is the observed runoff depth in mm, I_{max} is the maximum rainfall rate in mm/hr, and Dur. is the duration of the rainfall event in min. Data about E_o, L_o and rainfall for 21 rainfall events in 1987 and 1988 are obtained, in which sediment yields more than 100 kg are observed 12 times.

Table 1 Observed data.

Date (m/d/y)	E_o (kg)	L_o (mm)	I_{max} (mm/hr)	Dur. (min)
02/07/87	4.09	0.1910	96.0	50
02/12/87	19.19	0.0630	25.2	145
03/02/87	0.97	0.0560	33.0	87
05/02/87	1214.61	2.7410	31.7	122
06/01/87	0.94	0.0030	84.0	19
06/29/87	168.18	1.3710	27.0	79
07/04/88	5.27	0.0070	20.4	128
01/20/88	2061.86	5.7120	78.0	64
02/23/88	568.49	1.5770	22.0	159
03/12/88	4.38	0.1210	18.0	78
03/14/88	1875.53	5.6690	42.0	112
03/19/88	580.07	1.6890	37.5	76
03/24/88	4019.04	13.5210	120.0	58
04/05/88	3615.40	10.6000	108.0	114
04/08/88	1286.65	7.2310	108.0	71
04/19/88	3504.55	9.6330	90.0	92
04/30/88	887.47	5.3610	80.0	36
05/06/88	898.47	7.8190	90.0	63
07/13/88	40.73	0.4460	12.9	161
07/16/88	83.56	0.2190	12.0	133
07/25/88	0.49	0.0250	24.0	46

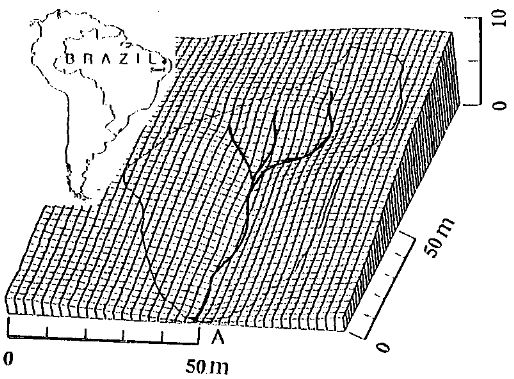

Fig. 2 Aeroview of test basin.

4. Division of River Basin into Elements

The test basin in Fig. 2 is divided into elements in three ways : 4, 10 and 23 elements. The 4 elements division of 3 overland flow elements and only one channel element is intended for a relatively large scale elements, and the 23 elements division of 16 overland flow elements and 7 channel flow elements is for a relatively small scale elements. The elements obtained by the 4 elements division based on soil, slope and surface cover characteristics are shown in Fig. 3 and the corresponding dimensions of the modeled elements are given in Table 2. The modeled elements and the corresponding dimensions for the 23 elements division are shown in Fig. 4 and Table 3, respectively. Median size elements are obtained by the 10 elements division and are shown in the previous paper [5] with

Table 2 Dimensions of modeled elements (4 elements).

Element	Area (m^2)	Length (m)	Width (m)	Slope	Lateral Slope
1	2169.00	41.20	52.64	0.085	–
2	1573.00	30.00	52.43	0.090	–
3	1458.00	35.00	41.66	0.105	–
4	–	62.50	–	0.050	0.25 :1

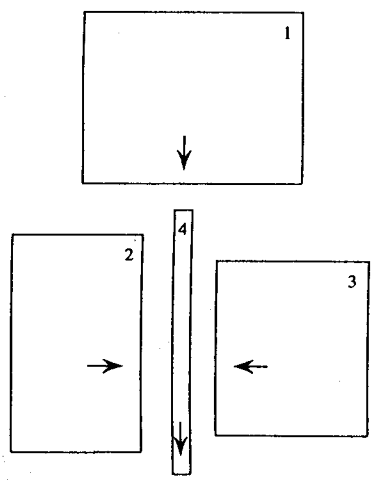

Fig. 3 Modeling of test basin (4 elements).

their dimensions. The shape of the cross section of channel elements is assumed to be triangular and lateral slope is 1/4, except for elements 17, 20 and 23 of the 23 elements division which have a trapezoid shape with lateral slope of 1/1 and flat bottom 0.10m, 0.10m and 0.20m wide, respectively. The minimum area in the 23 elements division and the maximum area in the 4 elements division of overland flow elements are 150 m^2 and 2169 m^2, respectively.

Table 3 Dimensions of modeled elements (23 elements).

Element	Area (m^2)	Length (m)	Width (m)	Slope	Lateral Slope
1	262.00	10.00	26.20	0.080	–
2	225.00	7.50	30.00	0.080	–
3	662.20	22.00	30.10	0.057	–
4	–	46.00	–	0.078	0.25:0.25
5	162.50	6.50	25.00	0.030	–
6	325.00	13.00	25.00	0.028	–
7	–	35.00	–	0.085	0.25:0.25
8	182.00	26.00	7.00	0.076	–
9	–	5.00	–	0.080	0.25:0.25
10	302.50	11.00	27.50	0.090	–
11	180.00	8.00	22.50	0.063	–
12	–	27.50	–	0.073	0.25:0.25
13	463.75	26.50	17.50	0.091	–
14	235.50	15.00	15.70	0.133	–
15	206.72	13.60	15.20	0.059	–
16	219.80	14.00	15.70	0.143	–
17	–	15.70	–	0.060	0.50:0.50
18	508.80	24.00	21.20	0.071	–
19	612.50	25.00	24.50	0.080	–
20	–	24.50	–	0.049	0.50:0.50
21	223.86	12.30	18.20	0.090	–
22	378.00	16.80	22.50	0.085	–
23	–	20.00	–	0.040	0.50:0.50

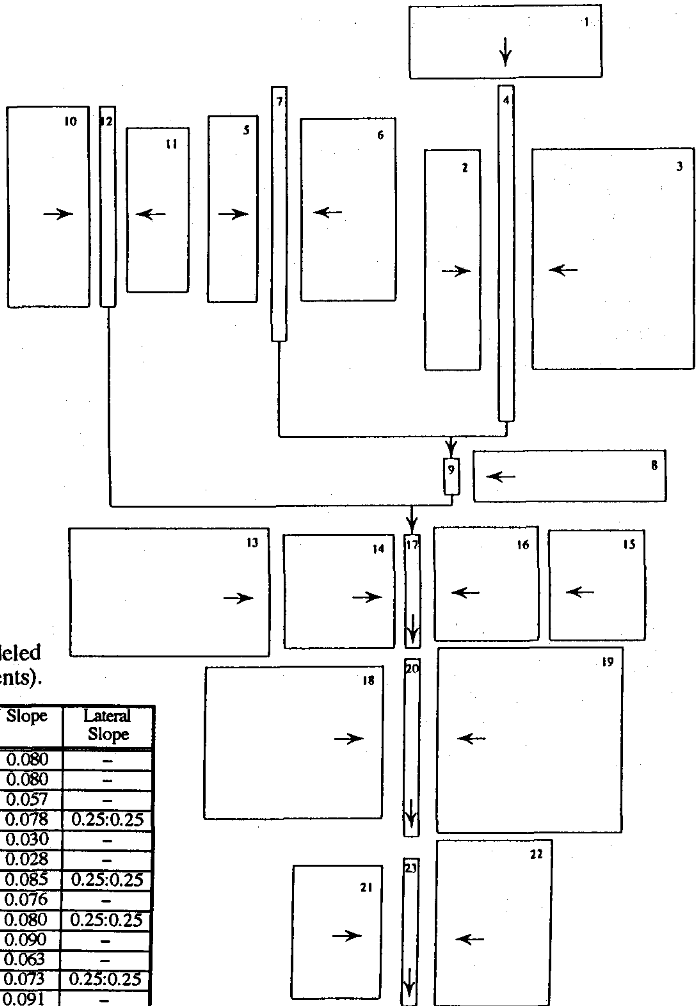

Fig. 4 Modeling of test basin (23 elements).

5. Results of Simulation

Table 4 shows the parameters a, K_L and K_I, optimized for 12 rainfall events with sediment yields E_o more than 100 kg, which are assumed to be more accurate than those less than 100 kg. Three parameters a, K_L and K_I should be constant for all rainfall events because they are characterized by sand and soil in the test basin. The orders of these optimized parameters for all the rainfall events seem to be equal for the 10 elements division, but for the 4 and 23 elements divisions variations of these values are relatively large. The average values of the parameters over the events can become the values for the specific test field. That is for the 10 elements division: $a = 0.015$ kg·$m^2/N^{1.5}$·s, $K_L = 2.2$ kg·m/$N^{1.5}$·s, $K_I = 4.0\times10^8$ kg·s/m^4. The

Table 4 Optimized values of parameters with data of $E_o > 100$ kg.

Rainfall Number	a (kg-m²/N$^{1.5}$-s)			K_L (kg-m/N$^{1.5}$-s)			K_I x 10^8 (kg-s/m⁴)		
	Total Number of Elements			Total Number of Elements			Total Number of Elements		
	4	10	23	4	10	23	4	10	23
4	0.0226	0.0215	0.0485	2.062	2.412	0.660	11.7	4.8	2.1
6	0.0163	0.0138	0.0064	2.034	1.937	2.252	0.3	1.5	2.7
8	0.0084	0.0128	0.0281	2.094	2.098	1.477	4.5	3.4	5.4
9	0.0240	0.0189	0.0469	2.499	2.760	0.336	10.5	1.5	2.5
11	0.0111	0.0139	0.0244	2.001	2.119	2.584	2.3	2.0	1.3
12	0.0159	0.0147	0.0327	2.159	2.363	0.338	6.6	1.5	0.6
13	0.0042	0.0144	0.0353	1.479	2.176	1.894	3.7	5.1	4.5
14	0.0041	0.0142	0.0320	1.924	2.145	1.540	3.1	3.6	4.6
15	0.0096	0.0139	0.0302	2.046	2.136	1.770	4.7	3.1	5.2
16	0.0092	0.0136	0.0301	2.134	2.194	1.980	3.4	6.7	6.0
17	0.0132	0.0139	0.0294	2.173	2.034	1.062	7.2	6.9	0.1
18	0.0158	0.0145	0.0322	2.140	2.232	0.292	3.4	8.1	0.7
Average	0.0129	0.0150	0.0313	2.062	2.217	1.349	5.1	4.0	3.0

initial moisture-tension parameter N_s changes largely with each rainfall event, because N_s is directly related to the moisture in the soil, therefore to the antecedent rainfall conditions. As the difference of the optimized values of N_s among three different ways of basin division was small, the relationship between optimized values of N_s and antecedent days without rainfall D is shown in Fig. 5 only for the 10 elements division, where data with blank circle in the figure are values optimized for $E_o < 100$ kg, using the above average value of a, K_L and K_I. The fitting curve in Fig. 5 can be used to estimate N_s, but it is only convenient because the values of N_s depend not only on the antecedent days without rainfall but also the antecedent rainfall intensity and other conditions.

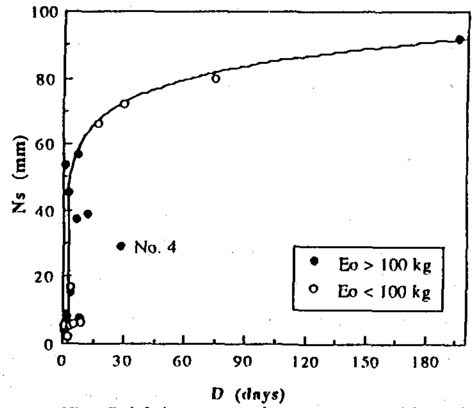

Fig. 5 Moisture-tension parameter N_s and antecedent days without rainfall D.

Fig. 6 and Fig. 7 show the comparison between observed and simulated runoff depth L and sediment yield E, respectively, for all the 21 rainfall events. Simulation is done with the average values of a, K_L, K_I

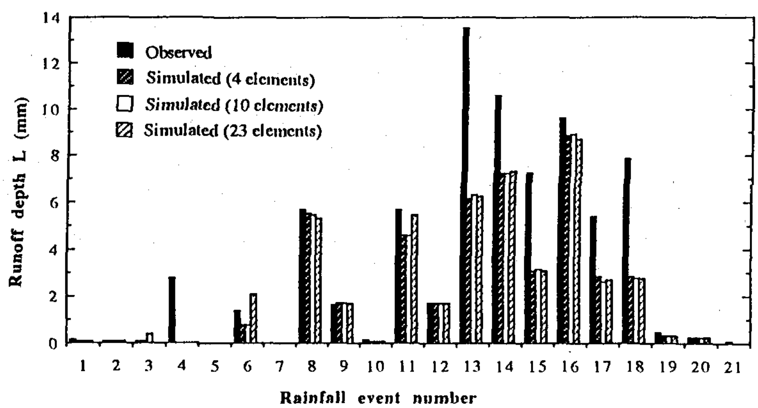

Fig. 6 Observed and simulated total runoff depths.

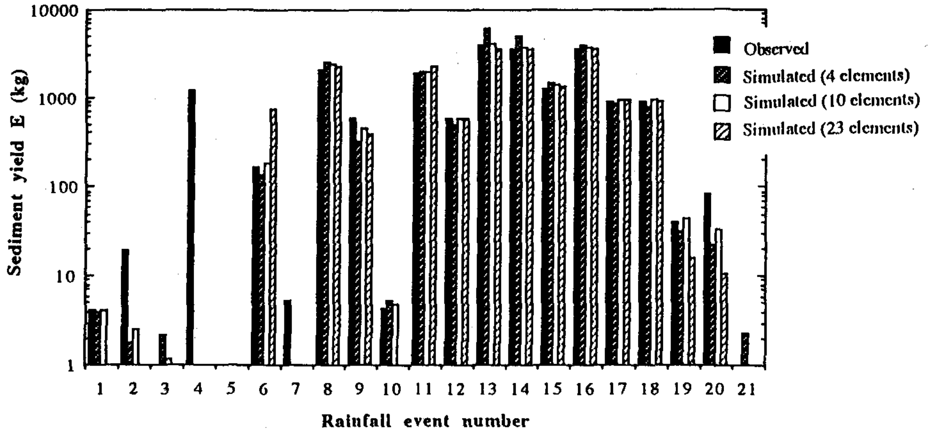

Fig. 7 Observed and simulated total sediment yields.

for each way of elements divisions and the above mentioned fitting curve for N_s. The simulated values for runoff depth L seem to be smaller than the observed data in several events, but those for sediment yield E follow the observed values for almost every rainfall event from weak to heavy rainfall except for event number 4. There are only small differences of simulated data among the three ways of basin division, although the 10 elements division seems to give the best results.

6. Conclusions

Parameters except for the moisture-tension parameter N_s in runoff-erosion model optimized by the Standardized Powell method change with the scale of the elements for the data of runoff and sediment yield observed in a test field with the area of 5200 m^2 in a typically semi-arid region of the north-east of Brazil. In this particular case, large total number of modeled elements which leads to precise division of the basin into small elements does not always give better simulation results. Although the medium scale of elements gave the best results among three ways of division of the basin, even the simplest division with 4 elements seems to give acceptable simulation results.

References

1) Bennett, J.P.: Concept of mathematical modeling of sediment yield, Water Resources Research, Vol. 10, No. 3, pp. 485-492, 1974.
2) Vanoni, V.A., Chairman: Task committee on preparation of manual on sedimentation, Sediment engineering, Jour. of Hydraulic Division (HY6), ASCE, pp. 1283-1331, 1970.
3) Lopes, V.L.: A numerical model of watershed erosion and sediment yield, Ph. D. Dissertation, University of Arizona, 1987.
4) Powell, M.J.D.: An efficient method for finding the minimum of several variables without calculating derivatives, Computer Journal, Vol. 7, pp. 155-162, 1964.
5) Srinivasan, V.S., C. Santos, K. Suzuki and M. Watanabe: Sediment yield observed in a small experimental basin and its simulation by runoff-erosion modeling, Proceedings of Hydraulic Engineering, JSCE, Vol. 37, pp. 717-722, 1993.
6) Nagai, A. and M. Kadoya: Optimization techniques for parameter identification of runoff models, DPRI Annuals, No. 23 B-2, Kyoto Univ., pp. 239-247, 1980 (in Japanese).