

Multiobjective Optimization of a Multireservoir System with Stochastic Inflows

By Yicheng WANG¹, Nobuyuki TAMAI², and Yoshihisa KAWAHARA³

This paper presents an optimization method for finding noninferior solutions of multiobjective multireservoir problem with stochastic inflows. Constraint technique, decomposition iteration, and simulation analysis are conjunctively used to deal, respectively, with multiobjective optimization (or vector optimization), large-scale multireservoir system, and stochasticity of inflows, which represent three difficult points in water resource system analysis. The effectiveness of the method is justified by applying it to the modeled multireservoir system in Tone river basin.

Keywords: multiobjective, multireservoir, stochastic inflows, decomposition

1. INTRODUCTION

Water resource systems tend to include multiple reservoirs which are operated to serve several objectives such as hydropower, water supply, and flood control. Moreover, the stochasticity of inflows into reservoirs is one of basic characteristics of water resources systems. It is multireservoir, multiobjective, and stochastic inflows that represent three difficult points in water resource systems analysis. In the recent decades, researchers have put their emphasis on one or two of these three difficult points, but no attempts have been made to deal simultaneously with all of them in a proper and efficient way.

Literature review indicates that, as far as stochastic inflows are taken into account, the choice of optimization techniques are restricted to dynamic programming. An ordinary dynamic programming approach is usually not feasible in computation when the number of state variables is more than four and/or the number of objectives is more than two, because computational time and space increase exponentially with the numbers of state variables and objectives. This phenomenon is thought of by Bellman and Dreyfus [1962] as the 'curse of dimensionality'. Some methods have been presented to alleviate the curse of dimensionality (for example, the discrete differential dynamic programming of Heidari et al. [1971], the successive approximation approach of Giles and Wunderlick [1981], and the principle of progressive optimality of Turgeon [1981a]), but they are effective only for cases where state variables are deterministic. Several attempts to solve large-scale stochastic problems are made by Roefs and Bodin [1970], Takeuchi and Moreau [1974], Gal [1979], and Turgeon [1980], but they take into account only a single objective.

The problem becomes much more complicated when one deals with multiobjective optimization, considering multiple reservoirs and the stochasticity of inflows. Changchit and Terrell [1989] presents chance-constrained goal programming method to solve multiobjective problem of multireservoir system under stochastic inflows. Its application to real-world problems is limited, however, because of the assumption of a linear release rule and the difficulties to determine the probability level for constraints. Mohan and Raipure [1992] investigates multiobjective problem of multireservoir system within a context of deterministic inflows. Wang and Tamai [1992a] conduct a multiobjective analysis with respect to a single reservoir, considering stochastic inflows.

This paper presents an optimization method for finding noninferior solutions of multiobjective multireservoir problem with stochastic inflows. Constraint technique, decomposition iteration, and simulation analysis are conjunctively used to deal with multiobjective optimization (or vector optimization), large-scale multireservoir system, and stochasticity of inflows, respectively. Detailed structures of models are explained in section 2. It should be pointed out that these models are developed primarily for the modeled multireservoir

¹. Student Member, Graduate Student, Department of Civil Engineering, University of Tokyo.

². Member, Professor, Department of Civil Engineering, University of Tokyo.

³. Member, Associate Professor, Department of Civil Engineering, University of Tokyo.

system in Tone river basin, which consists of three reservoirs in parallel. If reservoirs in series are considered, some simplified methods such as that presented by Turgeon [1980] may be used to aggregate them into a reservoir complex. The application of the method to the modeled multireservoir system in Tone river basin and results obtained are shown in section 3. Finally, conclusions are given in section 4.

2. MATHEMATICAL MODELS

Consider a system consisting of multiple reservoirs in parallel with stochastic inflows and multiple objectives (three objectives in this paper, that is, hydropower, water supply, and flood control).

A mathematical model for identifying the optimal operating policy of the system is written as

$$\text{Max} \left\{ E \left[\sum_{i=1}^m HP_{it}(S_{it}, R_{it}) \right], \sum_{i=1}^m WS_{it}(S_{it}, R_{it}), \sum_{i=1}^m FC_{it}(S_{it}, R_{it}) \right\} \quad (1)$$

s. t.

$$R_{it} = S_{it} + Q_{it} - S_{i,t+1} \quad (2)$$

$$S_i^{\min} \leq S_{it} \leq S_i^{\max} \quad (3)$$

$$R_i^{\min} \leq R_{it} \leq R_i^{\max} \quad (4)$$

$$HP_i^{\min} \leq HP_{it} \leq HP_i^{\max} \quad (5)$$

$$(i = 1, 2, \dots, m; t = 1, 2, \dots, T)$$

where R_{it} = the release from reservoir i during period t ; S_{it} = the initial storage volume of reservoir i at the beginning of period t ; Q_{it} = the inflow into reservoir i during period t ; HP_{it} = the hydropower generated by the hydropower plant corresponding to reservoir i ; WS_{it} = the firm water supply of reservoir i during period t ; FC_{it} = the reliability-based flood control capacity; E is expectation with respect to stochastic inflows; Superscripts $\min$ and $\max$ mean, respectively, lower bound and upper bound of physical parameters (for example, $S_i^{\max}$ refers to the storage capacity of reservoir i); m is the total number of reservoirs under consideration; T is the total number of time periods within a circle (for example, $T = 12$ if one month is taken as one period).

It is noted that expectation is not taken on items 2 and 3 of objective function (1), since the firm water supply WS_{it} and the reliability-based flood control capacity FC_{it} are defined within the context of reliability. For convenience of presentation, reliability-based flood control capacity is simply called flood control capacity in the remaining part of this paper.

The model described above is characterized by three difficult points, that is, vector optimization, large-scale system, and random variables. In the following, some techniques are presented for dealing with these difficulties.

Vector optimization is transformed into scalar one, using constraint technique. The scalar optimization model corresponding to expressions (1) to (5), with hydropower as objective and firm water supply and flood control capacity as constraints, is written as

$$\text{Max} \left\{ E \left[\sum_{i=1}^m HP_{it}(S_{it}, R_{it}) \right] \right\} \quad (6)$$

s. t.

$$R_{it} = S_{it} + Q_{it} - S_{i,t+1} \quad (7)$$

$$S_i^{\min} \leq S_{it} \leq S_i^{\max} - FC_{it} \quad (8)$$

$$\max\{WS_{it}, R_i^{\min}\} \leq R_{it} \leq R_i^{\max} \quad (9)$$

$$HP_i^{\min} \leq HP_{it} \leq HP_i^{\max} \quad (10)$$

$$\sum_{i=1}^m WS_{it} \geq WS_t \quad (11)$$

$$\sum_{i=1}^m FC_{it} \geq FC_t \quad (12)$$

$$(i = 1, 2, \dots, m; t = 1, 2, \dots, T)$$

where objective function (6) is to maximize the total expected hydropower of reservoirs; Equations (8) and (9) are modified forms of equations (3) and (4); $\max\{WS_{it}, R_i^{\min}\}$ is equal to the larger one of two values;

Equations (11) and (12) are new, representing objective constraints; WS_t and FC_t are, respectively, the total firm water supply and the total flood control capacity of reservoirs.

In theory, the model comprising (6) to (12) can be solved by means of some optimization techniques such as Stochastic Dynamic Programming (SDP). But in practice, it is impossible to solve it on computer because of the so-called curse of dimensionality.

In accordance with the curse of dimensionality, decomposition iteration technique is devised to break down the large-scale system consisting of m reservoirs in parallel into m subsystems containing only a single reservoir, which can be solved by some existing optimization techniques. In particular, decomposition iteration technique is to optimize the operating policy of one reservoir (or subsystem) while the operating policies of other reservoirs are kept unchanged. One iteration is finished after each reservoir is optimized once. Iterations continue until a constant operating policy for each reservoir is reached, i.e., the operating policy for each reservoir is unchanged as iterations continue. Obviously, an initial operating policy for each reservoir must be given in advance of iteration.

Assume that reservoir j ($j=1,2,\dots,m$) is optimized. An optimization model for reservoir j is written as

$$\text{Max}\{E[HP_j(S_{jt}, R_{jt})]\} \quad (13)$$

$$\text{s.t.} \quad R_{jt} = S_{jt} + Q_{jt} - S_{j,t+1} \quad (14)$$

$$S_j^{\min} \leq S_{jt} \leq S_j^{\max} - FC_{jt} \quad (15)$$

$$\max\{WS_{jt}, R_j^{\min}\} \leq R_{jt} \leq R_j^{\max} \quad (16)$$

$$HP_j^{\min} \leq HP_{jt} \leq HP_j^{\max} \quad (17)$$

$$WS_{jt} \geq WS_t - \sum_{i=1, i \neq j}^m WS_{it} \quad (18)$$

$$FC_{jt} \geq FC_t - \sum_{i=1, i \neq j}^m FC_{it} \quad (19)$$

$$(j=1,2,\dots,m; t=1,2,\dots,T)$$

The model comprising expressions (13) to (19) is different from the model comprising expressions (6) to (12) in two aspects. First, the objective function is reduced from m reservoirs in expression (6) to one reservoir in expression (13). Next, expressions (11) to (12) are modified so that WS_{it} and FC_{it} ($i=1,2,\dots,j-1,j+1,\dots,m$) are transferred to the right-hand side of inequalities, as shown in expressions (18) and (19), since WS_{it} and FC_{it} are known after the operating policy for reservoir i are found. It is expressions (18) and (19) that demonstrate that reservoirs are associated with rather than independent of each other.

It should be noted that the firm water supply WS_{jt} and the flood control capacity FC_{jt} are unknown before solving reservoir j , and thus values for WS_{jt} and FC_{jt} are assumed in advance as $WS_{jt}^{'}$ and $FC_{jt}^{'}$. After finishing the solution of reservoir j , firm water supply $WS_{jt}^{'}$ and flood control capacity $FC_{jt}^{'}$ may be derived from the operating policy of reservoir j . If $WS_{jt}^{'} \neq WS_{jt}^{'}$ or $FC_{jt}^{'} \neq FC_{jt}^{'}$, other values for WS_{jt} and FC_{jt} should be assumed again until the condition of $WS_{jt}^{'} = WS_{jt}^{'}$ and $FC_{jt}^{'} = FC_{jt}^{'}$ are satisfied.

By discretizing continuous reservoir storage volume and inflow, the stochastic single-reservoir optimization model comprising expressions (13) to (19) can be solved by the following SDP.

$$F_t^n(u, v) = \max_u [HP_{uv,t} + \sum_v P_{vv}^t \cdot F_{t+1}^{n-1}(u', v')] \quad (20)$$

where u is the u -th discrete value of initial storage volume in period t ; v is the v -th discrete value of the inflow in period t ; u' and v' represent, respectively, the u' -th storage volume and the v' -th inflow in period $t+1$; n is the total number of remaining periods in period t ; $F_t^n(\cdot)$ is the total expected hydropower with n periods to go,

including the current period t ; $HP_{uvu't}$ is the hydropower during current period t ; $P_{vv'}^t$ is the inflow transition probability in period t .

The optimal operating policy of a reservoir is found by iteratively solving the recursive equation (20). Then, the probability of the release from a reservoir $PR_{uvu't}$ is determined by solving the following simultaneous set of equations.

$$PR_{u',v',t+1} = \sum_u \sum_v PR_{uvu't} P_{vv'}^t, \forall u', v', t; u' = u'(u, v, t) \quad (21)$$

$$\sum_u \sum_v PR_{uvu't} = 1 \quad (22)$$

It should be noted that, after the optimal operating policy is found, u' is the function of u , v , and t as shown in (21). Hence, the probability $PR_{uvu't}$ in (21) and (22) is equivalent to the probability $PR_{uvu't}$.

Since the firm water supply W_{Sjt} and the flood control capacity FC_{jt} need to be derived from the release probability $PR_{uvu't}$, the simultaneous set of equations (21) and (22) must be solved several times for each reservoir due to the trial-and-error characteristic of W_{Sjt} and FC_{jt} , and thus many times for one iteration. As pointed out by Wang et al. [1992b], it takes substantial computational time to solve such a large simultaneous set of equations (21) and (22).

In order to avoid solving equations (21) and (22), synthetic inflows are employed to simulate the future operation of reservoirs, using the known operating policies. Since the firm water supply and the flood control capacity are determined by the simulation rather than the solution of equations (21) and (22), it can be expected that computational time is considerably reduced.

3. APPLICATION

The method devised in last section is applied to the modeled multireservoir system in Tone river basin, which consists of three reservoirs in parallel, that is, reservoirs Yagisawa, Kusaki, and Shimokubo. The geographical distribution of three reservoirs is schematically shown in Figure 1. The physical parameters of three reservoirs are listed in Table 1.

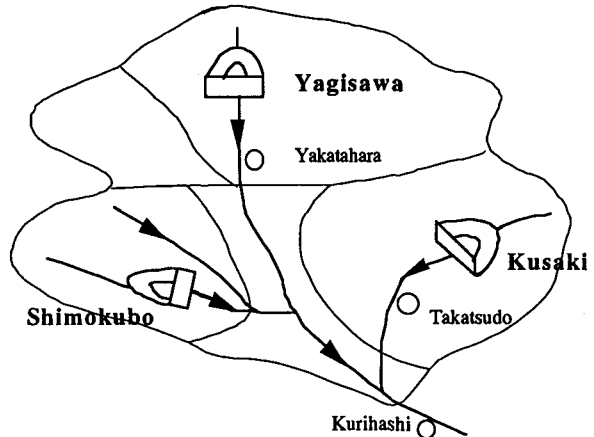


Figure 1. Geographic Distribution of Three Reservoirs

Table 1. Physical Parameters of Three Reservoirs

	Reservoir Storage Capacity $S_i^{\max}(10^6 \text{m}^3)$	Dead Storage Volume $S_i^{\min}(10^6 \text{m}^3)$	Maximum Release $R_i^{\max}(\text{m}^3/\text{s})$	Minimum Release $R_i^{\min}(\text{m}^3/\text{s})$	Hydropower Generation Capacity $HP_i^{\max}(\text{KW})$	Minimum Hydropower Generation $HP_i^{\min}(\text{KW})$
Yagisawa	204.3	28.5	300.0	0.0	240000.0	0.0
Kusaki	60.5	10.0	24.0 (1)*	0.0 (1)	20300.0 (1)	0.0 (1)
			24.29 (2)	0.0 (2)	36200.0 (2)	0.0 (2)
Shimokubo	130.0	10.0	12.0	0.0	15000.0	0.0

* Note: Two hydropower plants are considered in Kusaki. (1) and (2) indicate Higashi hydropower plant and Kodaira hydropower plant, respectively.

The historical monthly inflow data (including the extrapolated inflows) are used to determine the distribution which is the best fit for all months. The statistical parameters of inflows into these three reservoirs are listed in Table 2. With the Kolmogrov-Smirnov goodness-of-fit test, the lognormal is accepted as the proper distribution for all months [see, Wang et al., 1992b].

Table 2. Statistical Parameters of Annual Inflows into Three Reservoirs (m³/s)

	Yagisawa	Kusaki	Shimokubo
Mean	14.89	11.23	6.23
Standard Deviation	14.43	9.43	7.10
Cross Correlation Coefficients			
Yagisawa and Kusaki: 0.1015; Yagisawa and Shimokubo: 0.0794; Kusaki and Shimokubo: 0.2375			

Thomas-Fiering model (see, Loucks et al., 1981) is employed to generate 300 years' inflow for each reservoir without consideration of cross correlation, since cross correlation coefficients are small as shown in Table 2. For the purpose of illustration, the statistical parameters of historical and synthetic inflows into Yagisawa are listed in Table 3. It is clear from Table 3 that the statistical parameters of synthetic inflows are very close to those of historical inflows. This exhibits the appropriateness of Thomas-Fiering model for the problem. Synthetic inflows generated are input into simulation analysis model.

Table 3. Statistical Parameters of Historical and Synthetic Inflows into Yagisawa

	Mean (m ³ /s)		Standard Deviation(m ³ /s)		Lag One Correlation	
	Historical	Synthetic	Historical	Synthetic	Historical	Synthetic
Jan.	4.18	4.18	2.61	2.62	0.7847	0.7854
Feb.	3.71	3.71	2.05	2.06	0.1521	0.1514
Mar.	5.85	5.85	4.06	4.05	0.4759	0.4728
Apr.	32.17	32.17	9.08	9.08	-0.2935	-0.2945
May	45.44	45.44	12.32	12.32	0.5968	0.5962
Jun.	25.04	25.04	11.88	11.82	0.1237	0.1212
Jul.	19.00	19.00	9.01	8.99	0.3937	0.3938
Aug.	10.30	10.30	6.52	6.51	0.4681	0.4684
Sep.	7.85	7.85	4.17	4.15	0.4339	0.4344
Oct.	8.97	8.97	6.24	6.20	0.2836	0.2841
Nov.	9.63	9.63	4.74	4.71	0.3565	0.3565
Dec.	6.50	6.50	4.16	4.14	0.3722	0.3770

One month is taken as one period. The seasonal change of water supply is considered in such a way that the monthly firm water supply W_s in month t is derived from multiplication of total annual firm water supply, WS, by a constant coefficient(see, Wang and Tamai,1992a).

The reliabilities of the annual firm water supply, WS, and the total flood control capacity, FC, are given the values 90% and 95%, respectively.

The annual expected hydropower is achieved by iteratively solving the model comprising expressions (13) to (19). The solution results are listed in Table 4. For convenience of presentation, parameters WS and FC in Table 4 are only given 7 and 3 values, respectively. The values in column 4 are optimal annual expected hydropower corresponding to each combination of WS and FC. The results are also shown in Figure 2.

Table 4. Optimal Solution Results for Three Reservoirs

No.	Flood Control Capacity, FC(10 ⁶ m ³)	Annual Firm Water Supply, AFWS(m ³ /s)	Annual Expected Hydropower, AEHP(mwh)	No.	FC(10 ⁶ m ³)	AFWS(m ³ /s)	AEHP(mwh)
1	77.1	8.3	359457.41	12	98.65	20.4	345618.12
2	77.1	10.8	358496.09	13	98.65	21.4	342198.87
3	77.1	14.8	356481.84	14	98.65	22.2	336453.44
4	77.1	18.4	353455.47	15	115.65	8.3	349226.69
5	77.1	20.4	350456.09	16	115.65	10.8	348129.56
6	77.1	21.4	347754.94	17	115.65	14.8	345783.25
7	77.1	22.2	342051.06	18	115.65	18.4	341144.71
8	98.65	8.3	354686.03	19	115.65	20.4	337905.95
9	98.65	10.8	353724.72	20	115.65	21.4	334480.19
10	98.65	14.8	351378.41	21	115.65	22.2	328734.75
11	98.65	18.4	348856.87				

It is seen from Figure 2 that, when the flood control capacity is fixed, the annual expected hydropower decreases with the increase of annual firm water supply. On the other hand, with the increase of flood control capacity in direction AB, both annual expected hydropower and firm water supply decrease. These demonstrate that all the points marked in Figure 2 are noninferior solutions, and hence Table 4 is also called decision matrix, which is the basis for decision maker to make decision.

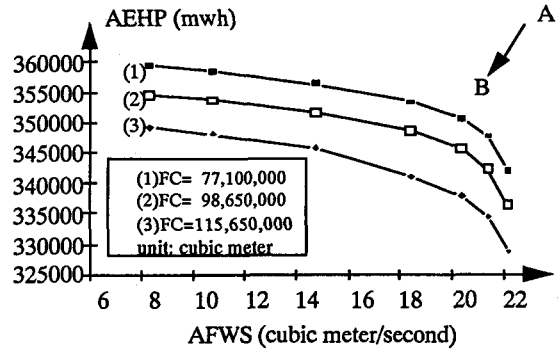


Figure 2. Noninferior Solutions for Three Reservoirs

4. CONCLUSIONS

In this paper, a new method is devised to deal with multiobjective problem of multireservoir system with stochastic inflows, and its effectiveness is justified by applying it to the modeled multireservoir system in Tone river basin. The following concluding points are made.

1. Constraint technique is effective in the sense that it transforms a multiobjective optimization problem into a uniojective one, which can be solved by scalar optimization approaches.

2. Decomposition iteration consists in breaking down a multireservoir system into multiple single-reservoir subsystems. Because state variables in decomposition iteration increase linearly rather than exponentially with the number of reservoirs, computational space is saved.

3. Simulation analysis is employed, instead of the solution of the large simultaneous set of equations, to solve for firm water supply and reliability-based flood control capacity, and thus computational time is substantially reduced.

4. A conjunctive use of constraint technique, decomposition iteration, and simulation analysis alleviates the curse of dimensionality considerably. Since this method is effective for solving the three-objective three-reservoir system, it is promising to apply it to the problem containing more objectives and reservoirs.

REFERENCES

- Bellman, R.E., and S.E. Dreyfus, *Applied Dynamic Programming*, Princeton University Press, Princeton, N.J., 1962.
- Changchit, C., and M.P. Terrell, CCGP model for multiobjective reservoir systems, *J. of Water Resour. Plann. and Manage.*, 115(5), 658-670, 1989.
- Gal, S., Optimal management of stochastic multireservoir water supply system, *Water Resour. Res.*, 15(4), 737-749, 1979.
- Giles, J.E., and W.O. Wunderlick, Weekly multipurpose planning model for TVA reservoir system, *J. of the Water Resour. Plann. and Manage. Div. ASCE*, 107(WR2), 495-511, 1981.
- Heidari, M., V.T. Chow, P.V. Kokotovic and D.P. Meredith, Discrete differential dynamic approach to water resources systems optimization, *Water Resour. Res.*, 7(2), 273-282, 1971.
- Loucks, D.P., J.R. Stedinger, and D.A. Haith, *Water Resource Systems Planning and Management*, Prentice-Hall, Englewood Cliffs, N.J., 1981.
- Mohan, S., and D.M. Raipure, Multiobjective analysis of multireservoir system, *J. of Water Resour. Plann. and Manage.*, 118(4), 356-370, 1992.
- Roefs, T.G., and L.D. Bodin, Multireservoir operation studies, *Water Resour. Res.*, 6(2), 410-420, 1970.
- Takeuchi, K., and D.H. Moreau, Optimal control of multi-unit interbasin water resource systems, *Water Resour. Res.*, 10(3), 407-414, 1970.
- Turgeon, A., Optimal operation of multireservoir power systems with stochastic inflows, *Water Resour. Res.*, 16(2), 275-283, 1980.
- Turgeon, A., Optimal short-term hydro scheduling from the principle of progressive optimality, *Water Resour. Res.*, 17(3), 481-486, 1981a.
- Wang, Y.C., and N. Tamai, Compromise programming for a stochastic reservoir system, *Proc. of the 47th Annu. Conf. of JSCE*, 1192-1193, 1992a.
- Wang, Y.C., N. Tamai, and Y. Kawahara, Comparison of stochastic dynamic- and linear- programming for reservoir operation, *Proc. of the 4th Symposium on Water Resources*, 351-356, 1992b.